Math 2415
Paper Homework #2

1. **[12.4, Cross Products]**

   Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} - 3\mathbf{j}$ and $\mathbf{c} = 3\mathbf{j} + \mathbf{k}$.

   (a) Find the length of $\mathbf{a}$.
   (b) Find a unit vector that is orthogonal to both $\mathbf{a}$ and $\mathbf{c}$.
   (c) Calculate the area of the parallelogram determined by the vectors $\mathbf{a}$ and $\mathbf{c}$.
   (d) Calculate the volume of the parallelipiped determined by the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

2. **[12.4, Cross Products]**

   (a) Explain why there must be at least one vector $\mathbf{v}$ so that $(1, 2, 4) \times \mathbf{v} = (2, -3, 1)$.
   (b) Find a vector $\mathbf{v}$ so that $(1, 2, 4) \times \mathbf{v} = (2, -3, 1)$.

3. **[12.4, Cross Products]** Suppose that $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$ and $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$. Use

   (a) the linearity property $\mathbf{u} \times (a\mathbf{v} + b\mathbf{w}) = a\mathbf{u} \times \mathbf{v} + b\mathbf{u} \times \mathbf{w}$,
   (b) the anti-symmmetry property $\mathbf{v} \times \mathbf{u} = -\mathbf{u} \times \mathbf{v}$, and
   (c) the formulae $\mathbf{i} \times \mathbf{j} = \mathbf{k}$,

   to derive the formula
   $$\mathbf{u} \times \mathbf{v} = (u_1 v_2 - u_2 v_1)\mathbf{k}.$$

4. **[12.5A, Lines]**

   (a) Find a vector parametrization for the line, $\mathcal{L}$, passing through the points $P = (1, 2, 3)$ and $Q = (9, -4, 7)$.
   (b) Which of the points are on the line $\mathcal{L}$? Which are on the line and are between P and Q? Why?
      i. $(17, 10, -11)$,
      ii. $(5, -1, 5)$,
      iii. $(17, -10, 11)$.
   (c) Determine whether the line, $\mathcal{L}$,
      i. intersects the yz-plane,
      ii. intersects with the x-axis.
   (d) Find a parametrization for a line whose intersection with the z-axis is one point.