Math 2415
Paper Homework #3

1. **[12.5B: Planes]** Let $A = (1, 2, 3)$, $B = (5, -1, 0)$ and $C = (4, 3, 7)$.

   (a) Find a parametrization, $(x, y, z) = \mathbf{r}(s, t)$, of the plane, $\mathcal{P}$, containing A, B, and C.

   (b) For each of the three points, A, B, and C, find values of the parameters (s, t) in the parameterization you found in (a).

   (c) Find a level set equation of the plane, $\mathcal{P}$, containing the points A, B, and C.

   (d) Check that A, B, and C each satisfy the equation you derived in (c).

   (e) Let $\mathcal{L}$ be the line passing through the point $(-1, 0, 2)$ that is parallel to the vector $(1, 0, 2)$. Find the point of intersection of this line with the plane, $\mathcal{P}$.

   (f) Let $\mathcal{Q}$ be the plane that contains the point $(3, 1, 2)$ and that is perpendicular to the line $\mathcal{L}$ in (e). Find a parametrization for the line of intersection of the planes $\mathcal{P}$ and $\mathcal{Q}$.

2. **[15.7A: Cylindrical Coordinates]**

   (a) Sketch the surface whose equation in cylindrical coordinates is given by $r = 2$.

   (b) Convert the equation $r = 2$ to spherical coordinates.

   (c) Convert the equation $x^2 + y^2 - 3z^2 = 1$ to cylindrical coordinates.

   (d) Sketch the solid where $3 \leq r \leq 4$, $\pi/4 \leq \theta \leq \pi/2$, $0 \leq z \leq 1$.

3. **[15.8A: Spherical Coordinates]** Consider the following points, curves, surfaces, and solids

   (i) The surface $\rho = 2$.

   (ii) The curve where $\rho = 2$ and $\theta = \pi/4$.

   (iii) The curve where $\rho = 2$ and $\phi = 3\pi/4$.

   (iv) The point $(\rho, \theta, \phi) = (2, \pi/4, 3\pi/4)$.

   (v) The solid where $\rho \leq 2$, $0 \leq \theta \leq \pi/4$ and $\pi/2 \leq \phi \leq 3\pi/4$.

   Now do the following problems:

   (a) Sketch (i)-(iv) *altogether in one plot, with labels*.

   (b) Convert the equation $\rho = 2$ to cylindrical coordinates.

   (c) Find the rectangular and cylindrical coordinates of the point in (iv).

   (d) Sketch (v). [Use a different plot from the one in (a).] **Hint:** This solid is enclosed by 5 surfaces. Sketch each surface and then visualize the solid region enclosed by them. The surfaces are $\rho = 2$, $\theta = 0$, $\theta = \pi/4$, $\phi = \pi/2$, and $\phi = 3\pi/4$. You can make a model of this solid by making 4 cuts in an apple. Try it! In fact, if you cut an apple and turn in photos of the solid you cut out which are taken from a couple of angles you can get full credit (instead of sketching the solid by hand).