Math 2415

Paper Homework #6

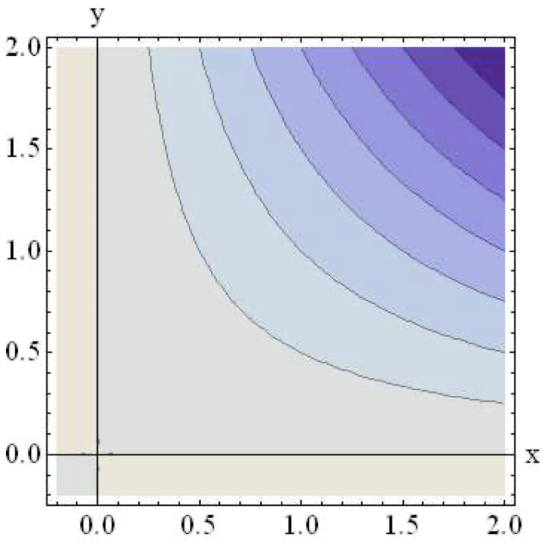
1. **14.3: First Partial Derivatives:** The plane $y = 3$ intersects the graph of $z = f(x, y) = xy^2 + 5x^3$ in a curve, C.

   (a) Calculate $\frac{\partial f}{\partial x}$ at $(x, y) = (2, 3)$.

   (b) Find a parametrization of the tangent line to the curve C at the point where $x = 2$.

2. **14.3: Mixed Partial Derivatives:** In this multiple choice problem provide a justification for your answer. **Hint:** Fix $x_0 = 1$ (for example). Compare the slope in the x-direction at (x_0, y_0) for two different values of y_0. Recall that wider spaced contours correspond to smaller slopes.

   In the contour plot below dark shades represent small values of the function and light shades represent large values of the function. What is the sign of the mixed partial derivative?



   (a) $f_{xy} > 0$

   (b) $f_{xy} < 0$

   (c) $f_{xy} \approx 0$

   (d) This cannot be determined from the figure.

3. **14.3: Second Partial Derivatives:** Verify that the function $u(x, y, z) = \log(x^2 + y^2)$ is a solution of the two dimensional Laplace equation $u_{xx} + u_{yy} = 0$ everywhere, except of course at the origin where f is not defined.

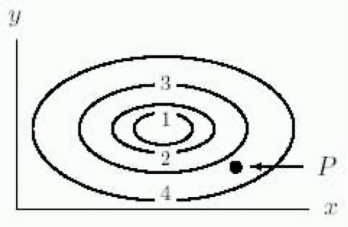
4. **14.4, Tangent Planes and Linear Approximation:** Let $f(x, y) = x^2y^2 - x$.

   (a) Find the equation for the tangent plane to the graph of f at $(2, 1, 2)$.

   (b) Use a linear approximation to find the approximate value of $f(2.1, 0.8)$.

5. In this multiple choice problem provide a justification for your answer.

The figure below shows level curves of the function $f(x, y)$. The tangent plane approximation to $f(x, y)$ at the point $P = (x_0, y_0)$ is $f(x, y) \approx c + m(x - x_0) + n(y - y_0)$. What are the signs of c, m, and n?



(a) $c > 0$, $m > 0$, $n > 0$

(b) $c < 0$, $m > 0$, $n < 0$

(c) $c > 0$, $m < 0$, $n > 0$

(d) $c < 0$, $m < 0$, $n < 0$

(e) $c > 0$, $m > 0$, $n < 0$