

Math 2415

Problem Section #1

Make sure you do some problems from each section.

12.1: 3D Coordinate Systems

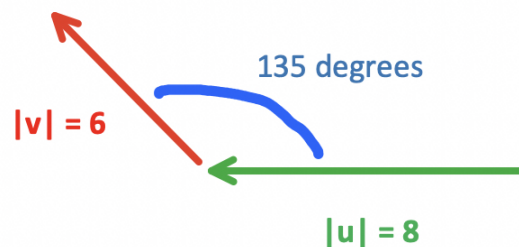
1. Draw a rectangular box with the origin and the point $(1, 2, 3)$ as opposite vertices and faces parallel to the coordinate planes. Label each vertex with its coordinates. Find the length of the diagonal of the box.
2. (a) What does the equation $x = 2$ represent in \mathbb{R}^2 ? Sketch!
(b) What does the equation $x = 2$ represent in \mathbb{R}^3 ? Sketch!
(c) What does the equation $z = 1$ represent in \mathbb{R}^3 ? Sketch!
(d) Describe the set of all points, (x, y, z) , in \mathbb{R}^3 for which $x = 2$ and $z = 1$. Sketch!
3. For what values of b and c do the points $(1, 2, 3)$, $(4, 5, 1)$, and $(10, b, c)$ all lie on the same line?
4. (a) Find the equation of the sphere with center $(1, 3, 5)$ and radius 4.
(b) What is the intersection of this sphere with the xz -plane? Argue algebraically and geometrically.
(c) What would the radius of the sphere have to be for the the intersection of the sphere and the xz -plane to be a single point. What are the coordinates of this point?

12.2: Vectors

1. Do not use coordinate representations of vectors to solve this problem. Just draw pictures.
 - (a) Draw two vectors that are not parallel and label them \mathbf{a} and \mathbf{b} .
 - (b) Sketch the vector $\mathbf{a} + \mathbf{b}$
 - (c) Sketch the vector $\mathbf{a} - \frac{1}{2}\mathbf{b}$
 - (d) Sketch the vector $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$
2. Sketch a parallelogram and label the vertices A , B , C , and D going around counter-clockwise from the bottom left vertex. Let E be the point obtained by intersecting the two diagonals of the parallelogram. Make sure the side lengths of your parallelogram are not all equal, ie you did not draw a rhombus.
 - (a) Name all pairs of equal vectors in your sketch.
 - (b) Write each combination of vectors as a single vector: $\vec{AB} + \vec{BC}$, $\vec{AE} - \vec{EB}$, $2\vec{AB} + \vec{BD}$.
3. Let $\mathbf{a} = 3\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. Find
 - (a) $\mathbf{a} + 2\mathbf{b}$
 - (b) $|\mathbf{b}|$
 - (c) $|\mathbf{a} - \mathbf{b}|$.
4. Suppose that $\mathbf{v} \in \mathbb{R}^2$ lies in the 2nd quadrant, makes an angle of 120° with the positive x -axis, and has length $|\mathbf{v}| = 2$. Find the coordinates of \mathbf{v} .

12.3: The Dot Product

- Find $\mathbf{a} \cdot \mathbf{b}$ if
 - $\mathbf{a} = (1, 2)$ and $\mathbf{b} = (-2, 3)$,
 - $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$,
 - $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$, and the angle between \mathbf{a} and \mathbf{b} is 120° .
- Let $\mathbf{u} = (3, -2, 1)$ and $\mathbf{v} = (2, 4, -1)$.
 - Find the scalar and vector projections of \mathbf{u} onto \mathbf{v} .
 - Find the angle between \mathbf{u} and \mathbf{v} to the nearest degree (use a calculator!)
 - Find three nonzero vectors that are orthogonal to \mathbf{u} .
- Answer this problem using the picture below. You are *not* allowed to calculate the components of the vectors \mathbf{u} and \mathbf{v} . *Warning:* Look carefully at the directions of the arrows on the vectors. Relate to theory from lectures!
 - Find $\mathbf{u} \cdot \mathbf{v}$
 - Use triangle geometry to find the scalar projection of \mathbf{v} onto \mathbf{u} .
 - Use triangle geometry to find the vector projection of \mathbf{u} onto \mathbf{v} . (Write your answer in terms of \mathbf{v} .)



- Let C be the point on the line segment AB that is twice as far from A and it is from B , and let O denote the origin. Let $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, and $\mathbf{c} = \overrightarrow{OC}$.
 - Make a sketch showing the relationships between all these points and vectors. Your sketch will help you solve the other parts of the problem.
 - Express \overrightarrow{AB} in terms of \mathbf{a} and \mathbf{b} .
 - Hence express \overrightarrow{AC} in terms of \mathbf{a} and \mathbf{b} .
 - Hence find a formula for \mathbf{c} in terms of \mathbf{a} and \mathbf{b} .
 - Now also suppose that $\mathbf{a} \perp \mathbf{b}$ and $|\mathbf{b}| = 1$. Find the scalar projection of \mathbf{c} onto \mathbf{b} . *Hint:* Your answer should be a number.
 - Calculate \mathbf{c} in the special case that $\mathbf{a} = \mathbf{i}$ and $\mathbf{b} = \mathbf{j}$. Is your answer consistent with your answer to (e)?

Extra Challenge Questions:

1. 12.3.56
2. 12.3.63
3. 12.3.47. In addition: The question asks you to find one vector \mathbf{b} with the property that $\text{comp}_a(\mathbf{b}) = 2$. However, there are lots of correct answers, \mathbf{b} . At this stage we don't know enough to easily find a formula for all solutions, but we can draw a picture of them. So: Draw a schematic diagram showing *all* possible vectors, \mathbf{b} for which $\text{comp}_a(\mathbf{b}) = 2$. Describe this set of vectors using an English sentence.