Math 2415

Problem Section #14

This week we will do problems from 16.3-16.5 as well as review for the Final Exam.

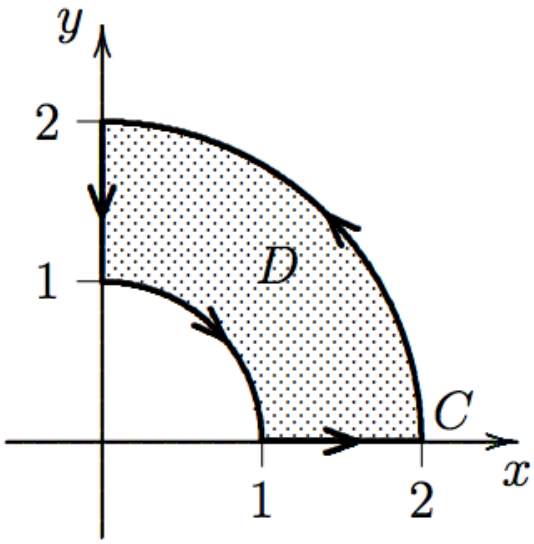
Based on past experience, about 50% of the points on the final exam will be on material from 15.3 onwards. In the previous problem session, we posted the same set of exam review problems.

16.3: FTC for line integrals, Conservative Vector Fields

1. Let $f(x,y) = xe^{x^2+y^2}$. Find $\int_C \nabla f \cdot d\mathbf{r}$, where C is any oriented curve from $(1,1)$ to $(2,2)$.
2. Let $\mathbf{F}_1(x,y) = (2y - x^2e^{-y})\mathbf{i} + 2xe^{-y}\mathbf{j}$ and $\mathbf{F}_2(x,y) = 2xe^{-y}\mathbf{i} + (2y - x^2e^{-y})\mathbf{j}$
   (a) One of these vector fields is conservative. Which one is it and why?
   (b) Find a potential function for the conservative vector field.
   (c) Evaluate $\int_C \mathbf{G} \cdot d\mathbf{r}$ where C is the line segment from $(1,0)$ to $(2,1)$ and $\mathbf{G}$ denotes the conservative vector field you identified in (a).

16.4: Green's Theorem

1. Use Green's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = (y - \cos y)\,\mathbf{i} + x \sin y\,\mathbf{j}$, and C is the circle $(x-3)^2 + (y+4)^2 = 9$., oriented counter clockwise.
2. Use Green's theorem to evaluate $\int_C xy^2\,dx - x^2y\,dy$ where C is given in the figure.



16.5, Curl and Divergence

Let $\mathbf{F}(x,y) = x^3\mathbf{i} + y^3\mathbf{j}$ be the velocity vector field of a fluid flowing in $\mathbb{R}^2$.

1. Calculate $\nabla \cdot \mathbf{F}$.

2. Calculate $\nabla \times \mathbf{F}$.
3. On average, is the fluid rotating clockwise, counter-clockwise, or not rotating at all about the point $(1, 2)$? Why?
4. On average, is the fluid flowing in, out, or neither in or out, of a small disc centered at $(1, 2)$? Why?

Final Exam Review

Here are a long list of problems you could work on, many of which are exam questions from past semesters.

Also see Dr. Makhijani's Final Exam Practice Problems, for which there are solutions past exams webpage.

1. Stewart, 15.6.21
2. Stewart, 15.7.21
3. Stewart, 15.7.25 (a)
4. Stewart, 15.8.23
5. Stewart, 15.Review.30
6. Spring 2014 Final Exam # 8
7. Fall 2009 Exam II # 4
8. Fall 2014 Final Exam # 6
9. Spring 2014 Final Exam # 6
10. Spring 2004 Final: 1
11. Spring 2004 Final: 2
12. Spring 2004 Final: 6
13. Spring 2004 Final: 7 (Part d is on 16.6)
14. Spring 2008 Final: 1
15. Spring 2008 Final: 3
16. Spring 2008 Final: 4
17. Spring 2008 Final: 6
18. Spring 2019 Final: 10 (Based on 16.5)
19. Fall 2009 Final: 4 (Based on 16,.6)
20. Fall 2009 Final: 5
21. Fall 2009 Final: 6
22. Fall 2009 Final: 9