Reviews

In 17th century Scotland and Switzerland, two inventors independently and almost simultaneously produced the concept of logarithm, constructed tables of logarithms for efficient arithmetic, and thus started the computing revolution. John Napier (1550–1617) of Edinburgh, Baron of Merchiston, studied mathematics and astronomy at St. Salvator’s College, St. Andrews. Jost Bürgi (1552–1632) of Lichtensteig, Toggenburg had only basic schooling in writing and arithmetic, never learned Latin, and thus could not read scientific literature. Nevertheless, he became an excellent mathematician, astronomer, and master craftsman of precise astronomical clocks and instruments. Subsequently, Henry Briggs (1561–1630) of London, England, derived an easier-to-use logarithm table used to this day.

For the 400-year celebration of the publication of the logarithm tables by Bürgi—developed already around 1600, but published in 1620—and the independent invention of the logarithm by Napier and Bürgi, Klaus Truemper has written a book that examines the work of these inventors. The author succeeds in bringing their thinking to life: How they decided on two very different ways to formulate the concept of logarithms and constructed quite different tables of logarithms that made efficient arithmetic possible, and how these results triggered the computing revolution that continues to this day.

The author first transports the reader to the start of the 17th century as both inventors begin their work. In particular, the reader looks over Bürgi’s shoulders, so to speak, as he comes up with the idea of a table of logarithms, decides various aspects, and then computes the table in just a few months—an astonishing achievement. In contrast, Napier’s and Briggs’s tables require years of computing effort.

The author narrates the developments in simple, non-technical language that reads more like a detective novel than a book about
mathematics. Indeed, the book provides a delightful and illuminating walk through that hugely important part of computing history.

—Fritz Staudacher, author of the Bürgi biography Jost Bürgi, Kepler und der Kaiser

The Daring Invention of Logarithm Tables takes a fresh look at the question “Who invented the concept of logarithm?”. So far, the answers have frequently asserted that John Napier is the sole inventor, and that Jost Bürgi is not an independent co-inventor. Klaus Truemper’s book explains the ideas of Jost Bürgi and John Napier, and the subsequent work of Henry Briggs, allowing the reader to trace their thinking. The text is easy to read and requires only elementary arithmetic as background.

The book invites the reader to know more about Jost Bürgi’s role in this process. Probably around 1600 Bürgi had the ingenious idea to tabulate the function $f(n) = 1.0001^n$ for $n = 0, 1, \ldots, 23027$ to the precision of 9 digits. He likely accomplished this in only a few months, and he did it with no systematic error. In Napier’s terminology—still used today—$n$ is the logarithm of $f(n)$ to base 1.0001.

For the simplification of multiplication and division, the tables of Napier and Bürgi are completely equivalent. After a detailed analysis that includes results since Archimedes, the author rightfully concludes: Napier and Bürgi are independent co-inventors of the logarithms.

—Jörg Waldvogel, Dept. of Mathematics, Eidgenössische Technische Hochschule Zürich, Switzerland
THE DARING INVENTION
OF LOGARITHM TABLES

How Jost Bürgi, John Napier, and Henry Briggs
simplified arithmetic and
started the computing revolution

KLAUS TRUEMPER
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Starting around 50,000 BCE, early humans used pebbles, scratches, or other marks to record quantities. These devices supported addition, subtraction, multiplication, and division.¹

Eventually, various symbols replaced these tools and simplified operations. Thus, mathematics was born.

During the last 5,000 years, mathematical concepts and models became more and more advanced, and computations became more complicated as well.

In particular, the 16th and 17th centuries produced sophisticated models for puzzling observations about the world such as the configuration of the heavenly bodies. Evaluation of these models required voluminous manual computations spanning months or even years of effort.

An ingenious new computing device then reduced that effort dramatically. Indeed, the tool was an order of magnitude more effective than anything invented before.

That device was the logarithm table, or rather several such tables in particular formats. Use of these tables compressed months of computing effort to weeks and sometimes just days.

1

Introduction
The tables triggered the invention of computing equipment where distances and angles represented the numbers of the logarithm tables. There were three such devices: the slide rule, the circular slide rule, and the slide cylinder. These tools were produced until the middle of the 20th century.

A downside of the logarithm tables was their arduous and error-prone construction. That aspect led to a sequence of groundbreaking inventions. First was a special purpose computing device in the 19th century that could compute such tables mechanically and error-free. It was called the difference engine. That design led to a general-purpose computer called the analytical engine and the first-ever computer program.

Unfortunately, both engine designs were so complicated that the difference engine was built for the first time in the late 20th and early 21st century, while the analytical engine never was—and likely never will be—constructed.

In the 1930s—about 100 years after the invention of the analytical engine—an ingenious new approach resulted in a general-purpose computer whose first version could be built using just sheet metal.

The ensuing electronic revolution created ever faster computing devices that have improved human life in almost miraculous ways. And all this started with the logarithm tables.

We have taken this warp-speed tour of mathematical and computing development to highlight the extraordinary impact of the invention of the logarithm table. Thus, it surely is worthwhile to study the who, when, where, and how of this invention.

This has been done in a number of articles and books, so you may wonder why we have written yet another book on the topic.

The existing material usually recounts in the language of modern mathematics how the logarithm table was invented.
Yes, the publications typically consider that certain concepts were not available at the time. But the interpretation of events then relies on modern mathematical concepts. That approach makes the invention of the logarithm table look like a much simpler and more natural step than it actually was.

Instead, we transport ourselves into the life of one of the inventors. We look over his shoulder, so to speak, as he thinks about and works on the problem of efficient computation.

In the process, we understand how difficult the work on logarithm tables really was: Tens of thousands of computing steps had to be performed with utter precision.

At the same time, we experience the magic of this invention as it comes together in the mind of one of its creators.

You surely have noticed that so far we haven’t mentioned a single name, let alone discussed the life of any person. That’s done on purpose: We learn about each player as we delve into his life.

Yes, "his" is correct here. With rare exception, women at that time were thought incapable of scientific thought. Which says something about the men of that time and nothing about the women.

There has been a major controversy involving logarithms: It concerns the question of who invented logarithms first and when. So far, a variety of conflicting answers have been given.

In the last part of this book we look at these answers and come up with an explanation how such a divergence of views is possible. We also offer our own view, as you are bound to expect.

The arguments rest on a particular interpretation of the brain’s reasoning process. That insight not only explains the divergence of opinions about the priority question, but also provides a reasonable justification for our answer.
One more remark about our interpretation. We are convinced that mathematics is created, and have written a book arguing the case. That conclusion is contrary to that of most mathematicians, who believe that mathematics is discovered.

If you are on that side of the fence, too, don’t let terms such as “invention” bother you. After all, the decision isn’t based on an objective reality but depends on the models of the world we have in our heads.

Indeed, during any discussion of creation versus discovery of mathematics, it is prudent to first talk about the different models that result in a particular conclusion instead of the conclusion itself. For example, one may want to examine:

- how a given model handles mathematical statements that are declared to be theorems, but that actually are flawed, with the error ranging from a spelling mistake to utter falsehood;
- how it classifies correct versus incorrect proofs, regardless of whether the statement to be proved is correct or not;
- how it compares with accepted models of other domains of human ingenuity;
- how it came to be;
- and lastly, how it influences teaching and research in mathematics.

This book does not require any mathematical background beyond everyday knowledge of numbers and the elementary operations of addition, subtraction, multiplication, and division. More complicated operations are rarely used. If so, they have been moved into the Notes section.