Reviews

In 17th century Scotland and Switzerland, two inventors independently and almost simultaneously produced the concept of logarithm, constructed tables of logarithms for efficient arithmetic, and thus started the computing revolution. John Napier (1550–1617) of Edinburgh, Baron of Merchiston, studied mathematics and astronomy at St. Salvator's College, St. Andrews. Jost Bürgi (1552–1632) of Lichtensteig, Toggenburg had only basic schooling in writing and arithmetic, never learned Latin, and thus could not read scientific literature. Nevertheless, he became an excellent mathematician, astronomer, and master craftsman of precise astronomical clocks and instruments. Subsequently, Henry Briggs (1561–1630) of London, England, derived an easier-to-use logarithm table used to this day.

For the 400-year celebration of the publication of the logarithm tables by Bürgi—developed already around 1600, but published in 1620—and the independent invention of the logarithm by Napier and Bürgi, Klaus Truemper has written a book that examines the work of these inventors. The author succeeds in bringing their thinking to life: How they decided on two very different ways to formulate the concept of logarithms and constructed quite different tables of logarithms that made efficient arithmetic possible, and how these results triggered the computing revolution that continues to this day.

The author first transports the reader to the start of the 17th century as both inventors begin their work. In particular, the reader looks over Bürgi's shoulders, so to speak, as he comes up with the idea of a table of logarithms, decides various aspects, and then computes the table in just a few months—an astonishing achievement. In contrast, Napier's and Briggs's tables require years of computing effort.

The author narrates the developments in simple, non-technical language that reads more like a detective novel than a book about

mathematics. Indeed, the book provides a delightful and illuminating walk through that hugely important part of computing history.

—Fritz Staudacher, author of the Bürgi biography *Jost Bürgi, Kepler und der Kaiser*

The Daring Invention of Logarithm Tables takes a fresh look at the question "Who invented the concept of logarithm?". So far, the answers have frequently asserted that John Napier is the sole inventor, and that Jost Bürgi is *not* an independent co-inventor.
Klaus Truemper's book explains the ideas of Jost Bürgi and John Napier, and the subsequent work of Henry Briggs, allowing the reader to trace their thinking. The text is easy to read and requires only elementary arithmetic as background.
The book invites the reader to know more about Jost Bürgi's role in this process. Probably around 1600 Bürgi had the ingenious idea to tabulate the function $f(n) = 1.0001^n$ for $n = 0, 1, \ldots, 23027$ to the precision of 9 digits. He likely accomplished this in only a few months, and he did it with no systematic error. In Napier's terminology—still used today—n is the logarithm of $f(n)$ to base 1.0001.
For the simplification of multiplication and division, the tables of Napier and Bürgi are completely equivalent. After a detailed analysis that includes results since Archimedes, the author rightfully concludes: Napier and Bürgi are independent co-inventors of the logarithms.

—Jörg Waldvogel, Dept. of Mathematics, Eidgenössische Technische Hochschule Zürich, Switzerland

Also by Klaus Truemper

Brain Science

Artificial Intelligence
Wittgenstein and Brain Science
Magic, Error, and Terror

History

The Construction of Mathematics

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Logic-based Intelligent Systems
Effective Logic Computation
Matroid Theory

Edited by Ingrid and Klaus Truemper

F. Hülster *Introduction to Wittgenstein's Tractatus Logico-Philosophicus* (English and German edition)

F. Hülster *Berlin 1945: Surviving the Collapse*

THE DARING INVENTION OF LOGARITHM TABLES

How Jost Bürgi, John Napier, and Henry Briggs simplified arithmetic and started the computing revolution

KLAUS TRUEMPER

Leibniz Company

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Softcover published by Leibniz Company
2304 Cliffside Drive
Plano, Texas, 75023
USA

Original edition 2020
Updated color edition 2023

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Cover Art:
Left column: Bürgi image and table. Center: Briggs table. Right column: Napier image and table. Cover design by Ingrid Truemper.

The book is typeset in LaTeX using the Tufte-style book class, which was inspired by the work of Edward R. Tufte and Richard Feynman.

Sources and licenses for all figures are listed in the Notes section. The licenses implicitly cover the figures of the front cover since they are derived from images in Chapters 5, 10, 13, and 15.

Library of Congress Cataloging-in-Publication Data
Truemper, Klaus, 1942–

The Daring Invention of Logarithm Tables
Includes bibliographical references and subject index.
ISBN 978-0-9991402-6-0
1. Mathematics. 2. Logarithm

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About the Color Edition

This book first appeared in 2020 in black and white due to the high cost of color printing. The goal was to keep the price at around two cups of coffee.

Ever since, we have had misgivings about that choice since Jost Bürgi (1552–1632), one of the key players in this book, used color printing for clarity in his logarithm tables all the way back in the 17th century. Yet we claimed that we couldn't do so in this book.

That dilemma has been solved: Color printing is now possible at the price of three cups of coffee.

1

Introduction

Starting around 50 000 BCE, early humans used pebbles, scratches, or other marks to record quantities. These devices supported addition, subtraction, multiplication, and division.[1]

Eventually, various symbols replaced these tools and simplified operations. Thus, mathematics was born.

During the last 5 000 years, mathematical concepts and models became more and more advanced, and computations became more complicated as well.

In particular, the 16th and 17th centuries produced sophisticated models for puzzling observations about the world such as the configuration of the heavenly bodies. Evaluation of these models required voluminous manual computations spanning months or even years of effort.

An ingenious new computing device then reduced that effort dramatically. Indeed, the tool was an order of magnitude more effective than anything invented before.

That device was the logarithm table, or rather several such tables in particular formats. Use of these tables compressed months of computing effort to weeks and sometimes just days.

The tables triggered the invention of computing equipment where distances and angles represented the numbers of the logarithm tables. There were three such devices: the slide rule, the circular slide rule, and the slide cylinder. These tools were produced until the middle of the 20th century.[2]

A downside of the logarithm tables was their arduous and error-prone construction. That aspect led to a sequence of groundbreaking inventions. First was a special purpose computing device in the 19th century that could compute such tables mechanically and error-free. It was called the difference engine.[3] That design led to a general-purpose computer called the analytical engine[4] and the first-ever computer program.[5]

Unfortunately, both engine designs were so complicated that the difference engine was built for the first time in the late 20th and early 21st century, while the analytical engine never was—and likely never will be—constructed.

In the 1930s—about 100 years after the invention of the analytical engine—an ingenious new approach resulted in a general-purpose computer whose first version could be built using just sheet metal.[6]

The ensuing electronic revolution created ever faster computing devices that have improved human life in almost miraculous ways.

And all this started with the logarithm tables.

We have taken this warp-speed tour of mathematical and computing development to highlight the extraordinary impact of the invention of the logarithm table. Thus, it surely is worthwhile to study the who, when, where, and how of this invention.

This has been done in a number of articles and books, so you may wonder why we have written yet another book on the topic.

The existing material usually recounts in the language of modern mathematics how the logarithm table was invented.

Yes, the publications typically consider that certain concepts were not available at the time. But the interpretation of events then relies on modern mathematical concepts. That approach makes the invention of the logarithm table look like a much simpler and more natural step than it actually was.

Instead, we transport ourselves into the life of one of the inventors. We look over his shoulder, so to speak, as he thinks about and works on the problem of efficient computation.

In the process, we understand how difficult the work on logarithm tables really was: Tens of thousands of computing steps had to be performed with utter precision.

At the same time, we experience the magic of this invention as it comes together in the mind of one of its creators.

You surely have noticed that so far we haven't mentioned a single name, let alone discussed the life of any person. That's done on purpose: We learn about each player as we delve into his life.

Yes, "his" is correct here. With rare exception, women at that time were thought incapable of scientific thought. Which says something about the men of that time and nothing about the women.

There has been a major controversy involving logarithms: It concerns the question of who invented logarithms first and when. So far, a variety of conflicting answers have been given.

In the last part of this book we look at these answers and come up with an explanation how such a divergence of views is possible. We also offer our own view, as you are bound to expect.

The arguments rest on a particular interpretation of the brain's reasoning process. That insight not only explains the divergence of opinions about the priority question, but also provides a reasonable justification for our answer.

One more remark about our interpretation. We are convinced that mathematics is created, and have written a book arguing the case.[7] That conclusion is contrary to that of most mathematicians, who believe that mathematics is discovered.

If you are on that side of the fence, too, don't let terms such as "invention" bother you. After all, the decision isn't based on an objective reality but depends on the models of the world we have in our heads. Chapters 9 and 23 of *Wittgenstein and Brain Science: Understanding the World*[8] discuss this aspect in detail.

This book does not require any mathematical background beyond everyday knowledge of numbers and the elementary operations of addition, subtraction, multiplication, and division.

More complicated operations are rarely used. If so, they have been moved into the Notes section.