Matroid Decomposition
Revised Edition

Klaus Truemper
University of Texas at Dallas
Richardson, Texas

Leibniz Company
Plano, Texas
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Preface

Matroids were first defined in 1935 as an abstract generalization of graphs and matrices. In the subsequent two decades, comparatively few results were obtained. But starting in the mid-1950s, progress was made at an ever-increasing pace. As this book is being written, a large collection of deep matroid theorems already exists. These results have been used to solve difficult problems in diverse fields such as civil, electrical, and mechanical engineering, computer science, and mathematics.

There is now far too much matroid material to permit a comprehensive treatment in one book. Thus, we have confined ourselves to a part of particular interest to us, the one dealing with decomposition and composition of matroids. That part of matroid theory contains several profound theorems with numerous applications. At present, the literature for that material is quite difficult to read. One of our goals has been a clear and simple exposition that makes the main results readily accessible.

The book does not assume any prior knowledge of matroid theory. Indeed, for the reader unfamiliar with matroid theory, the book may serve as an introduction to that beautiful part of combinatorics. For the expert, we hope that the book will provide a pleasant tour over familiar terrain.

The help of many people and institutions has made this book possible. P. D. Seymour introduced me to matroids and to various decomposition notions during a sabbatical year supported by the University of Waterloo. The National Science Foundation funded the research and part of the writing of the book through several grants. Most of the the writing was made possible by the support of the Alexander von Humboldt-Foundation and of the University of Texas at Dallas, my home institution. The University of Bonn and Tel Aviv University assisted the search for and verification of reference material.
M. Grötschel of the University of Augsburg made the resources of the Institute of Applied Mathematics available for the editing, typesetting, and proofreading. He also supported the project in many other ways. P. Bauer, M. Jünger, A. Martin, G. Reinelt, M. Stoer, and G. Ziegler of the University of Augsburg were of much assistance.

T. Konnerth most ably typeset the manuscript in \TeX. R. Karpelowitz and C.-S. Peng patiently prepared the numerous drawings.

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To all who so generously gave of their time and who lent support in so many ways, I express my sincere thanks. Without their help, the book would not have been written.

About the Revised Edition

In 1997, a transfer of the copyright from Academic Press, Inc., to the author made possible the issue of a revised edition. Since 1998, that edition has been distributed in electronic format; it can be printed for personal use without charge. Since 2016, it has been available as paperback.

We limited the changes to correction of technical and typographical errors and updating of the publication data of the references.

The change to an electronic version forced a reprocessing of the numerous drawings. R. L. Brooks, G. Qian, G. Rinaldi, and F.-S. Sun carried out much of that work. A. Bachem, F. Barahona, G. Cornuéjols, C. R. Coullard, A. Frank, A. M. H. Gerards, R. Hassin, D. Naddef, T. J. Reid, P. D. Seymour, R. Swaminathan, F.-S. Sun, and G. M. Ziegler assisted with the updating of the references. The final editing was done by I. Truemper.

G. Rinaldi and I. Truemper helped with the implementation of the paperback version.

We very much thank all who helped with the preparation of the revised edition. Without that help, we could not have accomplished that task.