(a) Truthfully write the phrase “I have read and understand the policies on the course website.”

**Solution:** I have read and understand the policies on the course website. ■

**Rubric:** 2 points total.

(b) Prove that the number of leaves in $F_n$ is precisely the $n$th Fibonacci number: $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$.

**Solution:** Let $n$ be an arbitrary non-negative integer. Assume that for any non-negative integer $k < n$, the number of leaves in $F_k$ is precisely $F_k$. There are several cases to consider:

- Suppose $n = 1$. Tree $F_n = F_1$ has $1 = F_1 = F_n$ leaves.
- Suppose $n = 2$. Tree $F_n = F_2$ has $1 = F_2 = F_n$ leaves.
- Suppose $n \geq 3$. In constructing $F_n$, we add one leaf as a child of each leaf of $F_{n-1}$, and we add one leaf for each node of $F_{n-1}$ that has one child; however, the nodes of $F_{n-1}$ with one child are precisely the leaves of $F_{n-2}$. The induction hypothesis implies that $F_{n-1}$ has $F_{n-1}$ leaves, and $F_{n-2}$ has $F_{n-2}$ leaves. Therefore, the number of leaves in $F_n$ is precisely $F_{n-1} + F_{n-2} = F_n$.

In each case, we conclude the number of leaves in $F_n$ is precisely $F_n$. ■

**Rubric:** 4 points total. –1 point for each missing base case (# of base cases may depend on the induction step). Students do not need to explicitly state the induction hypothesis if they use the natural one.

(c) How many nodes does $F_n$ have? Give an exact, closed-form answer in terms of Fibonacci numbers and prove your answer is correct.

**Solution:** Tree $F_n$ has $F_{n+2} - 1$ nodes.

**Proof:** Let $n$ be an arbitrary non-negative integer. Assume that for any non-negative integer $k < n$, tree $F_k$ has $F_{k+2} - 1$ nodes. There are two cases to consider:

- Suppose $n = 1$. Tree $F_n = F_1$ has $1 = F_3 - 1 = F_{n+2} - 1$ nodes.
- Suppose $n \geq 2$. Per part (a), tree $F_n$ is obtained from $F_{n-1}$ by adding $F_n$ leaves. The induction hypothesis implies $F_{n-1}$ has $F_{(n-1)+2} - 1 = F_{n+1} - 1$ nodes. Therefore, $F_n$ has $F_{n+1} - 1 + F_n = F_{n+2} - 1$ nodes.

In each case, we conclude $F_n$ has $F_{n+2} - 1$ nodes. ■

**Rubric:** 4 points total. –1 point for each missing base case (# of base cases may depend on their induction step). Students do not need to explicitly state the induction hypothesis if they use the natural one.
Sort the functions of $n$ from asymptotically smallest to asymptotically largest, indicating ties if there are any.

**Solution:**

\[
3 - \cos n \equiv 3345 \\
\ll \lg^{0.6} n \\
\ll \log_{9} n \equiv \lg(7n) \\
\ll \ln^{3} n \\
\ll \sqrt{n} \\
\ll 17n \equiv n + 500 \\
\ll n \log n \\
\ll n^{2} \\
\ll 2^{4 \lg n} \\
\ll 2^{n} \\
\ll 4^{n}
\]

**Rubric:** 10 points total. -1 point for each function that appears too soon or for which the wrong comparison is used.