CS 3345 HON

Kyle Fox (he/him)
Data structures: methods of organizing large amounts of data

```java
int n = 10000;
int[] bunchOfInts;
bunchOfInts = new int[n];
```

Array of n = 10,000 ints can read/write bunchOfInts[i] in constant time
insert in $\Theta(n)$ time
delete in $\Theta(n)$ time
+ algorithms!
https://personal.utdallas.edu/~kyle.fox/courses/cs3345.hon.22f/

↑

Reading, schedule, etc.

Learning, grades, turn-in
Grades:
assignments: 40% (written homework + programming)
drop lowest can use outside resources with citation - no penalty
mid-term: 25%
final: 35%
↑
cumulative
Math Review

Exponents:

\[ x \cdot x = x^{a+b} \]
\[ x / x = x^{a-b} \]
\[ (x^a)^b = x^{ab} \]
\[ \log_a y = x \iff a^x = y. \]

\[
\begin{align*}
\log_2 n & = \log_{10} n \\
\ln n & = \log_e n \\
\text{(e = 2.718)} \\
\log n & = \log_2 n \text{ (text)} \\
\log_{10} n \text{ (me)}
\end{align*}
\]
\[ a, b, x, y > 0 \]
\[ a, b \neq 1 \]

\[ \log_b x = \frac{\log_a x}{\log_a b} \]

\[ \log_a xy = \log_a x + \log_a y \]

\[ \log_a \frac{x}{y} = \log_a x - \log_a y \]

\[ \log_a x^c = c \log_a x \]
Prop: \( a, x, y > 0 \) \( a \neq 1 \) \( a^\log_a y = \log_a (x^{\log_a y}) \) \( = a^{\log_a (y^{\log_a x})} \) \( = a \).
\[
\sum_{i=0}^{n} a + id = a + \sum_{i=0}^{n} d + \sum_{i=0}^{n} 2d + \cdots + a + nd = \frac{(a + dn)(n+1)}{2}
\]
\[
\begin{align*}
\Rightarrow \quad & \quad \exists \quad \omega = 0 \\
\therefore \quad \omega = 0 & \quad \Rightarrow \quad \frac{n(n+1)}{2} = \Theta \left( n^2 \right) \\
\text{also} & \\
\exists \quad \omega^* & \quad \Rightarrow \quad \omega^* = \Theta \left( n^{c+1} \right) \\
\text{if} & \quad n > -1 \\
\text{nth Harmonic number:} & \\
H_n & \quad \Rightarrow \quad \sum_{i=1}^{n} \frac{1}{\omega} = \Theta (\log n)
\end{align*}
\]
If \( c < -1 \)

\[
\sum_{i=3}^{n} i \cdot c = \Theta(i)
\]

\[\omega = 0\]

geometric series

\[
\sum_{i=3}^{n} a r \cdot i = a + ar + ar^2 + \cdots
\]

\[\omega = 0\]

\[
= a \left(1 - r^{n+1}\right)
\]

\[\frac{a}{1-r}\]
If \( r > 0 \), \( r \neq 1 \), then \( r \) is a constant, and the sum is proportional to the largest term.

\[
\sum_{i=0}^{n} ar^i = \Theta(a) \quad \text{if} \quad 0 < r < 1
\]

\[
\sum_{i=0}^{n} ar^i = \Theta(ar^n) \quad \text{if} \quad r > 1
\]
Proofs:
Let $n$ be a positive integer.
A divisor of $n$ is an integer $d$ s.t.
$n/d$ is an integer.

$n$ is prime if it has exactly 2 divisors.
Thm: Every integer \( n > 1 \) has a prime divisor.
Direct proof:
Let \( n > 1 \) be an integer.
\( n \) has a prime divisor \( \Box \)

Proof by contradiction:
Assume there is an integer \( n > 1 \) with no prime divisor.
That's absurd!
Our assumption was wrong!
Proof: Assume there is an int \( n > 1 \) with no prime divisor. 
\( n \) is its own divisor so \( n \) is not prime. 
So there is an int \( 1 < d < n \) that divides \( n \). 
\( d \) is not prime, so
Some \( 1 \leq d' \leq d \) divides \( d \).

\[ \frac{n}{d'} = \left( \frac{n}{d} \right) \cdot \left( \frac{d}{d'} \right) \]

is an integer so \( d' \) divides \( n \).

There is a \( 1 \leq d'' \leq d' \) that divides \( d' \)...

\( \text{Ma}_{-1} \)
Proof by smallest counterexample.

Let \( n \geq 1 \) be the least integer with no prime divisor.

\( n \) is not prime.

Exist \( 1 < d < n \) that divides \( n \).
By assumption, \( d \) has a prime divisor. So some prime \( p \) divides \( d \), \( p \) divides \( n \). But \( n \) had no prime divisor?
Direct proof:

Let $n$ be any int $\geq 1$. Assume for every int $k$ s.t. $1 \leq k < n$, int $k$ has a prime divisor.

If $n$ is prime, it is its own prime.
Case 1: Suppose there is an integer $1 < d < n$ that divides $n$. By assumption, $d$ has a prime divisor $p$. Since $p$ divides $n$, $p$ is also a divisor of $n$. Thus, there is a prime divisor of $n$. The base case is proven.

Case 2: O.W. there is an integer $1 < d < n$ that divides $n$. By assumption, $d$ has a prime divisor $p$. Since $p$ divides $n$, $p$ is also a divisor of $n$. Thus, there is a prime divisor of $n$. The base case is proven.
In both cases, $n$ has a prime divisor. □

Proof by induction!
**Theorem:** $P(n)$ for every positive integer $n$.

**Proof by induction:** Let $n$ be an arbitrary positive integer. Assume that $P(k)$ is true for every positive integer $k < n$. There are several cases to consider:

- Suppose $n$ is \[ \ldots \text{blah blah blah} \ldots \]
  Then $P(n)$ is true.
- Suppose $n$ is \[ \ldots \text{blah blah blah} \ldots \]
  The inductive hypothesis implies that \[ \ldots \text{blah blah blah} \ldots \]
  Thus, $P(n)$ is true.

In each case, we conclude that $P(n)$ is true.