

# CS 4349.003 Homework 1

Due Wednesday September 4th on eLearning

August 26, 2019

Please answer the following **2** questions.

## Some important homework policies

- Each student must write their solutions in their own words and submit their solutions to eLearning. Clearly print your name, the homework number (Homework 1), and the problem number at the top of every page in case we print anything. Start each numbered homework problem on a new page.
- Unless the problem states otherwise, you must justify (prove) that your solution is correct.
- We strongly suggest you use  $\text{\LaTeX}$  to typeset your solutions. Any illegible solutions will be considered incorrect. The announcement for this homework links to a template for writing solutions in  $\text{\LaTeX}$ .
- If you use outside sources or write solutions in close collaboration with others, then you may cite that source or person and still receive full credit for the solution. Material from the lecture, the textbooks, or prerequisite courses need not be cited. Failure to cite other sources or failure to provide solutions in your own words, even if quoting a source, is considered an act of academic dishonesty.
- The homework is assigned to give *you* the opportunity to learn where your understanding is lacking and to practice what is taught in class. Its primary purpose *is not* for us to grade how well you paid attention in class. Read through the questions early. Do not expect to know the answers right away. Do expect to think hard about these questions and for a while, especially as we get deeper into algorithm design in future weeks. And *please, please, please* attend office hours or email Kyle and the TA so we can help you better understand the questions and class material.

See <https://personal.utdallas.edu/~kyle.fox/courses/cs4349.003.19f/about.shtml> and <https://personal.utdallas.edu/~kyle.fox/courses/cs4349.003.19f/writing.shtml> for more detailed policies. If you have any questions about these policies, please do not hesitate to ask in class, in office hours, or through email.

1. (a) Truthfully write the phrase *“I have read and understand the course policies.”*

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A couple years ago, Kyle started watching “Parks and Office Places”, a network television series that remains entertaining even after rewatching episodes multiple times. Each episode of the series is exactly 20 minutes long, and each season of the show features exactly 13 episodes. In anticipation for new seasons, Kyle likes to watch every episode of every previous season before beginning the new season’s episodes.

Suppose the show continues for  $n$  seasons and the title of each episode is stored in two-dimensional array  $titles[1..n, 1..13]$  where  $title[j, k]$  is the title of the  $k$ th episode of season  $j$ . Kyle’s viewing of the show’s episodes can be summarized by the following pseudocode.

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WATCHSERIES(titles[1..n, 1..13]):
  For each  $i \leftarrow 1$  to  $n$ 
    For each  $j \leftarrow 1$  to  $i$ 
      For each  $k \leftarrow 1$  to 13
        Watch episode  $titles[j, k]$ .

  Find a new show to watch.

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- (b) Give a tight asymptotic bound for the total amount of time Kyle spends watching “Parks and Office Places” according to the above pseudocode. Your bound should be given as  $\Theta(g(n))$  for some simple function  $g(n)$ , and you should *briefly* justify your answer. [Hint: The numbers 20 and 13 are both constants.]
- (c) “Parks and Office Places” has already been renewed for its 10th season, and Kyle cannot afford to spend all of his time rewatching every episode as explained above. Give a tight asymptotic bound for the total amount of time Kyle will spend watching “Parks and Office Places” if he only rewatches episodes of *one* previous season every time a new season comes out. Again, briefly justify your answer.

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The last part of this question has nothing to do with television.

- (d) Sort the functions of  $n$  listed below from asymptotically smallest to asymptotically largest, indicating ties if there are any. **Do not turn in proofs for this problem.** (But you may want to write the proofs for yourself anyway so you know if you’re correct.) To simplify your answers, write  $f(n) \ll g(n)$  to mean  $f(n) = o(g(n))$ , write  $f(n) \equiv g(n)$  to mean  $f(n) = \Theta(g(n))$ , and list all the functions in a sequence of these inequalities.

For example, if the given functions were  $n^2$ ,  $n$ ,  $\binom{n}{2}$ , and  $n^3$  then the two correct answers would be “ $n \ll n^2 \equiv \binom{n}{2} \ll n^3$ ” and “ $n \ll \binom{n}{2} \equiv n^2 \ll n^3$ ”.

$$\begin{array}{cccccc}
 3^n & n^2 & n & \log_{10} n & \sqrt{n} & \\
 6^n & \lg^2 n & 11n & n + 1000 & 2 - \sin n & \\
 2^{3 \lg n} & \lg 3n & 300 & \lg^{0.5} n & n \log n & 
 \end{array}$$

[Hint: You should be able to solve this problem using only what is written in the lecture notes on asymptotic analysis along with basic algebraic rules for manipulating logs, polynomials, and exponentials.]

2. This problem asks you to do two proofs by induction. **You will receive no points for either part if you do not follow the given templates.** We will not require you to follow a specific template in future assignments, although you may want to continue using this template anyway.

Recall the standard definition of the Fibonacci numbers:  $F_0 = 0$ ,  $F_1 = 1$ , and  $F_n = F_{n-1} + F_{n-2}$  for all  $n \geq 2$ .

- (a) Prove that  $\sum_{i=0}^n F_i = F_{n+2} - 1$  for every non-negative integer  $n$ . Your proof **must** follow the following template:

Let  $n$  be a non-negative integer.  
 Assume  $\sum_{i=0}^k F_i = F_{k+2} - 1$  for every non-negative integer  $k < n$ .  
 There are several cases to consider:

- Suppose  $n$  is...
- Suppose  $n$  is...
- ...
- Suppose  $n$  is...

The inductive hypothesis implies that...

In each case, we conclude  $\sum_{i=0}^n F_i = F_{n+2} - 1$ .

[Hint: Figure out the inductive case first so you know how many base cases you really need.]

- (b) The Fibonacci sequence can be extended backward to negative indices by rearranging the defining recurrence:  $F_n = F_{n+2} - F_{n+1}$ . Here are the first several negative-index Fibonacci numbers:

$n$	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
$F_n$	-55	34	-21	13	-8	5	-3	2	-1	1

Prove that  $F_{-n} = (-1)^{n+1}F_n$  for every non-negative integer  $n$ . Your proof **must** follow the following template:

Let  $n$  be a non-negative integer.  
 Assume  $F_{-k} = (-1)^{k+1}F_k$  for every non-negative integer  $k < n$ .  
 There are several cases to consider:

- Suppose  $n$  is...
- Suppose  $n$  is...
- ...
- Suppose  $n$  is...

The inductive hypothesis implies that...

In each case, we conclude  $F_{-n} = (-1)^{n+1}F_n$ .