1. (a) Suppose we are given an array $A[1..n]$ with the special property that $A[1] \geq A[2]$ and $A[n - 1] \leq A[n]$. We say that an element $A[x]$ is a local minimum if it is less than or equal to both its neighbors, or more formally, if $A[x - 1] \geq A[x]$ and $A[x] \leq A[x + 1]$. For example, there are six local minima in the following array:

```
9 7 7 2 1 3 7 5 4 7 3 3 4 8 6 9
```

We can find a local minimum in $O(n)$ time by scanning through the array. Describe and analyze an algorithm that finds a local minimum in $O(\log n)$ time. Your analysis should include an explanation for the algorithm’s running time.

[Hint: Suppose $A[\lceil n/2 \rceil]$ is not a local minimum. Is there a subarray of around $n/2$ elements you now know about that does contain a local minimum?]

(b) Suppose we are given two sorted arrays $A[1..n]$ and $B[1..n]$. The median element in the union of $A$ and $B$ is the $n$th smallest element in their union. Describe and analyze an algorithm that finds this median element in $O(\log n)$ time. You can assume the arrays contain no duplicate elements.

[Hint: Compare $A[\lfloor n/2 \rfloor]$ and $B[\lfloor n/2 \rfloor]$. How can you reduce the search space to two sorted arrays of size $\lceil n/2 \rceil$ each?]

2. Describe backtracking recursive algorithms for the following variants of the text segmentation problem. Assume that you have a subroutine IsWord that takes an array of characters as input and returns True if and only if that string is a “word”. You do not need to analyze the time complexity of your algorithms for this problem.

(a) Given two arrays $A[1..n]$ and $B[1..n]$ of characters, decide whether $A$ and $B$ can be partitioned into words at the same indices. For example, the strings BOTHEARTANDSATURNSPIN and PINSTARTRAPSANDRAGSLAP can be partitioned into words at the same indices as follows:

```
BOT · HEART · HAND · SAT · URNS · PIN
PIN · START · RAPS · AND · RAGS · LAP
```

(b) Given an array $A[1..n]$ of characters, compute the number of partitions of $A$ into words. For example, given the string ARTISTOIL, your algorithm should return 2 for the partitions ARTIST · OIL and ART · IS · TOIL.