CS 4349.003 Homework 9

Due Wednesday November 6th on eLearning

October 30, 2019

Please answer the following **2** questions.

1. You just discovered your best friend from elementary school on Twitbook. You both want to meet as soon as possible, but you live in two different cities that are far apart. To minimize travel time, you agree to meet at an intermediate city, and then you simultaneously hop in your cars and start driving toward each other. But where exactly should you meet?

   You are given a weight graph $G = (V, E)$, where the vertices V represent cities and the edges E represent roads that directly connect cities. Each edge e has a weight $w(e)$ equal to the time required to travel between the two cities. You are also given a vertex p, representing your starting location, and a vertex q, representing your friend's starting location.

   Describe and analyze an algorithm to find the target vertex t that allows you and your friend to meet as quickly as possible.

2. In this problem, we will discover how you, yes ***you***, can be employed by Wall Street to cause a major economic collapse! The *arbitrage* business is a money-making scheme that takes advantage of differences in currency exchange. In particular, suppose 1 US dollar buys 120 Japanese yen, 1 yen buys 0.01 euros, and 1 euro buys 1.2 US dollars. Then, a trader starting with \$1 can convert their money from dollars to yen, then from yen to euros, and finally from euros back to dollars, ending with \$1.44! The cycle of currencies \$→¥→€→\$ is called an ***arbitrage cycle***. Of course, finding and exploiting arbitrage cycles before the prices are corrected requires extremely fast algorithms.

   Suppose n different currencies are traded in your currency market. You are given a matrix $Exch[1 .. n, 1 .. n]$ of exchange rates between every pair of currencies; for each i and j, one unit of currency i can be traded for $Exch[i, j]$ units of currency j. (Do *not* assume that $Exch[i, j] \cdot Exch[j, i] = 1$.)

   (a) Describe and analyze an algorithm that returns a matrix $MaxAmt[1 .. n, 1 .. n]$, where $MaxAmt[i, j]$ is the maximum amount of currency j that you can obtain by trading, starting with one unit of currency i, assuming there are no arbitrage cycles.

   [Hint: Reduce to APSP. How can you turn a problem about maximizing a product into one about minimzing a sum?]

   (b) Describe and analyze an algorithm to determine whether the given matrix of currency exchange rates creates an arbitrage cycle.

   [Hint: Reduce to something else.]