CS 4349.006 Homework 5

Due Thursday, October 14 on eLearning

Please solve the following **2** problems.

1. Suppose we have n skiers with heights given in an array $P[1 .. n]$ and n skis with heights given in an array $S[1 .. n]$. Our goal is to assign a ski to each skier, so that the average difference between the height of a skier and their assigned ski is as small as possible. The algorithm should compute an assignment array $A[1 .. n]$, indicating that each skier i should be assigned ski $A[i]$, such that the expression

$$\frac{1}{n}\sum_{i=1}^{n} |P[i] - S[A[i]]|$$

   is as small as possible.

   (a) Describe two input arrays P and S that show the following greedy strategy *is not* always optimal:

   > Let skier i and ski j be such that $|P[i] - S[j]|$ is minimized. Assign ski j to skier i and recursively assign the remaining skis to the remaining skiers.

   Advice: The arrays you describe can be very very small.

   (b) Describe and analyze an efficient greedy algorithm that *does* minimize the average difference between the height of each skier and their assigned ski. Don't forget to prove your algorithm is correct!

   Advice: You may want to begin by sorting the skis and skiers by height.

2. You've been asked to test drive the latest mini electric vehicle from Teslazon Inc. You get to drive along a very long empty stretch of road, but your tiny vehicle can only go 100 miles on a single non-rechargeable battery. Unfortunately, you can only replace the batteries at official Teslazon Energy Stations, and your vehicle is so small that it can only carry the one battery it is using and no spares. Double unfortunately, your stingy bosses are making you pay for each brand new battery you want to install. You decide to study where all the Energy Stations are along your route so you can minimize the amount of money you spend on batteries.

   Formally, you are given an array $D[1 .. n]$ describing stations along your route where $D[i]$ is the distance from the start of your route to the ith station. Assume your test drive starts and ends at battery stations (so $D[1] = 0$, and $D[n]$ is the total length of the trip) and your vehicle starts with an empty battery (so you must purchase a new battery at station 1, but you don't need to buy a fresh battery at station n).

(a) Describe and analyze a greedy algorithm that computes the minimum *number* of batteries you need to purchase to complete your test drive. Don't forget to prove your algorithm is correct!

(b) While studying the location of the Energy Stations, you learn each station charges a different amount for its batteries. *What a pain!* Now, along with $D[1 .. n]$, you are also given an array $C[1 .. n]$ where $C[i]$ is the cost to replace your battery at the ith station (again, you'll have to spend at least $C[1]$ just to get moving). Describe and analyze an efficient algorithm to compute the minimum total cost of the batteries you need to complete your test drive.

Advice: Greed won't help you now, so try dynamic programming. Either (a) try to decide for each station if you should purchase a battery while remembering the last station from which you made a purchase, or (b) try to repeatedly decide on the next *station from which you* will *purchase a battery. Either approach should lead to the same running time.*