CS 4349.006 Homework 6

Due Thursday, October 21 on eLearning

Please solve the following **2** problems.

1. A ***number maze*** is an $n \times n$ grid of positive integers. A token starts in the upper left corner. Your goal is to move the token to the lower-right corner. On each turn, you are allowed to move the token up, down, left, or right; the distance you may move the token is determined by the number on its current square. For example, if the token is on a square labeled 3, then you may move the token exactly three steps up, three steps down, three steps left, or three steps right. *However, you are never allowed to move the token off the edge of the board.*

   Describe and analyze an efficient algorithm that determines if a given maze has a solution. For example, given the number maze shown below, your algorithm should return TRUE.

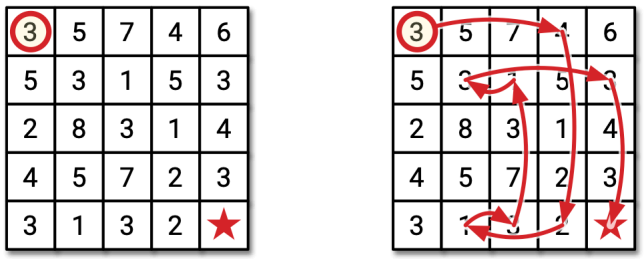
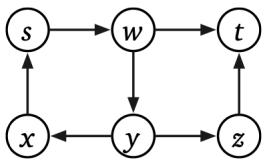


Figure 1. A 5×5 number maze that can be solved in eight moves.

   Advice: Model the number maze as a graph where vertices are token locations. What should you use for the edges? What graph algorithm from class should you call? How do you use the results of that graph algorithm? What is the running time in terms of n?

2. Suppose you are given a directed graph $G = (V, E)$ and two vertices s and t. Describe and analyze an algorithm to determine if there is a walk in G from s to t (possibly repeating vertices and/or edges) whose length is divisible by 3.

   For example, given the graph shown below, with the indicated vertices s and t, your algorithm should return TRUE, because the walk $s \to w \to y \to x \to s \to w \to t$ has length 6.



Advice: Build a new graph to model your progress during a walk. For the vertices, describe some way to encode your current location in G along with a little more information to help you know how many steps ($\bmod 3$) you've taken.