CS 4349.006 Homework 9

Due Thursday, November 11 on eLearning

Please solve the following **2** problems.

1. After moving to a new city, you decide to choose a walking route from your new office to home. Knowing you'll be tired after a long day of work, you decide your route must be entirely downhill. You also want to take the *shortest* downhill path, because you'll be hungry and wanting to get home to eat as quickly as possible.

   Your input consists of an undirected graph G whose vertices represent intersections and whose edges represent road segments along with a start vertex s (your office) and a target vertex t (your home). Every vertex v has an associated value $h(v)$ which is the height of that intersection above sea level, and each edge uv has an associated value $\ell(uv)$ which is the length of that road segment.

   (a) Suppose we allow some or all vertex heights to be equal. Describe and analyze an algorithm to find the shortest downhill walk from s to t; you may use flat edges in your walk, but you should never use an edge that goes uphill.

   Advice: Build a **directed** *graph and run an appropriate shortest paths algorithm.*

   (b) Now suppose we assume all vertex heights are *distinct*; i.e., no two heights are equal. Describe and analyze an algorithm to find the shortest downhill walk from s to t.

   Advice: You should be able to use a faster shortest paths algorithm than in part (a).

2. Let $G = (V, E)$ be a directed graph with edge weights $w : E \to \mathbb{R}$; edge weights can be positive, negative, or zero, but there are no negative weight cycles.

   (a) Describe and analyze an algorithm that reports whether or not there is a cycle of length zero in G.

   Advice: Compute all pairs shortest path distances. How can you quickly tell if an edge $u \to v$ belongs to a length zero cycle?

   (b) ***This part was more difficult than Kyle expected. To make up for lost time, everybody will get 4 points extra credit and part (b) will not be graded.***

   ~~Describe and analyze an algorithm that constructs the subgraph H of G with the following properties:~~

   - ~~Every vertex of G is a vertex of H.~~
   - ~~Every edge in H belongs to a directed cycle in H.~~
   - ~~Every directed cycle in H has length 0.~~
   - ~~Every directed cycle of length 0 in G is also a cycle in H.~~

   ~~In particular, if there are no length zero cycles in G, then H has no edges.~~