CS 4349.501 Homework 10

Due Wednesday November 15th, in class

November 8, 2017

Please answer both questions. ***There is an extra credit problem on the second page.*** Each student must write their solutions in their own words and submit their solutions on paper at the beginning of class. ***Include your name and/or Net ID at the top of each page.*** And please staple your papers together.

1. Let $G = (V, E)$ be a directed graph with ***integer*** edge capacities $c : E \to \mathbb{Z}_{\geq 0}$. Suppose you have already computed a maximum flow f^* in G.

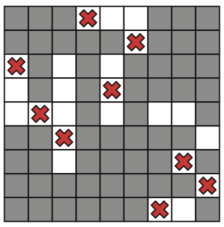
   (a) Describe and analyze an algorithm to update the maximum flow after the capacity of a single edge is increased by 1.

   (b) Describe and analyze an algorithm to update the maximum flow after the capacity of a single edge is decreased by 1.

   You algorithm for both parts should be faster than computing a new maximum flow from scratch.

2. Suppose you are given an $n \times n$ grid, some of whose squares are colored black and the rest white. Describe and analyze an algorithm to determine whether tokens can be placed on the grid so that

   - every token is on a white square;
   - every row of the grid contains exactly one token; and
   - every column of the grid contains exactly one token.



An example grid with a proper placement of tokens.

Your input is a two dimensional array $IsWhite[1 .. n][1 .. n]$ of booleans, indicating which squares are white. Your output is a single boolean. For example, given the grid above as input, your algorithm should return TRUE. *[Hint: Compute a maximum matching in an appropriate bipartite graph. What pairs of objects are you trying to match, and how do you know if you can match them? Be sure to give your running time in terms of n. You* **really** *should learn to solve this type of problem.]*

3. **Extra credit** (worth 1/2 a normal question): You're organizing the First Annual UTD Erik Jonsson School 72-Hour Dance Exchange, to be held all day Friday, Saturday, and Sunday. Several 30-minute sets of music will be played during the event, and a large number of DJs have applied to perform. You need to hire DJs according to the following constraints.

   - Exactly k sets of music must be played each day, and thus $3k$ sets altogether.
   - Each set must be played by a single DJ in a consistent music genre (ambient, bubblegum, dubstep, horrorcore, hyphy, trip-hop, J-pop, swing, Nashville country, . . .).
   - Each genre must be played at most once per day.
   - Each candidate DJ has given you a list of genres they are willing to play.
   - Each DJ can play at most three sets during the entire event.

   Suppose there are n candidate DJs and g different musical genres available. Describe and analyze an efficient algorithm that either assigns a DJ and a genre to each of the $3k$ sets, or correctly reports that no such assignment is possible.