

CS 4349.501 Homework 4

Due Wednesday September 27th, in class

September 20, 2017

Please answer each of the following questions. Each student must write their solutions in their own words and submit their solutions on paper at the beginning of class. **Include your name and/or Net ID at the top of each page.**

1. Let $A[1 .. n]$ be an array/sequence. Recall from lecture that a subsequence of A is any sequence obtained by extracting elements from A in order; the elements need not be contiguous in A . For example, the strings C, DAM, YAIIOAI, and DYNAMICPROGRAMMING are all subsequences of DYNAMICPROGRAMMING. The sequence $\langle 5, 9, 4 \rangle$ is a subsequence of $\langle 1, 5, 45, 9, 34, 42, 4, 6 \rangle$.

Call a sequence $X[1 .. n]$ of numbers **weakly bitonic** if there is an index i with $1 \leq i \leq n$, such that the prefix $X[1 .. i]$ is increasing and the suffix $X[i .. n]$ is decreasing. In other words, $X[1] < X[2] < \dots < X[i]$ and $X[i] > X[i + 1] > \dots > X[n]$. Both $\langle 3, 56, 92, 34, 0, -5 \rangle$ and $\langle 45, 23, 1 \rangle$ are weakly bitonic. Describe and analyze an $O(n^2)$ time algorithm to compute the length of the longest weakly bitonic subsequence of an arbitrary array A of integers. Your analysis should explain how much time **and space** your algorithm uses.

2. A palindrome is any string that is exactly the same as its reversal, like I, or DEED, or RACECAR, or AMANAPLANACATACANALPANAMA.

Describe and analyze an efficient algorithm to find the length of the *longest palindrome subsequence* of a given string/array $A[1 .. n]$. For example, the longest palindrome subsequence of MAHDYNAMICPROGRAMZLETMESHOWYOUTHEM is MHYMRORMYHM. Yes, the palindrome subsequence may have odd length. Your analysis should explain how much time **and space** your algorithm uses.

3. **Extra credit** (worth 1/2 a normal question): Any string can be decomposed into a sequence of palindromes. For example, the string BUBBASEESABANANA can be broken into palindromes in the following ways (and many others):

BUB • BASEESAB • ANANA
B • U • BB • A • SEES • ABA • NAN • A
B • U • BB • A • SEES • A • B • ANANA
B • U • B • B • A • S • E • E • S • A • B • A • N • ANA

Describe and analyze an $O(n^2)$ time algorithm to find the smallest number of palindromes that make up a given input string/array $A[1 .. n]$. For example, given the input string BUBBASEESABANANA, your algorithm would return the integer 3. Your analysis should explain how much time **and space** your algorithm uses.