CS 4349.501 Homework 5

Due Wednesday October 11th, in class

September 27, 2017

Please answer each of the following questions. Each student must write their solutions in their own words and submit their solutions on paper at the beginning of class. ***Include your name and/or Net ID at the top of each page.***

1. Consider a weighted version of the class scheduling problem seen during lecture, where different classes offer different numbers of credit hours (totally unrelated to the duration of the class lectures). The goal now is to choose a set of non-conflicting classes that give you the largest possible number of credit hours, given an array of start times, end times, and credit hours as input.

   Formally, you're given three arrays $S[1..n]$, $F[1..n]$, and $C[1..n]$ where $S[i]$, $F[i]$, and $C[i]$ respectively give the start time, end time, and credit hours for course i. You must compute a ***credit-maximal conflict-free schedule*** defined as a subset $X \subseteq \{1, 2, \ldots, n\}$ that maximizes $\sum_{i \in X} C[i]$ so that for each $i, j \in X$, either $S[i] > F[j]$ or $S[j] > F[i]$.

   (a) Prove that the greedy algorithm given in class—choose the class that ends first and recurse—does *not* always return a credit-maximal conflict-free schedule.

   (b) Describe an algorithm to compute a credit-maximal conflict-free schedule in $O(n^2)$ time.

2. Let X be a set of n intervals on the real line. A subset of intervals $Y \subseteq X$ is called a ***full path*** if the intervals in Y cover the intervals in X, that is, any real value that is contained in some interval in X is also contained in some interval in Y. The ***size*** of the full path is the number of intervals it contains.

   Describe and analyze a greedy algorithm to compute the smallest full path of X as quickly as possible. Assume that your input consists of two arrays $X_L[1..n]$ and $X_R[1..n]$, representing the left and right endpoints of the intervals in X. Don't forget to prove your greedy algorithm is correct!

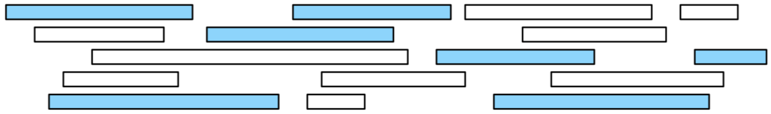

Figure 1. From Erickson 7. A set of intervals. The seven shaded intervals form a full path.