

CS 4349.501 Homework 6

Due Wednesday October 18th, in class

October 11, 2017

Please answer **both** of the following questions. Each student must write their solutions in their own words and submit their solutions on paper at the beginning of class. **Include your name and/or Net ID at the top of each page.** And please staple your papers together if you can.

1. A **number maze** is an $n \times n$ grid of positive integers. A token starts in the upper left corner; your goal is to move the token to the lower-right corner. On each turn, you are allowed to move the token up, down, left, or right; the distance you may move the token is determined by the number on its current square. For example, if the token is on a square labeled 3, then you may move the token three steps up, three steps down, three steps left, or three steps right. However, you are never allowed to move the token off the edge of the board.

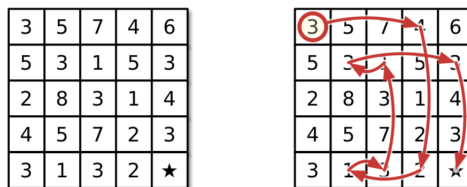


Figure 1. A 5×5 number maze that can be solved in eight moves.

Describe and analyze an efficient algorithm that either returns the minimum number of moves required to solve a given number maze, or correctly reports that the maze has no solution. Your running time should be in terms of n . [Hint: Build a configuration graph. What are its vertices? What are its edges? What problem are you solving in your graph?]

2. A **polygonal path** is a sequence of line segments joined end-to-end; the endpoints of these line segments are called the **vertices** of the path. The **length** of the polygonal path is the sum of the lengths of its segments. A polygonal path with vertices $(x_1, y_1), (x_2, y_2), \dots, (x_k, y_k)$ is **monotonically increasing** if $x_i < x_{i+1}$ and $y_i < y_{i+1}$ for every index i —informally, each vertex of the path is above and to the right of its predecessor.

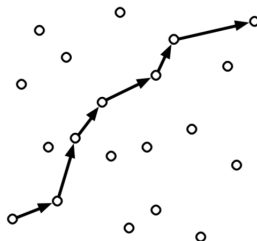


Figure 2. A monotonically increasing polygonal path with seven vertices through a set of points.

Suppose you are given a set S of n points in the plane, represented as two arrays $X[1 .. n]$ and $Y[1 .. n]$. Describe and analyze an algorithm to compute the length of the maximum-length monotonically increasing path with vertices in S . Assume you have a subroutine $\text{LENGTH}(x, y, x', y')$ that returns the length of the segment from (x, y) to (x', y') .

You may use the following fact: Given a **directed acyclic graph** $G = (V, E)$ with lengths on the edges, we can compute the length of the maximum-length directed path in G in $O(V + E)$ time. **However, your final running time should still be in terms of n .**