CS 4349.501 Homework 9

Due Wednesday November 8th, in class

November 1, 2017

Please answer ***both*** questions (Problem 2 appears on the second page). Each student must write their solutions in their own words and submit their solutions on paper at the beginning of class. ***Include your name and/or Net ID at the top of each page.*** And please staple your papers together if you can.

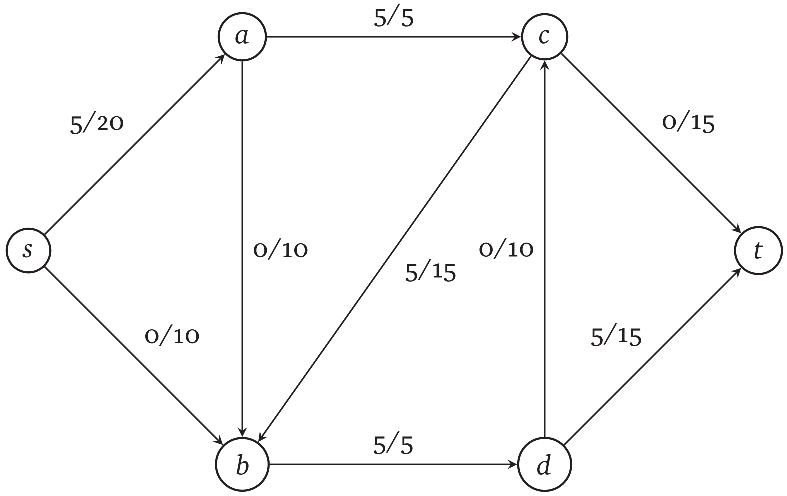
1. Let $G = (V, E)$ be a directed graph with edge weights $w : E \to \mathbb{R}$ (which may be positive, negative, or zero). Assume there are no negative length cycles in G.

   (a) How could we delete an arbitrary vertex v from graph G, without changing the shortest-path distance between any other pair of vertices? Describe and analyze an algorithm that constructs a directed graph $G' = (V', E')$ with edge weights $w' : E' \to \mathbb{R}$, where $V' = V \setminus \{v\}$, and the shortest-path distance between any two nodes in G' is equal to the shortest-path distance between two nodes in G. The algorithm should run in $O(V^2)$ time.

   (b) Now suppose we have already computed all shortest-path distances in G'. Describe and analyze an algorithm to compute the shortest-path distances from v to every other vertex, and from ever other vertex to v, in the original graph G. Again, the algorithm should run in $O(V^2)$ time.

   (c) Finally, combine parts (a) and (b) to describe and analyze another all-pairs shortest path algorithm. This algorithm should run in $O(V^3)$ time. (The resulting algorithm is *not* the same as Floyd-Warshall!)

2. Consider the directed graph $G = (V, E)$ below with non-negative capacities $c : E \to \mathbb{R}_{\geq 0}$ and an (s, t)-flow $f : E \to \mathbb{R}_{\geq 0}$ that is feasible with respect to c. Each edge is labeled with its flow/capacity.



An (s, t)-flow f. Each edge is labeled with its flow/capacity.

(a) Draw the residual graph $G_f = (V, E_f)$ for flow f. Be sure to label every edge of G_f with its residual capacity.

(b) Describe an augmenting path $s = v_0 \to v_1 \to \ldots \to v_r = t$ in G_f by either drawing the path in your residual graph or listing the path's vertices in order.

(c) Let $F = \min_i c_f(v_i \to v_{i+1})$ and let $f' : E \to \mathbb{R}_{\geq 0}$ be the flow obtained from f by pushing F units through your augmenting path. Draw a new copy of G, and label its edges with the flow values for f'. Is your new flow a maximum flow in G?

(d) For this last part, let $G = (V, E)$ be an arbitrary directed graph (not necessarily the one given above) with non-negative capacities $c : E \to \mathbb{R}_{\geq 0}$ on the edges and two special vertices s and t. Suppose we assign a non-negative ***limit*** $\ell : V \setminus \{s, t\} \to \mathbb{R}_{\geq 0}$ for the amount of flow that can pass through each vertex other than s or t. Formally, a flow $f : E \to \mathbb{R}_{\geq 0}$ is feasible *with respect to both c and ℓ* if for all edges $e \in E$ we have $f(e) \leq c(e)$ and for all vertices $v \in V \setminus \{s, t\}$ we have $\sum_u f(u \to v) \leq \ell(v)$.

Describe and analyze an algorithm to compute a graph $G' = (V', E')$ with non-negative edge capacities $c' : E' \to \mathbb{R}_{\geq 0}$ *but no vertex limits* so that the value of the maximum feasible flow in G' with respect to c' is equal to the value of the maximum feasible flow in G with respect to both c and ℓ.