Please answer the following 3 questions, some of which have multiple parts. If asked for an $O(T(n))$ time algorithm, you may give a randomized algorithm with expected time $O(T(n))$.

1. (From Mount) You are given three sets of points $R$, $G$, and $B$ (red, green, and blue) in $\mathbb{R}^2$. A tricolor strip is a pair of parallel lines such that the closed region bounded between these two lines contains exactly three points, one from each of $R$, $G$, and $B$. Define the strip’s height to be the vertical distance between these lines.

(a) Explain what a tricolor strip of height $h$ corresponds to in the dual plane.

(b) If a tricolor strip is of minimum height, what additional conditions must be satisfied? Explain briefly.

(c) Present an algorithm, which given inputs $R$, $G$, and $B$, computes the minimum height tricolor strip. Your algorithm should run in time $O(n^2 \log n)$, where $n = |R| + |G| + |B|$.
2. **(From Mount)** Let us consider a motion planning problem in the plane where the ground is the $x$-axis. Consider a robotic crane, whose base is anchored at the origin. The crane can stretch vertically up to any height $h \geq 0$ above the $x$-axis, and it can extend horizontally at its highest point to the right of the $y$-axis by any distance $w \geq 0$. There is a hook dangling down at a distance 1 from the tip of the crane. Defining the points $p_0 = (0, 0)$, $p_1 = (0, h)$, $p_2 = (w, h)$ and $p_3 = (w, h-1)$, we require that none of the three line segments $p_0p_1, p_1p_2, p_2p_3$ intersects any obstacles in the robot’s workspace.

Suppose that you are given a workspace consisting of $n$ disjoint axis-parallel rectangular obstacles $\mathcal{R} = \{R_1, \ldots, R_n\}$, where the $i$th rectangle is defined by its lower corner $r^-_i$ and its upper right corner $r^+_i$. You may assume that all these rectangles lie above the $x$-axis, but they may lie on either side of (or overlap) the $y$-axis.

(a) Given a rectangle $r^- = (x^-, y^-)$ and $r^+ = (x^+, y^+)$, describe the shape of the resulting C-obstacle in the $(w, h)$ configuration space of the crane. *[Hint: There will be a few cases depending on whether the rectangle lies to the left, right, or overlaps the $y$-axis.]*

(b) Given the set $\mathcal{R}$ of rectangles, and given starting and target configurations $s = (w_s, h_s)$ and $t = (w_t, h_t)$, sketch an algorithm for determining whether there is a collision-free motion of the crane between these configurations. Don’t worry about running time. *[Hint: A high-level sketch of the algorithm is sufficient. To make your life simpler, you may assume that you are given a procedure that will input the C-obstacles from part (a), compute the union of these C-obstacles, and return a convinient decomposition of free-space (e.g., as a trapezoidal map).]*
3. *(From 3Marks)*

(a) Let $S$ be a set of $n$ axis-parallel rectangles in the plane. We want to be able to report all rectangles in $S$ that are completely contained in a query rectangle $Q = [x_{lo}, x_{hi}] \times [y_{lo}, y_{hi}]$. Describe a data structure for this problem that uses $O(n \log^3 n)$ space and has $O(\log^3 n + k)$ query time, where $k$ is the number of reported rectangles.

*Hint: Transform the problem into some orthogonal range searching problem in a higher dimensional space. You may assume orthogonal range trees can support both points and ranges that include $-\infty$ or $+\infty$ in some components (so 1D range $[\infty, 5]$ would include all real numbers less than or equal to 5 and point $(2, +\infty)$ would lie higher than any bounded rectangular range in the plane).*

(b) Let $P$ be a set of $n$ points in the plane. We want to be able to report all points in $P$ that are completely contained in a query triangle. However, the triangle is guaranteed to have one horizontal edge, one vertical edge, and one edge of slope $-1$ or $+1$. Describe a data structure for this problem that uses $O(n \log^3 n)$ space and has $O(\log^3 n + k)$ query time, where $k$ is the number of reported points.