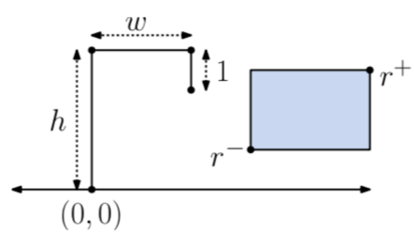
CS/SE 6301.008 Homework 4

Due Thursday April 5th at 11:30am, on eLearning

Please answer both questions. You may form groups of up to three students. Each group should write a single set of solutions with each group member's name and Net ID on the front page. Each group member should then submit a copy through eLearning.

1. **(From Mount)** Let us consider a motion planning problem in the plane where the ground is the x-axis. Consider a robotic crane, whose base is anchored at the origin. The crane can stretch vertically up to any height $h \geq 0$ above the x-axis, and it can extend horizontally at its highest point to the right of the y-axis by any distance $w \geq 0$. There is a hook dangling down at a distance 1 from the tip of the crane. Defining the points $p_0 = (0,0)$, $p_1 = (0,h)$, $p_2 = (w,h)$ and $p_3 = (w,h-1)$, we require that non of the three line segments $\overline{p_0p_1}, \overline{p_1p_2}, \overline{p_2p_3}$ intersects any obstacles in the robot's workspace.



(From Mount). The robotic crane.

Suppose that you are given a workspace consisting of n disjoint axis-parallel rectangular obstacles $\mathscr{R} = \{R_1, \ldots, R_n\}$, where the ith rectangle is defined by its lower corner r_i^- and its upper right corner r_i^+. You may assume that all these rectangles lie above the x-axis, but they may lie on either side of (or overlap) the y-axis.

(a) Given a rectangle $r^- = (x^-, y^-)$ and $r^+ = (x^+, y^+)$, describe the shape of the resulting C-obstacle in the (w,h) configuration space of the crane. *[Hint: There will be a few cases depending on whether the rectangle lies to the left, right, or overlaps the y-axis.]*

(b) Given the set $\mathscr{R}$ of rectangles, and given starting and target configurations $s = (w_s, h_s)$ and $t = (w_t, h_t)$, sketch an algorithm for determining whether there is a collision-free motion of the crane between these configurations. Don't worry about running time. *[Hint: A high-level sketch of the algorithm if sufficient. To make your life simpler, you may assume that you are given a procedure that will input the C-obstacles from part (a), compute the union of these C-obstacles, and return a convinient decomposition of free-space (e.g., as a trapezoidal map).]*

2. Recall from the lecture on Fréchet distance, a ***reparameterization*** is a continuous and bijection function $\alpha : [0,1] \to [0,1]$ where $\alpha(0) = 0$ and $\alpha(1) = 1$. Given two (polygonal) curves in the plane $P : [0,1] \to \mathbb{R}^2$ and $Q : [0,1] \to \mathbb{R}^2$, we defined the ***Fréchet distance*** as

$$Fr(P,Q) := \inf_{\alpha,\beta} \max_{t \in [0,1]} ||P(\alpha(t))\, Q(\beta(t))||$$

where α and β are taken from the set of reparameterizations.

Consider the following variant of Fréchet distance we'll call the ***wandering Fréchet distance***. It is defined as

$$wFr(P,Q) := \inf_{\gamma,\zeta} \max_{t \in [0,1]} ||P(\gamma(t))\, Q(\zeta(t))||$$

where $\gamma : [0,1] \to [0,1]$ and $\zeta : [0,1] \to [0,1]$ are taken from the set of all continuous and *surjective* functions with $\gamma(0) = \zeta(0) = 0$ and $\gamma(1) = \zeta(1) = 1$. In other words, a person on P and a dog on Q are allowed to wander back and forth along their curves as much as they'd like as long as they both start at the beginning and finish at the ending of their respective curves. We still want to minimize the length of their leash. The decision version of the wandering Fréchet distance problem is to decide, given P, Q, and r, whether or not $wFr(P,Q) \leq r$.

(a) Recall the free-space diagram used for computing the decision version of regular Fréchet distance. Reparameterizations α and β corresponded to monotone paths in the free-space diagram. What do valid functions γ and ζ as used in the wandering Fréchet distance definition correspond to in the free-space diagram?

(b) Given polygonal curves $P = \langle p_1, p_2, \ldots, p_m \rangle$ and $Q = \langle q_1, q_2, \ldots, p_n \rangle$ as well as a value $r \geq 0$, sketch an algorithm for determining if $wFr(P,Q) \leq r$ and give its running time. You may assume the free-space diagram can be computed in $O(mn)$ time and afterward you have access to all cells $\square_{ij}$ as determined by P and Q along with each cell's respective free space intervals.