Please solve the following 2 problems, some of which have multiple parts.

1. Let us consider a motion planning problem in the plane where the ground is the x-axis. Consider a robotic crane, whose base is anchored at the origin. The crane can stretch vertically up to any height \( h \geq 0 \) above the x-axis, and it can extend horizontally at its highest point to the right of the y-axis by any distance \( w \geq 0 \). There is a hook dangling down at a distance 1 from the tip of the crane. Defining the points \( p_0 = (0, 0) \), \( p_1 = (0, h) \), \( p_2 = (w, h) \) and \( p_3 = (w, h-1) \), we require that none of the three line segments \( p_0p_1, p_1p_2, p_2p_3 \) intersects any obstacles in the robot’s workspace.

![Figure 1. The robotic crane.](image)

Suppose you are given a workspace with a single axis-parallel regular obstacle with lower left corner \( r^- = (x^-, y^-) \) and upper right corner \( r^+ = (x^+, y^+) \). You may assume the rectangle lies above the x-axis, but it may lie either side of (or overlap) the y-axis. Describe the shape of the resulting C-obstacle in the \((w, h)\) configuration space of the crane.

Advice: There will be a few cases depending on whether the rectangle lies to the left, right, or overlaps the y-axis.

2. (a) Let \( S \) be a set of \( n \) axis-parallel rectangles in the plane. We want to be able to report all rectangles in \( S \) that are completely contained in a query rectangle \( Q = [x_{lo}, x_{hi}] \times [y_{lo}, y_{hi}] \). Describe a data structure for this problem that uses \( O(n \log^3 n) \) space and has \( O(\log^3 n + k) \) query time, where \( k \) is the number of reported rectangles. [Hint: Transform the problem into some orthogonal range searching problem in a higher dimensional space. You may assume orthogonal range trees can support both points and ranges that include \(-\infty\) or \(+\infty\) in some components (so 1D range \([-\infty, 5]\) would include all real numbers less than or equal to 5 and point \((2, +\infty)\) would lie higher than any bounded rectangular range in the plane).]
(b) Let $P$ be a set of $n$ points in the plane. We want to be able to report all points in $P$ that are completely contained in a query triangle. Fortunately, the triangle is guaranteed to have one horizontal edge, one vertical edge, and one edge of slope $-1$ or $+1$. Describe a data structure for this problem that uses $O(n \log^3 n)$ space and has $O(\log^2 n + k)$ query time, where $k$ is the number of reported points.

Extra Credit: Describe a data structure that uses $O(n \log^2 n)$ space and has $O(\log n + k)$ query time instead.