HW 1 Available

due Fri, Feb. 12th

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Mondays 12pm - 2pm

```
QuickSort(A[1..n]):
    if (n > 1)
        Choose a pivot element A[p]
        r ← Partition(A, p)
        QuickSort(A[1..r − 1])      ⟨⟨Recurse!⟩⟩
        QuickSort(A[r + 1..n])      ⟨⟨Recurse!⟩⟩
```

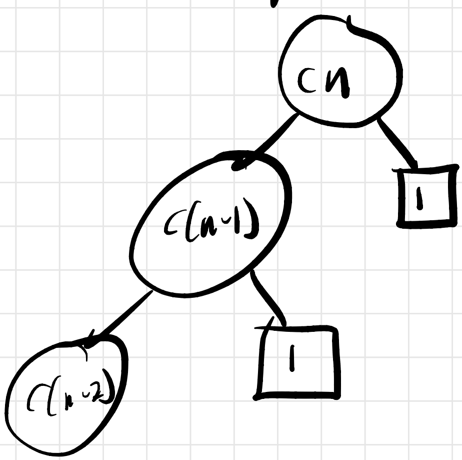
```
Partition(A[1..n], p):
    swap A[p] ↔ A[n]
    ℓ ← 0                  ⟨⟨#items < pivot⟩⟩
    for i ← 1 to n − 1
        if A[i] < A[n]
            ℓ ← ℓ + 1
            swap A[ℓ] ↔ A[i]
    swap A[n] ↔ A[ℓ + 1]
    return ℓ + 1
```

$T(n)$: Time to sort $A[1..n]$.

$$T(n) = \max_{1 \le r \le n} \{T(r-1) + T(n-r)\} + \Theta(n)$$

r: rank of pivot element

(index of pivot sorted order)



$\Sigma \le cn$

$\Sigma \le c(n-1)$

$\Sigma \le c(n-2)$

$\Sigma \le c(n-3)$

At most n levels.

So $T(n) \leq O(n) \cdot n = O(n^2)$

Better would be pivoting on <u>median</u> (rank $\lceil n/2 \rceil$ for this class)

$$T(n) = T(\lceil n/2 \rceil - 1) + T(\lfloor n/2 \rfloor) + \Theta(n)$$

$$\leq 2T(n/2) + \Theta(n)$$

$$= \Theta(n \log n) \quad (\text{if } r = \lceil n/2 \rceil)$$

If larger subproblem had size $\leq 2n/3$.

$\Rightarrow \log_{3/2} n$ levels in recursion tree

$\Rightarrow T(n) = O(n \log n)$

Element Selection

Given an array $A[1..n]$ (may not be sorted) + integer k s.t. $1 \le k \le n$.

Goal: Return the element of rank k. (kth smallest)

Quickselect [Hoare]

```
QuickSelect(A[1..n], k):
  if n = 1
      return A[1]
  else
      Choose a pivot element A[p]
      r ← Partition(A[1..n], p)
      if k < r
          return QuickSelect(A[1..r−1], k)
      else if k > r
          return QuickSelect(A[r+1..n], k−r)
      else
          return A[r]
```

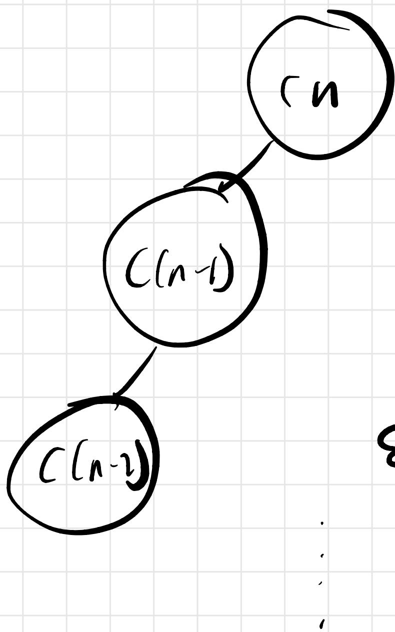
rank

$k-r$ ↓ element of second call's array is

$(k-r+r) = (k)$th of $A[1..n]$

worst-case

$$T(n) = \max_{1 \le r \le n} \max\{T(r-1), T(n-r)\} + \Theta(n)$$



$\Sigma = cn$

$\Sigma = c(n-1)$

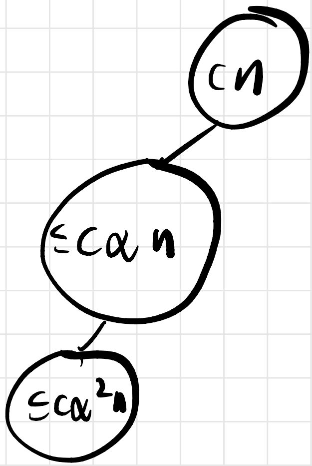
$\Sigma = c(n-2)$

$\le n$ levels

$T(n) = O(n^2)$

Suppose we always recurse on $\le \alpha n$ elements for some $\alpha < 1$.

$$T(n) \le T(\alpha n) + \Theta(n) = \Theta(n)$$



$\Sigma = cn$

$\Sigma = c\alpha n$

$\Sigma \le c\alpha^2 n$

We'd like an Approximate Median Fairy to find our pivot.

Blum et al. ['7x]

AMF using RF!

Idea: Partition A into $\lceil n/5 \rceil$ blocks of size 5.

- compute median of each block via "brute force"
- use recursion to find the median of these medians for our pivot

```
MomSelect(A[1..n], k):
  if n ≤ 25 ⟨⟨or whatever⟩⟩
        use brute force
  else
        m ← ⌈n/5⌉
        for i ← 1 to m
              M[i] ← MedianOfFive(A[5i − 4..5i]) ⟨⟨Brute force!⟩⟩
        mom ← MomSelect(M[1..m], ⌊m/2⌋) ⟨⟨Recursion!⟩⟩

        for j ← 1 to n ⟨⟨Find pivot index.⟩⟩
              if A[j] = mom
                    p ← j
        r ← Partition(A[1..n], p)

        if k < r
              return MomSelect(A[1..r − 1], k) ⟨⟨Recursion!⟩⟩
        else if k > r
              return MomSelect(A[r + 1..n], k − r) ⟨⟨Recursion!⟩⟩
        else
              return mom
```

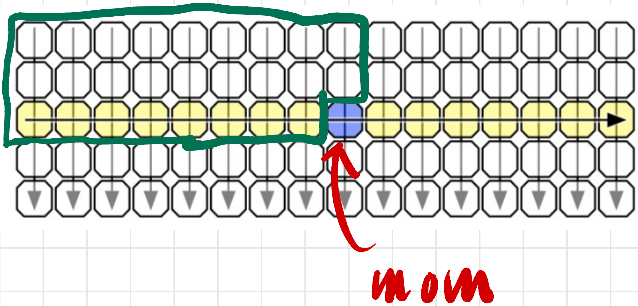
"median of medians"

Imagine...

put elements of A in a

$5 \times \lceil n/5 \rceil$ grid

- sort each column top-down
- sort columns by their median elements



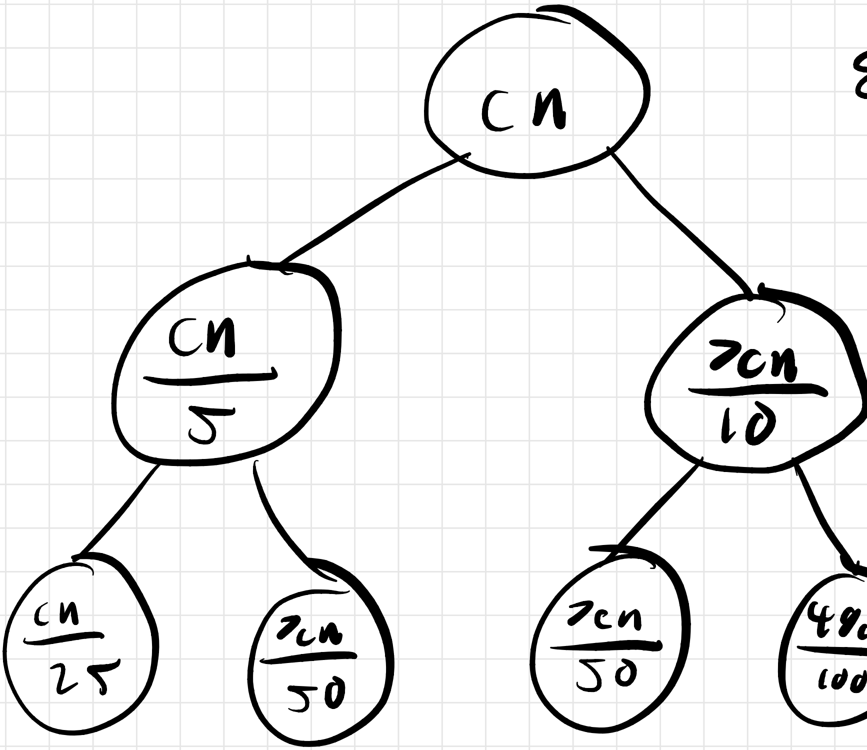
Suppose element of rank k is $>$ mom

$\sim (n/5)/2 = n/10$ medians $<$ mom

3 elements per column $\leq$ the column median

so $\geq \sim \frac{3n}{10}$ elements $<$ mom

$\Rightarrow \leq \frac{7n}{10}$ elements $>$ mom so... $T(n) \le T(n/5) + T(7n/10) + \Theta(n)$



$\Sigma = cn$

$\Sigma \le \frac{9cn}{10}$

$\Sigma \le \frac{81cn}{100}$

$(9/10)^i cn$

so $T(n) \le \Theta(n)$

If we use 3...

$$T(n) \leq T(n/3) + T(2n/3) + \Theta(n)$$

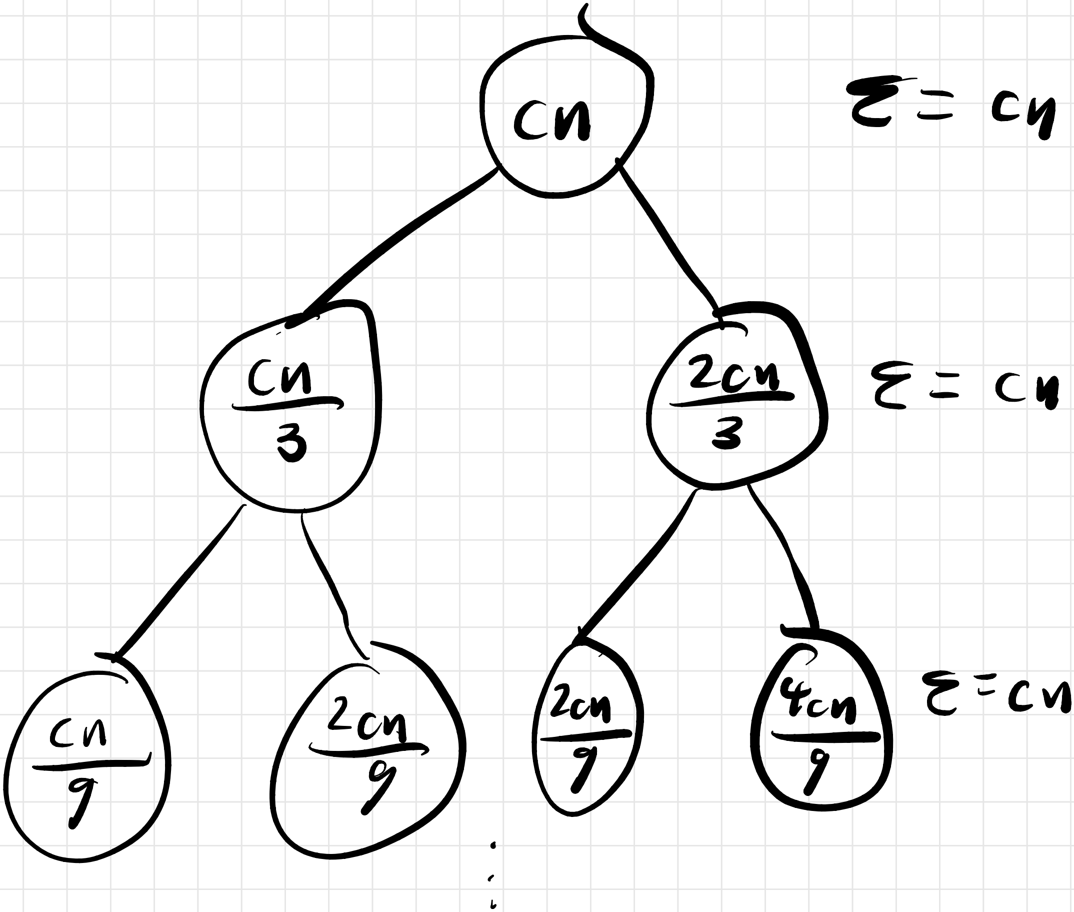
$$= \Theta(n \log n)$$

not really practical :

in practice, use

a <u>random</u> pivot

$$T(n) \leq T(n/3) + T(2n/3) + \Theta(n)$$



$$T(n) \leq cn \log_{3/2} n = O(n \log n)$$

$$T(n) \geq cn \log_3 n = \Omega(n \log n)$$

$$T(n) = \Theta(n \log n)$$

$= n/6$

$\frac{1}{2} \cdot \frac{n}{3}$ hessors medians

2 smaller elements per ~~column~~ column

so $\geq 2 \cdot \frac{n}{6} = \frac{n}{3}$ elements smaller than mom

$\Rightarrow \leq \frac{2n}{3}$ larger elements