

CS 6363.500 Homework 5

Due Tuesday April 30th on eLearning

April 17, 2019

Please answer each of the following questions.

1. A given flow network G may have more than one minimum (s, t) -cut. Let's define the **best** minimum (s, t) -cut to be any minimum cut (S, T) with the smallest number of edges crossing from S to T .

Describe an efficient algorithm to find the best minimum (s, t) -cut. You may assume the capacities are integers. [Hint: Change the capacities.]

2. The Academic Senate at the University of Texas at Dallas has decided to convene a committee to decide on the official ice cream flavor of the university. Exactly one faculty member must be chosen from each academic department to serve on this committee. Some faculty members have appointments in multiple departments, but each committee member can represent only one department. For example, if Prof. Turing is affiliated with both the Department of Computer Science and the Department of Mathematics, and they are chosen as the Compute Science representative, then someone else must represent Mathematics. Finally, University policy requires that every faculty committee must contain exactly the same number of assistant professors, associate professors, and full professors. Fortunately, the number of departments is a multiple of 3.

Describe and analyze an algorithm to choose a subset of UTD faculty to staff the ice cream flavor committee or correctly report that no valid committee is possible. Your input is a bipartite graph indicating which professors belong to which departments; each professor is labeled with that professor's rank (assistant, associate, or full). Your running time should be expressed in terms of the number of faculty members n and the number of departments k . [Hint: Reduce to computing a maximum (s, t) -flow. The flow network will likely have more vertices than the input graph describing department membership.]

3. Suppose you are given a magic black box that can determine **in polynomial time**, given an arbitrary graph G , the number of vertices in the largest complete subgraph of G . Describe and analyze a **polynomial time** algorithm that computes, given an arbitrary graph G , a complete subgraph of G of maximum size, using this magic block box as a subroutine.
4. Prove the following problems are NP-hard. You may assume NP-hardness of CIRCUITSAT, 3SAT, MAXINDSET, MAXCLIQUE, MINVERTEXCOVER, 3COLOR, HAMILTONIANCYCLE (in directed or undirected graphs), HAMILTONIANPATH (in directed or undirected graphs), and SUBSETSUM. (You *do not* necessarily need to understand the NP-hardness reductions to those problems to start work on this question.)

- (a) Given an undirected graph G , does G contain a simple path that visits all but 17 vertices?
- (b) Given an undirected graph G , can we color each vertex of G with one of 19 colors so that every edge touches two different colors?
5. Let $G = (V, E)$ be a graph. A *dominating set* in G is a subset S of the vertices such that every vertex in G is either in S or adjacent to a vertex in S . The DOMINATINGSET problem asks, given a graph G , for a dominating set of minimum size. Prove that this problem is NP-hard. You may assume NP-hardness of CIRCUITSAT, 3SAT, MAXINDSET, MAXCLIQUE, MINVERTEXCOVER, 3COLOR, HAMILTONIANCYCLE (in directed or undirected graphs), HAMILTONIANPATH (in directed or undirected graphs), and SUBSETSUM.