

Computer-Aided Generation of Mandala Thangka Patterns

Jiajing Zhang

The State Key Lab of CAD&CG
Zhejiang University
Hangzhou, 310058, China
stou@zju.edu.cn

Ren Peng

Department of Computer Science and Technology
Zhejiang University
Hangzhou, 310027, China
pengren@zju.edu.cn

Kang Zhang

Department of Computer Science
The University of Texas at Dallas
Richardson, Texas 75082, USA
kzhang@utdallas.edu

Jinhui Yu

The State Key Lab of CAD&CG
Zhejiang University
Hangzhou, 310058, China
jhyu@cad.zju.edu.cn

ABSTRACT

The mandala thangka, as a religious art in Tibetan Buddhism, is an invaluable cultural and artistic heritage. However, drawing a mandala is both time and effort consuming and requires mastery skills due to its intricate details. Retaining and digitizing this heritage is an unresolved research challenge to date. In this paper, we propose a computer-aided generation approach of mandala thangka patterns to address this issue. Specifically, we construct parameterized models of three stylistic patterns used in the interior mandalas of Nyingma school in Tibetan Buddhism according to their geometric features, namely the star, crescent and lotus flower patterns. Varieties of interior mandalas are successfully generated using these proposed patterns based on the hierarchical structures observed from hand drawn mandalas. The experimental results show that our approach can efficiently generate beautifully-layered colorful interior mandalas, which significantly reduces the time and efforts in manual production and, more importantly, contributes to the digitization of this great heritage.

CCS CONCEPTS

• **Computing methodologies** → **Shape analysis**; *Model development and analysis*; • **Applied computing** → **Fine arts**;

KEYWORDS

Mandala thangka, Computer-aided generation, Geometric features, Parameterized models, Hierarchical structure

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions@acm.org.

VINCI '17, August 14-16, 2017, Bangkok, Thailand

© 2017 Association for Computing Machinery.

ACM ISBN 978-1-4503-5292-5/17/08...\$15.00

<https://doi.org/10.1145/3105971.3105974>



Figure 1: Examples of mandala thangka patterns. The images are scanned from the published book [28].

ACM Reference format:

Jiajing Zhang, Kang Zhang, Ren Peng, and Jinhui Yu. 2017. Computer-Aided Generation of Mandala Thangka Patterns. In *Proceedings of VINCI '17, Bangkok, Thailand, August 14-16, 2017*, 8 pages.

<https://doi.org/10.1145/3105971.3105974>

1 INTRODUCTION

The thangka, or scroll painting, usually depicting a Buddhist deity, scene, or mandala, is a special art of Tibetan Buddhism. As a religious art, thangka retains high cultural and artistic values. In 2006 the Tibetan Thangka was recorded in the first list of national intangible cultural heritages in China and, in 2009, Regong Arts was inscribed on the Representative List of the Intangible Cultural Heritage of Humanity by the UNESCO. Mandala is an important subject in thangka which is a spiritual and ritual symbol representing the universe [14, 22]. In various spiritual traditions, mandalas may be employed as a spiritual guidance tool for focusing attention of practitioners and for mediation. In addition to its religious significance, mandalas have favorable psychological effects, as noted in [12].

In its most common form, a mandala appears as a series of concentric circles. It depicts deities which are enclosed in the square structure situated concentrically within these circles, as shown in Figure 1 [28]. Drawing a mandala is both time and effort consuming, since it first needs to be sketched onto the canvas in the right proportions following the ancient grid

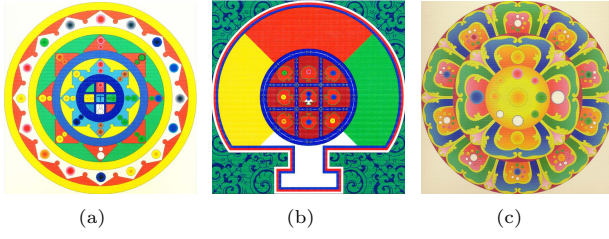


Figure 2: Examples of interior mandala patterns: (a) Star pattern; (b) Crescent pattern; (c) Lotus flower pattern. The images are scanned and cropped from the published book [28].

patterns, after which the long painting procedure starts, with fine brushes. The time required to finish drawing a mandala usually ranges from one month to several months, depending on the complexity of the mandala.

Mandalas on one hand are beautiful and intricate, on the other hand are highly geometric and regular, thus it is possible to model mandala mathematically. The modelled mandalas can be further applied to multiple areas: structural analysis in image analysis and retrieval; modern artistic pattern and card design; mental clarity and spiritual growth in psychology.

In Buddhism there are more than 700 deities, each having his or her own mandala. Modeling a large number of mandalas is beyond the scope of this work. We focus on modeling the central part, called *the interior mandala*, which is the most important part in any mandala. Specifically, in this paper we propose a computer-aided generation approach by modeling three stylistic patterns used in the interior mandalas of Nyingma school in Tibetan Buddhism [28]: star (Figure 2(a)), crescent (Figure 2(b)) and lotus flower patterns (Figure 2(c)). The key idea of our method consists of two steps: first, we construct models of parameterized motifs according to the geometric features in each pattern; second, we place these parameterized motifs in a hierarchical structure of concentric circles outside-in to generate final interior mandalas, where the parameter values are determined by a uniformly-spaced reference grid. Comparative results show that our approach can efficiently generate beautifully-layered colorful interior mandalas comparable with the hand drawn mandalas.

To our knowledge, this study is the first attempt in constructing parameterized models of interior mandala patterns. Our work provides a reference for computer-aided generation of both traditional mandala patterns used in Buddhism and mandala-like patterns used in modern art design, which also contributes to the digitization of this great heritage.

2 RELATED WORK

Previous works in computer generation of traditional art patterns can be classified mainly into the following

categories: Islamic geometric pattern, Indian kolam pattern, Chinese paper-cut pattern and Uyghur fabric pattern.

Islamic geometric patterns are built on stars, squares and circles, typically repeated, overlapped and interlaced to form the overall intricate connected patterns. Earlier works [9, 15, 21] used symmetry groups to analyze forms of organization and structures of islamic star patterns. The strap work approach to create shapes which were formed by straightedge and compasses was used in [26, 36]. By using polygonal network as the base, the polygons-in-contact technique was adopted to create a variety of islamic geometric patterns [3, 19, 20]. The modular design system based on the star, cross and traditional islamic pattern modules was introduced in [1, 4–6] to generate different family of geometric star and rosette patterns. Based on shape rules, [2, 10, 18, 33, 34] applied the parametric shape grammar approach to generating islamic geometric patterns.

Indian kolam patterns consist of an symmetrical matrix of dots and curving lines which wind around the dots on the geometry. A tiling-based approach using diamond-shaped tiles placed corner to corner to construct square loop kolam was presented in [31]. Gopalan et al. [13] proposed a topological method which can generate all possible kolams for any spatial configuration of dots. Other approaches include encoding kolams using graph, picture and array grammars [7, 8, 30], converting Kolam patterns into numbers and linear diagrams [39], L- and P-systems [35], extended pasting schemes [32], gestural lexicons [37], stroke chain-code [27] and knot theory [17].

For Chinese paper-cut patterns and Uyghur fabric patterns, Liu et al. [25] studied the cyclic and dihedral symmetries of different annuli in paper-cut designs, and synthesized new designs with different rotational orders. Li et al. [24] designed a set of tools for annotating animated 3D surfaces with holes derived from traditional paper cutting motifs. By using independent patterns as basis, a library of complex paper-cut patterns was established in [23]. Zhao et al. [42] proposed a method of automatic generation of Uyghur fabric patterns based on configuration styles such as hexagonal, brick-shaped and diamond structure tile.

For other traditional art patterns, Persian floral patterns were created by using NURBS and circle packing [11, 16]. Yoon et al. [40] constructed Korean Danchong patterns based on triangular, hexagon and circle. Zhang and Yu [41] generated Kandinsky art based on styled patterns. The Miro style of surrealism was generated by extracting and coding pictorial elements in Miro's paintings [38].

Patterns in the interior mandalas of Buddhism differ from aforementioned patterns both in geometric shape and composition structure. Poelke et al. [29] created mandala like patterns by creating polynomials with L-symmetric zero set and applying the classical Schwarz reflection principle, the resultant patterns however differ dramatically from those in the interior mandala patterns both in shape variation regularity and hierarchical structure. It is therefore necessary to construct parameterized motifs according to geometric

features in each pattern and place them in a hierarchical structure to generate different interior mandalas.

3 STAR PATTERNS

Star patterns are widely used in interior mandalas, which may contain 4, 8 or 16 isosceles triangles inscribed in a circle (red lines in Figure 3(a)), with their vertices pointing outward. Two adjacent isosceles triangles are connected by a small circular arc called *the connecting arc* (blue curves in Figure 3(a)).

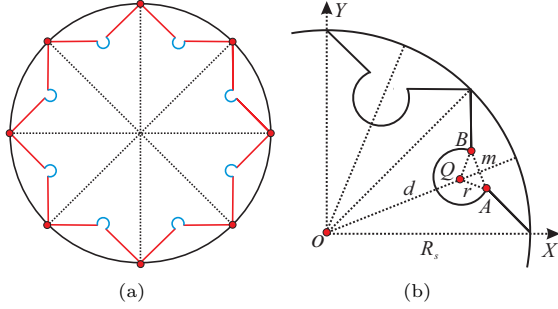


Figure 3: A star pattern: (a) Basic structure; (b) Geometric model.

To model the star pattern, we first place a circumscribed circle centered at the origin O and set its radius as R_s , and then calculate the coordinates (V_{ix}, V_{iy}) of the isosceles triangle's vertex V_i (red points in Figure 3(a)) with the following formulae:

$$\begin{cases} V_{ix} = R_s \cdot \cos((i-1) \cdot 2\pi/n_s) \\ V_{iy} = R_s \cdot \sin((i-1) \cdot 2\pi/n_s) \end{cases} \quad (1)$$

where $i \in [1, n_s]$, and n_s is the number of isosceles triangles.

Assume that the distance between O and the connecting arc's center Q is $d = |OQ|$, the coordinates (Q_x, Q_y) of Q are calculated as:

$$\begin{cases} Q_x = d \cdot \cos(\pi/n_s) \\ Q_y = d \cdot \sin(\pi/n_s) \end{cases} \quad (2)$$

As indicated in Figure 3(b), the altitude of an isosceles triangle can be increased or decreased when d is decreased or increased, thus we can change the shape of isosceles triangles by adjusting d .

Next we determine the size of connecting arcs and their end points coinciding with end points of the base in two adjacent isosceles triangles. To clearly illustrate, we enlarge the connecting arcs in Figure 3(b). Setting the arc radius as r and chord length $|AB| = m$, the deviation angle from \overline{OQ} to \overline{QA} is calculated as $\alpha = \arcsin(m/2r)$.

The connecting arc \widehat{AB} can be defined by the following equations:

$$\begin{cases} \widehat{AB}_x(\theta) = Q_x + r \cdot \cos(\theta + \pi/n_s) \\ \widehat{AB}_y(\theta) = Q_y + r \cdot \sin(\theta + \pi/n_s) \end{cases} \quad (3)$$

where $\theta \in [\alpha, 2\pi - \alpha]$.

Once the arc \widehat{AB} is obtained, we simply rotate \widehat{AB} around O with $2\pi/n_s$ to produce $n_s - 1$ arcs so that all adjacent isosceles triangles can be connected by those arcs to form the final star pattern. Also, by tuning parameters R_s , n_s , d , r and m , variants of star patterns can be obtained, as shown in Figure 4.

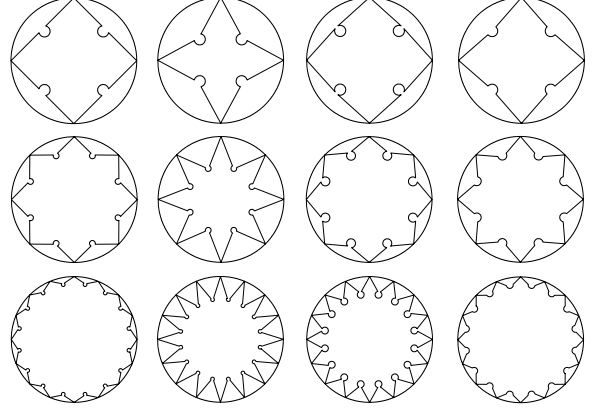


Figure 4: Variants of star patterns.

4 CRESCENT PATTERNS

Crescent pattern is usually used in mandalas for dakinis. It consists of a relatively large arc (red curve in Figure 5(a)) and an inverted T-shaped structure at the bottom (blue line in Figure 5(a)).

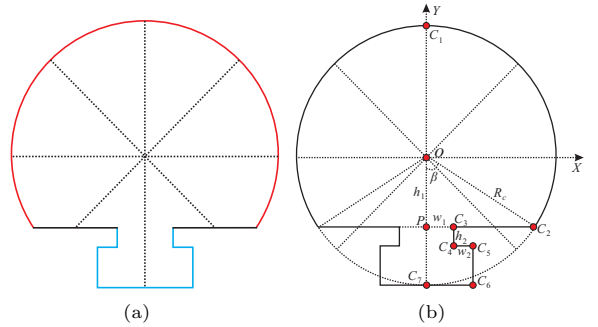


Figure 5: A crescent pattern: (a) Basic structure; (b) Key points in the geometric model.

We label several key points on the large arc and the T-shaped structure with $C_1 \sim C_7$, as shown in Figure 5(b). Assume that the arc center is at the origin O and the arc radius is R_c , the distance from O to the chord is $|OP| = h_1$, as shown in Figure 5(b), the deviation angle from \overline{OP} to \overline{OC}_2 is calculated as $\beta = \arccos(h_1/R_c)$.

We set the following two parameters $w_1 = |PC_3|$ and $h_2 = |C_3C_4|$, with which the width and height of the vertical rectangle in the T-shaped structure can be determined. The

width of the protruded horizontal rectangle in the T-shaped structure is determined by $w_2 = |C_4C_5|$. All key points can be determined by the formulae given in Table 1.

Table 1: Coordinates of key points in the crescent pattern.

Key points	Coordinates of key points
$\widehat{C_1C_2}(\theta)$	$(R_c \sin(\theta), R_c \cos(\theta)), \theta \in [\beta, \pi]$
C_3	$(w_1, R_c \cos(\beta))$
C_4	$(w_1, R_c \cos(\beta) + h_2)$
C_5	$(w_1 + w_2, R_c \cdot \cos(\beta) + h_2)$
C_6	$(w_1 + w_2, R_c)$
C_7	$(0, R_c)$

We connect $\widehat{C_1C_2}$, C_3 , C_4 , C_5 , C_6 and C_7 to generate the right part of the crescent pattern, and then flip it over horizontally to obtain the left part of the pattern. Moreover, variants of crescent patterns can be generated by tuning parameters R_c , h_1 , h_2 , w_1 and w_2 , as shown in Figure 6.

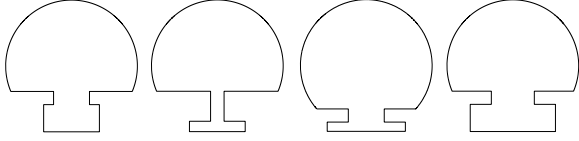


Figure 6: Variants of crescent patterns.

5 LOTUS FLOWER PATTERNS

Lotus flower patterns are used in mandalas for deities in the lotus flower family. They may contain 4, 8, 10 or 16 petal patterns from the inner core to the outer rings (Figure 2(c)). Here we take the 4-petal pattern as an example to describe how to model the lotus flower patterns. The model is divided into skeletal and shape levels: the skeletal pattern globally defines the structure of the entire petal pattern and locally defines petal contours, as shown by Figure 7(a), where each petal pattern has an outer line depicting petal's contour (red curves) and an internal line depicting petal's texture (blue curves). Decorative shapes are then added inside the outer and internal lines in the skeletal pattern to form the final petal pattern, as indicated by red and blue curves in Figure 7(b), respectively. Small decorative shapes are finally added between every two adjacent petal patterns (green curves in Figure 7(b)). Next, we describe the construction of the skeletal patterns and decorative shapes in detail.

5.1 Skeletal patterns

In the 4-petal pattern, a petal is restricted to a quarter circle in a unit circle, as shown in Figure 8(a). Since the petal

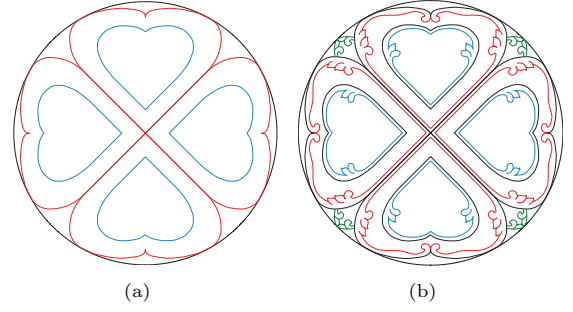


Figure 7: Model of a lotus flower pattern: (a) Skeletal patterns; (b) Decorative shapes.

shape is symmetric about its central axis, we can model the right half of the petal by first specifying 9 control points as the original set of control points (red points in Figure 8(b)) and then interpolating those points with a B-spline. The positions of 9 control points are defined with polar coordinates. We use a three dimensional array to store the radius r_i , the angle θ_i and the relative angle $\Delta_i = |\theta_{ref} - \theta_i|$, where $i = 1, 2, \dots, 9$, and θ_{ref} is the angular coordinate of a reference control point taken from the given control points, in this example we take the 9th point as the reference point. Using a relative angle allows us to flip control points about the given axis and control the petal width with great ease.

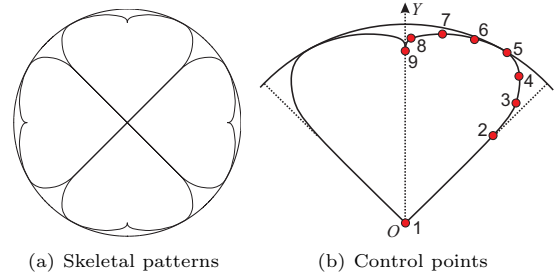


Figure 8: Construction of 4-petal skeletal patterns.

The left half of the petal can be obtained by flipping all control points about petal's central axis, i.e., the line goes through the center O and the 9th control point, as shown in Figure 8(b). This can be achieved simply by adding angular coordinate of the petal's central axis to Δ_i to obtain the angular coordinates of control points flipped over, and keeping the radius coordinates of control points flipped over unchanged. For the petals in the remaining 3 quarter circles, we repeat the above procedure to obtain a complete 4-petal pattern.

For patterns with $n_p > 4$ petals, we modify relative angles Δ_i in the original control point set by $\Delta'_i = \Delta_i * 4/n_p$, a new set of control points with polar coordinates $(r_i, \theta_{ref} - \Delta'_i)$ define petal's shape which is narrower than that defined with the original set of control points. We can use this new set of

control points to generate the patterns with n_p petals in a similar manner. Figure 9(a)-9(c) show skeletal patterns with 8, 10 and 16 petals.

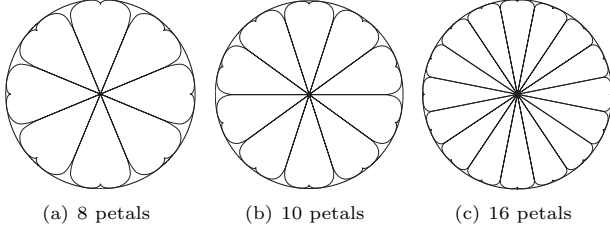


Figure 9: Skeletal patterns with different number of petals (please zoom in to see details).

5.2 Decorative shapes

Skeletal patterns described in the previous section define the structure of lotus flower patterns. Decorative shapes need to be added inside the skeletal patterns to obtain the final patterns. As shown in Figure 10(a), decorative shapes are irregular thus can be modeled using splines. Again, we take the 4-petal pattern as an example to demonstrate how to add decorative shapes onto skeletal patterns. The right half of the decorative shape is divided into 4 segments, as the purple, green, blue and orange curves in the right half of the quarter circle, as indicated by the red rectangle in Figure 10(a) and its enlarged portion in Figure 10(b). We specify 31 control points all together for those 4 segments, and then proceed to interpolate those points associated with each segment with the B-spline to generate corresponding decorative shapes. We take the 31th point as the reference point and repeat the same procedure described in Section 5.1 to obtain decorative shapes in the remaining 3 quarter circles. Figure 11(a)-11(c) show decorative shapes with 8, 10 and 16 petals.

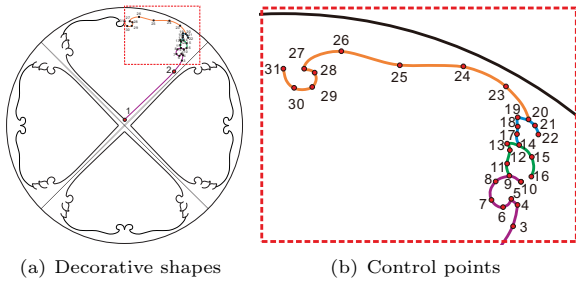


Figure 10: Construction of 4-petal decorative shapes.

5.3 Complete lotus flower patterns

In addition to patterns in Section 5.1 and 5.2, there are other patterns in the local parts of the lotus flower pattern, such as the internal pattern in the petal with a heart shape,

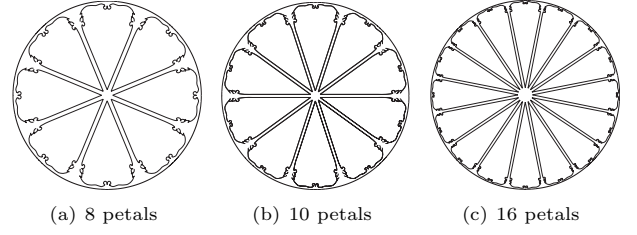


Figure 11: Decorative shapes with different number of petals (please zoom in to see details).

and small decorative shapes between every two adjacent petals near the outer part of the entire pattern. They are all generated with B-splines that interpolate control points specified, we omit the trivial description of their construction.

With all models constructed for different patterns in the lotus flower patterns, we can generate a complete lotus flower pattern with the following procedure. First, we specify n_p as the petals number in the pattern, and then divide the unit circle into n_p sectors. In the first sector, we proceed to generate the skeletal patterns for both outer and inner patterns in the petal and add decorative shapes inside skeletal patterns to obtain a complete petal pattern. The petal patterns in the remaining sectors of the unit circle can be generated by rotating the petal pattern in the first sector around the center with $2\pi/n_p$. Finally we add small decorative shapes between every two adjacent petals to obtain a complete lotus flower pattern.

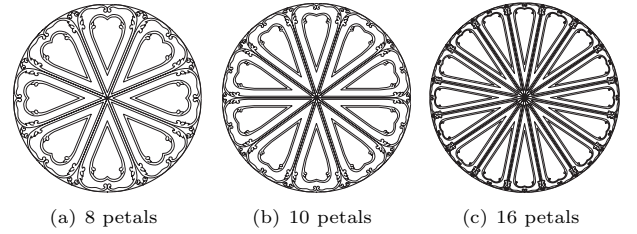


Figure 12: Lotus flower patterns with different number of petals (please zoom in to see details).

By changing n_p , lotus flower patterns with different petals can be generated, as shown in Figure 12. Note that when n_p increases, the petal width decreases correspondingly, which requires decrease of small decorative shapes between every two adjacent petals in both width and height, so that they can be added at the outer part of petals. Since each small decorative shape spans from the 4th control point of its left to right petals in the outer skeletal pattern (Section 5.1), it also touches a vertex shared by two neighboring sectors on the unit circle. We take the triangle determined by those three points as a reference triangle and define control points of small decorative shapes inside the reference triangle

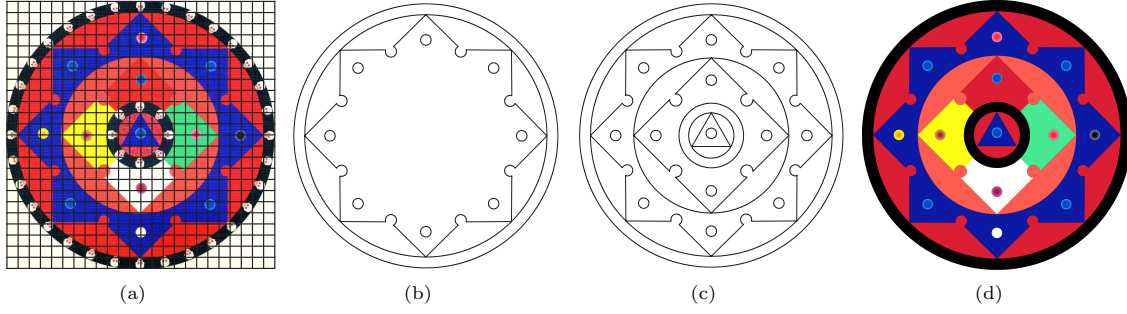


Figure 13: Generation process of central part of Black Yamaraja mandala (please zoom in to see details): (a) Hand drawn mandala with a grid overlayed; (b) 1st and 2nd ring; (c) 3rd~5th ring; (d) Colored mandala.

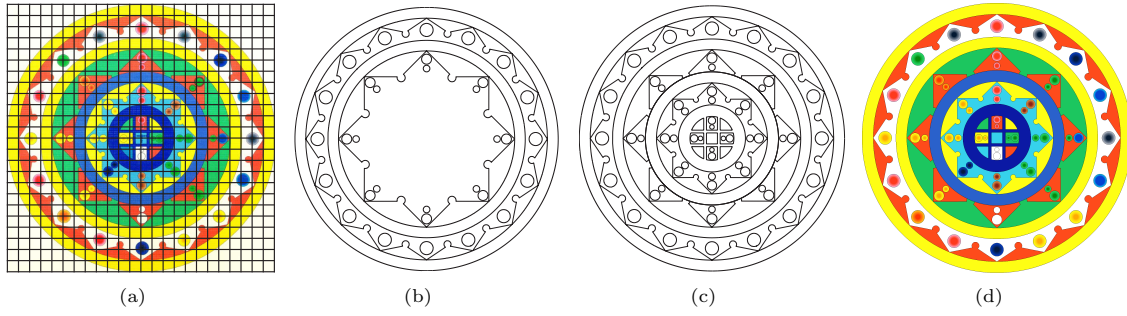


Figure 14: Generation process of central part of Zhitro Narag Dongprug mandala (please zoom in to see details): (a) Hand drawn mandala with a grid overlayed; (b) 1st~4th ring; (c) 5th~8th ring; (d) Colored mandala.

using local coordinates, thus, small decorative shapes change accordingly when the petal's width changes.

Since the lotus flower pattern is restricted to a unit circle, we can scale up the radial coordinates of all control points in the entire pattern by R_f to obtain the lotus flower pattern within the circumscribed circle of radius R_f .

6 GENERATION OF INTERIOR MANDALAS

With models available for the star, crescent and lotus flower patterns, it is possible for us to generate corresponding interior mandalas composed of related patterns. Globally the interior mandala has a hierarchical structure of concentric rings. On each ring, corresponding patterns are added.

To generate the interior mandalas within a specified area, such as a square of side length W , we need to first determine the distance between neighboring rings in the square. Since the radius of each ring varies from mandala to mandala, we first overlay a uniformly-spaced grid of size $w = W/24$ as the reference grid over a mandala, as thangka artists do when they draw mandalas manually, and then generate each ring according to its radius denoted by R_r in the reference grid from the outer rings to the inner core. We choose this outer-in order to generate concentric rings because we can color

patterns associated with each ring conveniently during the rendering phase.

Once all rings in an interior mandala are generated, we add proper patterns with parameter values indicated by the reference grid on the corresponding rings and color them according to the hand drawn interior mandalas. Finally, small circles depicting different deities are added inside patterns, with flat or gradient colors specified interactively. Our system is implemented in Matlab R2013b and performed on a platform of Core i5-4590 3.30GHz CPU and PC with 8GB memory. We choose 4 interior mandalas from [28] as the hand drawn images and simulate them with our models. The generation time for the 4 interior mandalas ranges from 22.02 to 46.75 seconds, which indicates that our approach can efficiently generate colorful interior mandala patterns compared with the hand drawn mandala images.

Figures 13 and 14 present the generation process of the central parts of Black Yamaraja and Zhitro Narag Dongprug mandalas with star patterns, respectively. The pattern in Black Yamaraja mandala consists of 5 rings counted outside-in, with a 8 pointed stars on the 2nd ring and 4 pointed stars on the 3rd ring. The pattern in Zhitro Narag Dongprug mandala consists of 8 rings counted outside-in, with a 16 pointed stars on the 2nd ring, a 8 pointed stars on the 4th

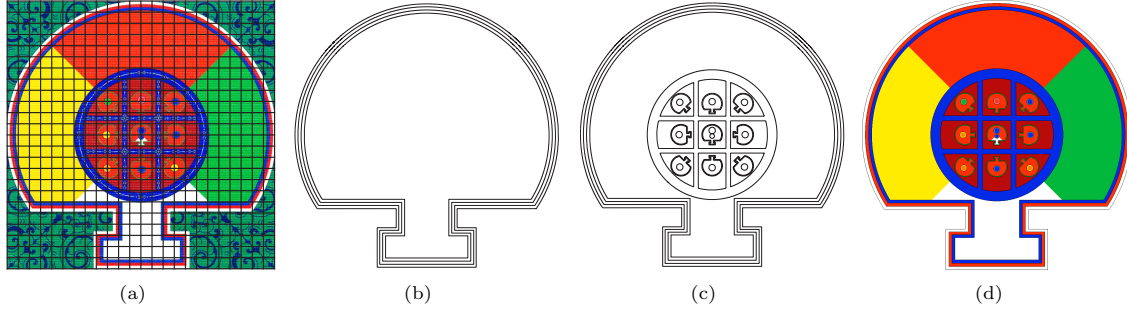


Figure 15: Generation process of central part of Yangdag Nine Crescent mandala (please zoom in to see details): (a) Hand drawn mandala with a grid overlayed; (b) 4 concentric large crescent patterns; (c) 9 crescent patterns with different orientations; (d) Colored mandala.

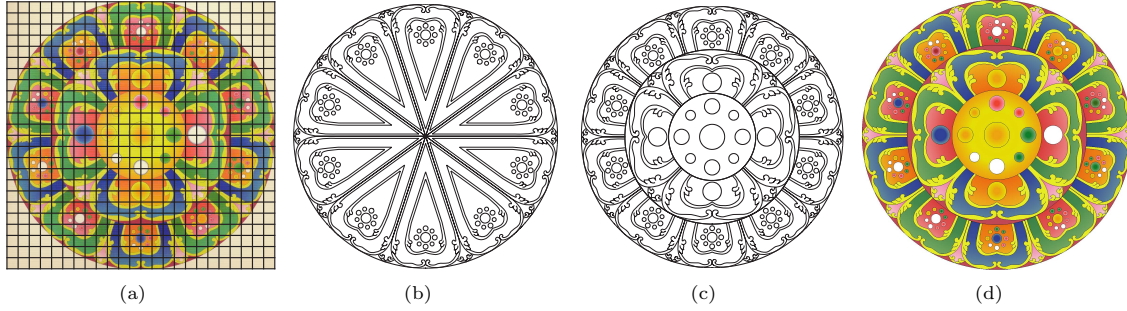


Figure 16: Generation process of central part of Dumdo's Auxiliary Yangsem mandala (please zoom in to see details): (a) Hand drawn mandala with a grid overlayed; (b) 1st ring; (c) 2nd and 3rd rings; (d) Colored mandala.

and 6th rings. The area surrounded by the 8th ring is divided into 9 parts.

Figure 15 shows the generation process of the central part of Yangdag Nine Crescent mandala with crescent patterns. There are 4 concentric large crescent patterns and 9 crescent patterns with different orientations in the 9 parts divided in the central circle.

Figures 16 presents the generation process of the central parts of Dumdo's Auxiliary Yangsem and Dumdo's Auxiliary Nyanthos mandala pattern, respectively. The pattern in Dumdo's Auxiliary Yangsem mandala consists of 3 rings counted outside-in, with lotus flower pattern of 10 petals on the 1st ring and 4 petals on the 2nd ring.

7 CONCLUSION AND FUTURE WORK

This paper has introduced a computer-aided generation approach of mandala thangka patterns. The experimental results show that our method can efficiently generate beautifully-layered colorful interior mandalas compared with the hand drawn mandalas. In addition to mandalas used in Buddhism, mandala patterns that are in any number of shapes, sizes and colors can be modelled for a wide range of applications, including coloring book for children and adults, textile pattern, card and package design, etc. Thus,

our work provides a reference for computer-aided generation of both traditional mandala patterns used in Buddhism and mandala-like patterns used in modern art design. In the future, we plan to explore the generation methods for other types of decorative patterns in surrounding regions other than the interior area. Moreover, investigating aesthetic rules of coloring mandala patterns is another interesting research topic.

ACKNOWLEDGMENTS

We thank the support of the National Natural Science Foundation of China under Grant No.61379069.

REFERENCES

- [1] V. Beatini. 2017. Kinetic rosette patterns and tessellations. *International Journal of Computational Methods and Experimental Measurements* 5, 4 (2017), 631–641.
- [2] S. Cenani and G. Cagdas. 2007. A shape grammar study: generation with geometric Islamic patterns. In *Proceedings of 10th Generative Art Conference*. 1–7.
- [3] P. R. Cromwell. 2009. The search for quasi-periodicity in Islamic 5-fold ornament. *The Mathematical Intelligencer* 31, 1 (2009), 36–56.
- [4] P. R. Cromwell. 2012. A modular design system based on the Star and Cross pattern. *Journal of Mathematics and the Arts* 6, 1 (2012), 29–42.

- [5] P. R. Cromwell. 2015. Cognitive bias and claims of quasiperiodicity in traditional Islamic patterns. *The Mathematical Intelligencer* 37, 4 (2015), 30–44.
- [6] P. R. Cromwell. 2016. Modularity and hierarchy in Persian geometric ornament. *Nexus Network Journal* 18, 1 (2016), 7–54.
- [7] N. G. David, T. Robinson, D. G. Thomas, B. J. Balamurugan, and S. C. Samuel. 2007. Graph grammars for kolam patterns and honeycomb patterns. *International Journal of Mathematical Science* 6 (2007), 355–367.
- [8] E. D. Demaine, M. L. Demaine, P. Taslakian, and G. T. Toussaint. 2007. Sand drawings and Gaussian graphs. *Journal of Mathematics and the Arts* 1, 2 (2007), 125–132.
- [9] M. O. Djibril and R. O. H. Thami. 2008. Islamic geometrical patterns indexing and classification using discrete symmetry groups. *Journal on Computing and Cultural Heritage* 1, 2 (2008), 1–14.
- [10] U. Ebru. 2009. A Shape Grammar Model To Generate Islamic Geometric Pattern. In *Proceedings of 12th Generative Art Conference*. 290–297.
- [11] K. Etemad, F. F. Samavati, and P. Prusinkiewicz. 2008. Animating persian floral patterns. In *Proceedings of the Fourth Eurographics conference on Computational Aesthetics in Graphics, Visualization and Imaging*. ACM, 25–32.
- [12] D. Fontana. 2005. *Meditating with mandalas*. Wiley Series in Telecommunications.
- [13] V. Gopalan and B. K. Vanleeuwen. 2015. A Topological Approach to Creating Any Pulli Kolam, an Artform from South India. *FORMA* 30, 1 (2015), 35–41.
- [14] M. Grey. 2001. Encountering the mandala: the mental and political architectures of dependency. *Culture Mandala: The Bulletin of the centre for East-West cultural and economic studies* 4, 2 (2001), 1–13.
- [15] B. Grünbaum and G. C. Shephard. 1992. Interlace patterns in Islamic and Moorish art. *Leonardo* 25, 3 (1992), 331–339.
- [16] N. Hamekasi and F. Samavati. 2012. Designing Persian Floral Patterns using Circle Packing. In *GRAPP/IVAPP*. 135–142.
- [17] Y. Ishimoto. 2007. Solving infinite kolam in knot theory. *Forma* 22 (2007), 15–30.
- [18] I. Jowers, M. Prats, H. Eissa, and J. H. Lee. 2010. A study of emergence in the generation of Islamic geometric patterns. In *New Frontiers: Proceedings of the 15th International Conference on Computer-Aided Architectural Design Research in Asia*. 39–48.
- [19] C. S. Kaplan. 2005. Islamic star patterns from polygons in contact. In *Proceedings of Graphics Interface*. ACM, 177–185.
- [20] C. S. Kaplan and D. H. Salesin. 2004. Islamic star patterns in absolute geometry. *ACM Transactions on Graphics* 23, 2 (2004), 97–119.
- [21] H. Karam and M. Nakajima. 1999. Islamic symmetric pattern generation based on group theory. In *1999 Proceedings Computer Graphics International*. IEEE, 112–119.
- [22] N. Kumar. 2006. The Mandala: Sacred Geometry and Art. *Exotic India Art* (2006).
- [23] Y. Li and D. Tang. 2010. Design Method for Paper-cut Picture Based on Decorative Pattern. *Computer Engineering* 36, 21 (2010), 234–238.
- [24] Y. Li, J. Yu, K. L. Ma, and J. Shi. 2007. 3D Paper-Cut Modeling and Animation. *The Journal of Computer Animation and Virtual Worlds* 18, 4-5 (2007), 395–403.
- [25] Y. Liu, J. Hays, Y. Q. Xu, and H. Y. Shum. 2005. Digital papercutting. In *ACM SIGGRAPH 2005 Sketches*. ACM, 99.
- [26] P. J. Lu and P. J. Steinhardt. 2007. Decagonal and quasicrystalline tilings in medieval Islamic architecture. *Science* 315, 5815 (2007), 1106–1110.
- [27] S. Nagata. 2007. Digitalization and analysis of traditional cycle patterns in the world, and their contemporary applications. *Forma* 22, 1 (2007), 119–126.
- [28] T. L. Nima. 2012. *Mandalas of Tibetan Buddhism*. Tibet Renmin Press.
- [29] K. Poelke, Z. Tokoutsis, and K. Polthier. 2014. Complex Polynomial Mandalas and their Symmetries. In *Proceedings of Bridges 2014: Mathematics, Music, Art, Architecture, Culture*. 433–436.
- [30] M. Pradella, A. Cherubini, and S. C. Reghizzi. 2011. A unifying approach to picture grammars. *Information and Computation* 209, 9 (2011), 1246–1267.
- [31] S. Raghavachary. 2004. Tile-based kolam patterns. In *Proceedings of ACM SIGGRAPH Sketch*. ACM, 58.
- [32] T. Robinson. 2007. Extended pasting scheme for kolam pattern generation. *Forma* 22, 1 (2007), 55–64.
- [33] Z. Sayed, H. Ugail, I. Palmer, and J. Purdy. 2015. Parameterized Shape Grammar for Generating n-fold Islamic Geometric Motifs. In *International Conference on Cyberworlds*. IEEE, 79–85.
- [34] Z. Sayed, H. Ugail, I. Palmer, J. Purdy, and C. Reeve. 2016. Auto-Parameterized Shape Grammar for Constructing Islamic Geometric Motif-Based Structures. *Transactions on Computational Science XXVIII* 9590 (2016), 146–162.
- [35] K. G. Subramanian, R. Saravanan, and T. Robinson. 2007. P Systems for Array Generation and Application to Kolam Patterns. *Forma* 22 (2007), 47–54.
- [36] R. Tennant and A. Dhabi. 2008. Medieval Islamic architecture, quasicrystals, and Penrose and Girih tiles: questions from the classroom. *Symmetry: Culture and Science* 19, 2-3 (2008), 113–125.
- [37] T. M. Waring. 2012. Sequential encoding of Tamil kolam patterns. *Forma* 27 (2012), 83–92.
- [38] L. Xiong and K. Zhang. 2016. Generation of Miro's Surrealism. In *Proceedings of the 9th International Symposium on Visual Information Communication and Interaction*. ACM, 130–137.
- [39] K. Yanagisawa and S. Nagata. 2007. Fundamental study on design system of kolam pattern. *Forma* 22, 1 (2007), 31–46.
- [40] K. Yoon, H. Kim, and R. Sarhangi. 2014. Geometric Constructions of Korean Danchong Patterns and Building Platonic Solids. In *Proceedings of Bridges 2014: Mathematics, Music, Art, Architecture, Culture*. 525–532.
- [41] K. Zhang and J. Yu. 2016. Generation of Kandinsky art. *Leonardo* 49, 1 (2016), 48–55.
- [42] H. Y. Zhao, Z. G. Pan, Y. Y. Fan, and Z. G. Xu. 2013. Automatic generation of Xinjiang ethnic fabric patterns based on configuration style. *Journal of Graphics* 34, 1 (2013), 18–21.