Science of Energy II
OPRE 6389 Lecture Notes by Metin Çakanyıldırım
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1 Fuel-based Energy Forms

1.1 Chemical Energy

When molecules and atoms chemically react with each other, chemical bonds (such as ionic or covalent bonds) between the atoms of a molecule can be broken, reorganized and re-established. For example, hydrogen gas molecule is made of two hydrogen atoms bonded together with a covalent bond and it weighs only 2 grams per mole [1]. Chemists depict this covalent bond (sharing of two electrons by two hydrogen atoms) as H:H, where the dots represent the electrons. When the hydrogen molecule is burned (hydrogen combustion) with oxygen gas, the bonds between two hydrogen atoms are broken as well as those between oxygen atoms to make up bonds between hydrogen and oxygen atoms. In layman's terms, burning hydrogen results in water:

$$H_2 + \frac{1}{2}O_2 \longrightarrow H_2O + 286,000 \text{ joules.}$$

This combustion reaction also releases 286,000 joules of energy per mole of hydrogen gas burned.

We can also write a combustion reaction for gasoline without referring to exact chemical formulas:

$$\text{1 gallon gasoline} + \text{oxygen} \longrightarrow \text{carbon dioxide} + \text{water} + 132,000,000 \text{ joules.}$$

Energy content of 1 gallon of gasoline is 132 mega joules (Hofstrand 2007).

Example Suppose that 1 gallon of gasoline costs \$4. How many moles of hydrogen needs to be burned to obtain the energy in 1 gallon of gasoline? To avoid an energy price arbitrage, what should the cost of hydrogen gas be?
ANSWER To obtain 132 mega joules, approximately 460 (=132,000,000/286,000) moles of hydrogen gas must be burned. The weight of 462 moles of hydrogen is 920 grams, whose cost must be \$4 to avoid an arbitrage. Or the price of hydrogen must be approximately \$4.34 per kilogram. Note that the cost of hydrogen produced from water by using electrolysis and wind energy can range from \$3.74 to 5.86 per kilogram (Saur and Ainscough 2011). ⋄

Unfortunately, hydrogen gas often does not naturally occur. Methane gas CH_4 naturally occurs and constitutes a significant portion of natural gas that is extracted in gas fields. Methane can also be burned to obtain energy:

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O + 890,000 \text{ joule.}$$

This combustion reaction yields carbondioxide gas, a greenhouse gas, in addition to water, so it is not as clean as hydrogen combustion reaction. Weight of 1 mole of methane is 16 grams (12 for one carbon atom and 4 for four hydrogen atoms).

Example Suppose that hydrogen gas is sold at \$4 per kilogram, what should be the price of methane gas so that the price of energy whether obtained from hydrogen or methane is the same.
ANSWER From the previous exercise \$4 buys 500=1000/2 moles of hydrogen and $500 \times 286,000$ joules. To

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[1] Recall from Science of Energy I that 1 mole of a substance has 6.02×10^{23} atoms or molecules.

obtain this amount of energy from methane combustion, $160 \approx 500 \times 286,000/890,000$ methane molecules are necessary. The weight of 160 methane molecules is 2,560 grams, so the cost of 2.56 kilogram of methane should be \$4, or the price of methane should be \$1.56 per kilogram. Hydrogen gas is 8 times lighter than methane gas and yields about 1/3 of the energy. Hydrogen is a more efficient fuel in terms of energy provided per weight. ⋄

The unit of transaction for methane, when sold as natural gas, is energy content. Suppose that gasoline is priced at \$4 per gallon and methane is sold at \$4 per million BTU. 1 million (mega) BTU is 1/ 0.00095 $\times 10^6$ joules or approximately 1,052 mega joules. A single dollar buys 33 mega joules in terms of gasoline and 263 mega joules in terms of methane at the prices above. Energy bought as natural gas is much cheaper than energy bought as gasoline.

Human body also performs combustion reactions to obtain energy. For example, it burns sugar (such as glucose $C_6H_{12}O_6$):

$$C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O + 2,800,000 \text{ joule}.$$

One mole of glucose has 6 carbons weighing 6(12), 12 hydrogens weighing 12(1) and 6 oxygen weighing 6(16), so 1 glucose molecule has weight of 180 grams.

Example How many grams of sugar needs to be eaten to get 1,000 kilocalories?
ANSWER 180 grams of glucose yields 2,800,000 joules or 666,652 (=2,800,000 × 0.23809) calories or 666.652 kilocalories. Then 1 gram of glucose gives 3.7 (=666.652/180) kilocalories. Dieticians typically shorten this and say 1 gram of carbohydrates provide 4 calories but they should say 4 kilocalories. 270 (=1000/3.7) grams of sugar approximately gives 1,000 kilocalories. ⋄

Example Is methane or glucose a more efficient fuel in terms of energy provided per weight? Explain.
ANSWER 16 grams of methane yields 890,000 joule or methane yields 55,625 (=890,000/16) joules per gram. 180 grams of glucose yields 2,800,000 joules or glucose yields 15,555 (=2,800,000/180) joules per gram. Although glucose is a larger molecule and gives more energy per molecule, it is less efficient than methane on a weight basis. Note that glucose has 6 oxygen atoms, that is 96/180 oxygen by weight. These oxygens in glucose cannot be burned with the oxygen in the environment to release energy and they decrease the efficiency of the glucose as a fuel. ⋄

1.2 Nuclear Energy

Similar to the bonds of a molecule containing chemical energy, nucleus of an atom contains nuclear energy. To obtain this nuclear energy, we should break, reorganize or unite nuclei. So nuclear reactions happen at a much smaller scale than chemical reactions but they involve rearrangement of fundamental particles (protons and neutrons). Hydrogen is a simple element with only 1 proton in its nucleus and no proton. It is customary to write the number of protons as subscript and the total number of protons and neutrons as superscript. For example H_1^1 is the regular hydrogen atom with 1 proton and no neutron. Sometimes, hydrogen atoms can be found with a neutron in their nucleus, this variety denoted by H_1^2 is called deuterium. Very rarely, there can be two neutrons in the nucleus giving rise to tritium H_1^3.

To understand the source of nuclear energy, we can compute the mass of a nucleus in two ways. The direct method is to compute the mass of a nucleus experimentally. This experimental mass can be compared against a theoretical mass based on the number of protons and neutrons in the nucleus:

$$\text{Theoretical mass} = \text{Number of protons} \times \text{Mass of a proton} + \text{Number of neutrons} \times \text{Mass of a neutron}.$$

Interestingly, there is a mass deficit in each nucleus:

$$\text{Mass deficit} = \text{Theoretical mass} - \text{Experimental mass} \geq 0.$$

A mass equivalent to the mass deficit disappears when protons and nucleons are pulled together in a nucleus. This mass turns into nuclear energy at the rate of the square of speed of light. This energy is analogous to the energy required to compress a coil spring. A nucleus can be thought as a source of stored (nuclear) energy, as a compressed spring is a source of mechanical (kinetic) energy.

Nuclear energy is a separate form of energy. It is far stronger to be explained by gravitational pull of protons or nucleons. It causes attraction among positively charged protons as opposed to repulsion predicted by electrical energy. Moreover, nuclear energy is active only in the nucleus, it does not reach outside the nucleus to pull electrons, so it is short-ranged.

Presence of more or fewer neutrons in the nucleus does not affect the chemical properties of an element. Chemical properties such as bonding and ionization are driven by the electrons rotating in an orbit around the nucleus. The number of neutrons affect the physical properties, most clearly the weight of the nucleus. Two types of the same element with different number of neutrons are called isotopes. For example, hydrogen, deuterium and tritium are all isotopes.

Some isotopes of an atom are stable some are not. Stability is a consequence of a balance between nuclear and electric forces in a nucleus. In the case of instability, some particles can be pushed away from the nucleus to achieve stability. If the emitted (pushed away) particle is a Helium He_2^4 atom, an alpha decay is said to occur. Alpha decay is a form of nuclear radiation, as are beta decay and gamma decay. Beta decay involves emitting of an electron (minimal mass, negative electric charge) while gamma decay has emitting of a photon (minimal mass, no charge, high energy).

Some unstable large isotopes split into smaller atoms rather than emitting much smaller particles. To help with this split, an unstable isotope can be hit with some particles. To split the nucleus, the particle needs to reach the nucleus so it should not be positively charged. This is why a neutron is a good choice to hit the nucleus with. An isotope with even number of protons and even number of neutrons are more stable than an isotope with odd number of protons and neutrons. There are some pairing-caused stability theories as to why this might be the case. According to Das and Ferbel (2004), empirically 156 isotopes with even number of protons and nucleons are stable whereas the corresponding number for isotopes with odd number of protons and nucleons is only 5. For example, Uranium has unstable isotope U_{92}^{235} and stable isotope U_{92}^{238}, which is common in the nature.

The unstable (radioactive) isotope U_{92}^{235} can be bombarded with slow-moving neutrons to have a fission (split) reaction:

$$U_{92}^{235} + n_0^1 \implies U_{92}^{236} \implies Ba_{56}^{144} + Kr_{36}^{89} + 3n_0^1 + 2.07 \times 10^{13} \text{ joules},$$

where U_{92}^{236} is Uranium isotope with 144 neutrons, Ba_{56}^{144} is Barium isotope with 88 neutrons, Kr_{36}^{89} is Krypton isotope with 53 neutrons and n_0^1 is a neutron. Barium-144 isotope goes through a series of beta decays and an alpha decay to turn into Lanthanum, Cerium-144, Praseodymium, Neodymium and eventually to Cerium-140:

$$Ba_{56}^{144} \implies La_{57}^{144} + e_{-1}^0 \implies Ce_{58}^{144} + 2e_{-1}^0 \implies Pr_{59}^{144} + 3e_{-1}^0 \implies Nd_{60}^{144} + 4e_{-1}^0 \implies Ce_{58}^{140} + He_2^4 + 4e_{-1}^0.$$

Final products Cerium-140 Ce_{58}^{140} and Helium-4 He_2^4 of this decay chain are both stable. On the other hand, Krypton-89 isotope goes through beta decays to turn into Rubidium, Strontium and eventually Yttrium:

$$Kr_{36}^{89} \implies Rb_{37}^{89} + e_{-1}^0 \implies St_{38}^{89} + 2e_{-1}^0 \implies Y_{39}^{89} + 3_{-1}^0.$$

The final product Yttrium-89 Y_{39}^{89} is stable.

The energy released by the fission reaction above of a single Uranium atom is 215 MeV (Mega electron volts), which is $215 \times 10^6 \times 1.6 \times 10^{-19} = 344 \times 10^{-13}$ joules. 1 mole of Uranium or 235 gram Uranium has 6.02×10^{23} atoms and releases 2.07×10^{13} joules. 1 gram of U_{92}^{235} can release 10^{11} ($\approx 2.07 \times 10^{13}/235$) joules.

Example In comparison with glucose, how efficient fuel source is Uranium?

ANSWER 180 gram of glucose gives 2,800,000 joules, that is 15,555 joules per gram. 235 gram of Uranium gives 2.07×10^{13} joules, that is 8.8 $\times 10^{10}$ joules per gram. Uranium is more efficient by a factor of 5,100,000 (=$0.51 \times 10^7 = 8.8 \times 10^{10}/15,555$). Uranium is more efficient than glucose by 5.1 million times and more efficient than methane by 1.43 ($\approx 5.1 \times 15,555/55,625$) million times. ⋄

Example 10^{11} joules is about 1 mega watt day and provided by the fission of 1 gram of U_{92}^{235}. To operate a 1,200 mega watt nuclear reactor throughout a year, how much U_{92}^{235} is necessary?
ANSWER 1,200 mega watt reactor yields 1,200 mega watt year of energy, that is 365 × 1200 mega watt day of energy. This much of energy requires 438,000 (=365 × 1200) grams of U_{92}^{235}. That is, 438 kilograms or 0.438 tons of U_{92}^{235}.

Example A 1,200 mega watt reactor requires enriched uranium fuel rods that are 6% U_{92}^{235} and 94% U_{92}^{238}. To operate the reactor throughout the year, how many tons of enriched uranium are necessary?
ANSWER From above, the reactor requires 0.438 tons of U_{92}^{235}, which is only 6% of the enriched fuel. All of the enriched fuel is 7.3 (=0.438/0.06) tons.

For a nuclear reactor, the burn up out rate is amount of energy obtained from 1 unit of mass of enriched fuel. Typically, energy is measured in terms of mega watt days and mass is measured in kilogram, so the units of burn up rate is mega watt day / kilogram.

Example What is the burn up rate of a nuclear reactor that generates 1200 mega watt day by burning 20 (=7300/365) kilograms of enriched fuel?
ANSWER Burn up rate is 60 mega watt day per kilogram. This number is more like an upper bound on the burn up rate, the actual burn up rate in practice can be lower say 40-50 mega watt day per kilogram.

The fission reaction above resulted in the release of 3 neutrons, only 2 neutrons can be released in a slightly different version:

$$U_{92}^{235} + n_0^1 \implies U_{92}^{236} \implies Ba_{56}^{144} + Kr_{36}^{90} + 2n_0^1 + \text{energy},$$

where one of the neutrons remains in Krypton to give it a mass of 90. Krypton-90 isotope can further go through a series of beta decays to turn into Rubidium, Strontium, Yttrium, and eventually to Zirconium: $Kr_{36}^{90} \implies Rb_{37}^{90} + e_{-1}^{0} \implies St_{38}^{90} + 2e_{-1}^{0} \implies Y_{39}^{90} + 3e_{-1}^{0} \implies Zr_{40}^{90} + 4e_{-1}^{0}$. The final product Zirconium-90 Zr_{40}^{90} is stable. The number of neutrons released by a fission reaction can be 2 or 3, so it can be taken as a random variable. Neutrons generated by reactions can be thought as generations, where the initial neutron starts the first generation which may have 2 or 3 neutron offsprings. Each of these neutrons start the second generation and so on. The number of neutrons in the nth generation will be at least 2^n and most 3^n. It is possible to study the expected number of neutrons in a generation, variance of this number and the probability that a given number of U_{92}^{235} is depleted in some number of generations; these issues are studied under the umbrella of Branching Processes in Probability.

Stars such as sun generate energy from fusion (union) of nuclei. The following fusion reaction is possible on the sun:

$$4H_1^1 \implies He_2^4 + \text{nuclear particles} + \text{gamma ray} + 0.24 \times 10^{13} \text{ joules},$$

where He_2^4 is the regular Helium gas. The energy created by sun is emitted to the space (including earth) in terms of sunlight and gamma rays, both of which are electromagnetic waves that constitute the source of solar energy.

2 Exercises

1. On Thanksgiving day, average turkey consumed by Americans weigh about 15 pounds and requires 20 minutes of cooking per pound. A typical gas oven burns about 11,000 btus per hour. Suppose that 50 million turkeys are cooked on Thanksgiving either at gas ovens or at electric ovens whose electricity is generated at gas fired power plants. What is the btu demand for cooking turkeys on Thanksgiving day? Suppose that 1 cubic feet of gas has 1000 btus and that U.S. annual production is about 20,000 billion cuibicfeet of gas. Is the gas demanded on Thanksgiving to cook turkeys a significant portion of annual U.S. gas production? If your answer is yes, you can use swing spreads to buy gas in advance to sell it during Thanksgiving.
   ANSWER Each turkey is cooked for 300 minutes or 5 hours by consuming 55,000 btus. 50 million turkeys require 2750 billion btus. Turkeys require 2.75 billion cubic feet of gas. 2.75 billion is about 1 thousandth of 20,000 billion, so it is not a significant portion of annual production. Said differently, Cameron's gas liquefaction plant has daily capacity of 1.7 billion cubic feet [2], so the plant by itself can cover the Thanksgiving demand in less than 2 days of operation. Unfortunately, Thanksgiving does not increase the demand significantly to have an effect on prices.

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[2] http://www.cameronlng.com/expansion-update.html