Photovoltaic Solar Energy
OPRE 6389 Lecture Note
Compiled at 18:48 on Wednesday 1st March, 2017

# 1 Science of Photovoltaics

A photovoltaic device contains special materials that generate electricity from light. Materials used for a photovoltaic device are called semiconductors whose conducting properties can be deliberately altered by controlling the type and quantity of impurities added into the original semiconducting material. Semiconducting materials can be classified as n(egative)-type and p(ositive)-type. Semiconductors that have an excess of electrons are called n-type semiconductors and semiconductors that have holes (lack of electrons) are called p-type semiconductors. When an n-type semiconductor is placed appropriately next to a p-type semiconductor and light is shined on them, electrons can flow from n-type to p-type. This phenomenon is known as *photovoltaic effect*.

**Conductors, Semiconductors and Insulators:** The elements such as copper and aluminum are conductors, which have a sea of free-roaming electrons somewhat independently of their nuclei. Moving these electrons away from their nuclei requires relatively less energy that can be supplied by light. Removal of the electrons from a metal by using light is called *photoelectric effect*. To generate electric current from light, what we want is controlled flow of electrons rather than their arbitrary dispersion into the space. Electrons of conductors are too free to control. At the other extreme, electrons of insulators such as plastics are bonded too strongly to their nuclei, hence their removal requires too much energy. Neither conductors nor insulators have the appropriate level of electron freedom to generate electricity from light.

Electrons of semiconductors have less freedom than those of conductors and more freedom than those of insulators. Semiconductors such as silicon crystal (made up of silicon $Si_{14}^{28}$) have electrons that can be moved with light without causing these electrons to leave the surface of the semiconductors. This light-caused electron movement is an example of *photovoltaic effect*. Similar to silicon, elements of boron, germanium and arsenic are often used in semiconductor manufacturing; see Figure 1.

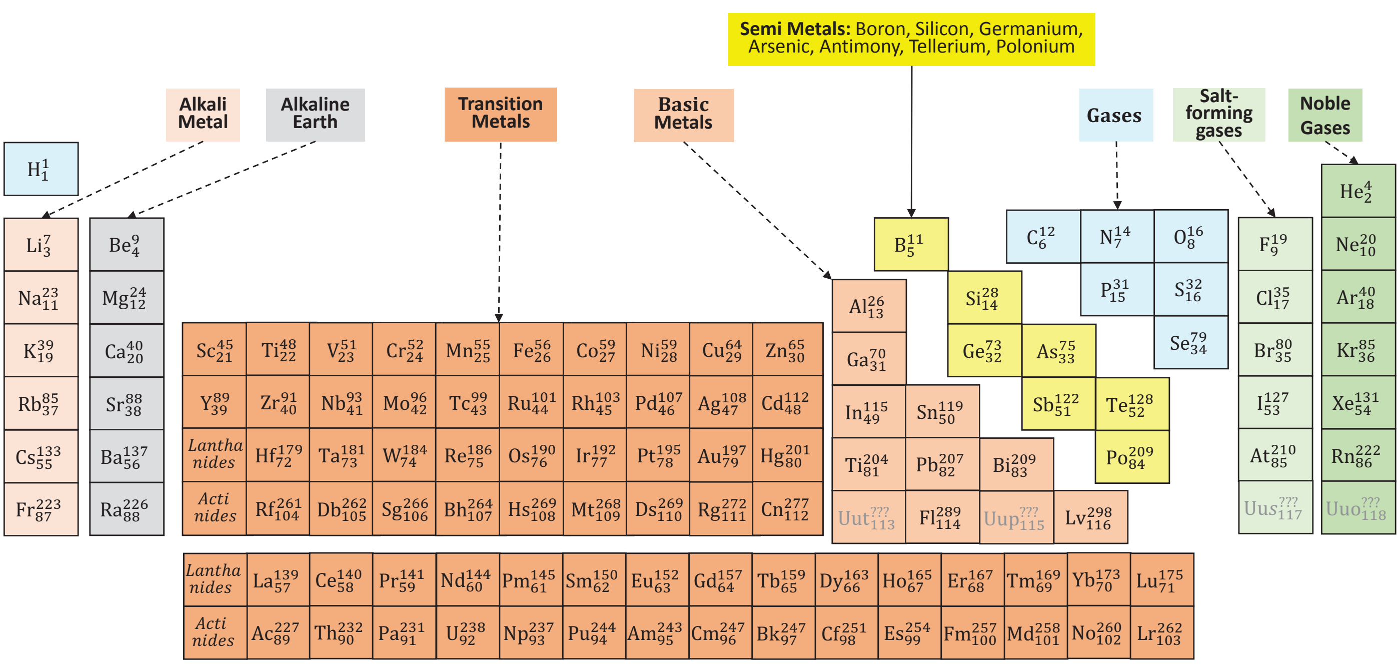


Figure 1: Periodic table of elements shows semimetals between metals and gases.

In a semiconductor material, each electron resides in its regular orbit (*valence band*) around its nucleus. While residing in its valence band, an electron has a certain amount of energy. Energy of an electron increases upon receiving photovoltaic energy through a light photon; see Figure 2. A rise in an electron's energy, if small, warms up the material containing the electron. If the energy rises by a large amount, it moves an electron from its valence band to the conduction band. Conduction band is further away from the nucleus and accommodates electrons that are ready to break away from their nucleus. If an electron in the conduction band is not directed through a circuit to come out away from its nucleus, it can drop back to its valence band. In the process of falling back, the electron may emit light. For example, fluorescent lamps excite gas electrons in the lamp with electricity and the electrons go up to their conduction band, fall back to their valence band and emit light. *Fluorescent effect* is the opposite of the *photovoltaic effect*.

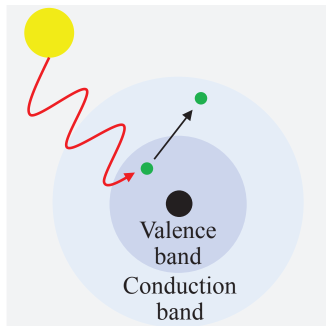


Figure 2: Light increases the energy of an electron possibly from valence band to conduction band.

To facilitate a single directional movement of electrons under photovoltaic effect, electron-rich (n-type) and electron-lacking (p-type) regions of a semiconductor can be manufactured. Silicon $Si_{14}^{28}$ has 14 electrons; 10 are in lower stable orbits while 4 are in the external orbit. These 4 electrons for each silicon atom makes up chemical bonds among the atoms in the silicon crystal. It is not easy to move electrons in this pure silicon crystal structure, so the crystal is unpurified slightly. Negative and positively charged regions of crystal can be obtained by using some other elements than silicon. Here are two examples:

• Fosfor (phosphorus $P_{15}^{31}$) immediately to the right of silicon in Figure 1 is an ideal element to load up the crystal with extra electrons because fosfor has 5 electrons in its external orbit. If silicon is replaced with fosfor in a region of the crystal, that region has extra electrons and becomes relatively n(egative)-type.

• Boron ($B_5^{11}$) to the left of silicon in Figure 1 is an ideal element to reduce the number of electrons because boron has 3 electrons in its external orbit. If silicon is replaced with boron in a region of the crystal, that region lacks electrons and becomes relatively p(ositive)-type.

The elements used instead of silicon are called *dopants*. The concentration of dopant atoms can range from $10^{12}$ to $10^{20}$ per $cm^3$ and affect the conductivity of the semiconductor. The most fundamental electronic device constructed using semiconductor materials is a diode. A diode is essentially an electrical device that allows current to move through in primarily one direction. A photovoltaic device is the most basic type of diode, where the photo-electric effect is used in combination with a p-n junction to build a photo-diode. In this diode, a p-type semiconducting material is brought in contact with an n-type semiconducting material to form a p-n junction, which is then exposed to sunlight. This is described in Figure 3 in steps.

• In Step 1, we observe a p-n junction where the n-type material has an excess of electrons and the p-type material has excess holes in the absence of light.

• In Step 2, the excess electrons present in the p-type material combine with the holes in the interface region due to opposite charge attraction and create the *depletion region* observed in Step 2 of Figure 3. This depletion region is well defined and it occurs only at the interface of the P-N junction.

• In Step 3, when light shines on this semiconductor device, mobile electrons are created as a result of the photoelectric effect. The energy to mobilize electrons come from the photons in the light. Some electrons accelerate towards the n-type material or, said differently, some holes accelerate towards the p-

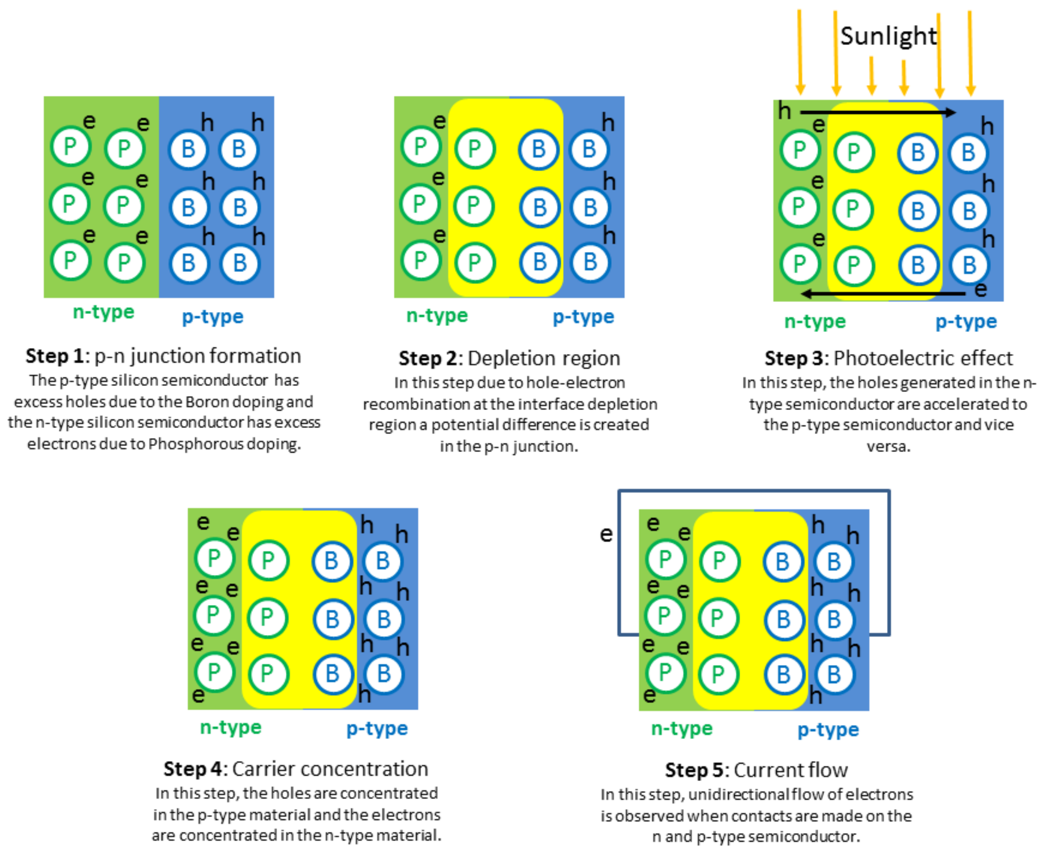


Figure 3: Multi-step physics of a photovolatic device. A better name for Step 3 is photovoltaic effect. P: Phosphorus, B: Boron, e: electron, h: hole.

type material.

• In Step 4, the acceleration in the previous step forms a high concentration of mobile electrons in the n-type semiconductor and mobile holes in the p-type semiconductor.

• In Step 5, by merely connecting the p-type and the n-type with a conductor, a unidirectional flow of electrons is observed across the p-n junction.

The efficiency of this photovoltaic device is directly dependent on the wavelength of the incident light, the intensity of the incident light and the bandgap of the semiconducting material used to build the photovoltaic device. Besides this, the reflectance of the material, the thermodynamic efficiency of the solar cell and conductive efficiency of the solar cell also play a strong interdependent role in defining the efficiency of a photo-voltaic device.

Semiconductor materials commonly used to build photovoltaic devices need to have bandgaps that match the solar spectrum. Silicon is the most commonly used semiconducting material in solar energy harvesting. Affordability and the processing experience from the electronic device industry (transistors, LEDs etc.) have given silicon a competitive advantage over other materials. Some other materials used in photovoltaic devices are Galium Arsenide (GaAs), Cadmium Telluride, Copper Indium Gallium Selenide (CIGS), etc.

## 2 Photovoltaic Cell Types and Manufacturing

Each photovoltaic (PV) device is called a photovoltaic cell or a solar cell. Depending on the technology used, solar cells are better classified as follows.

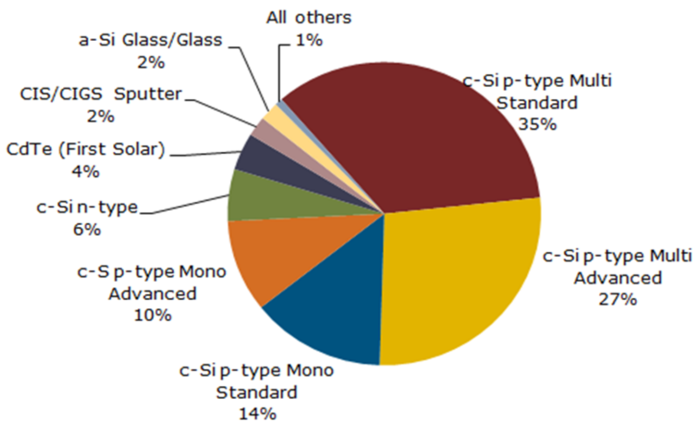


Figure 4: Photovoltaic Production - Technology Distribution circa 2014.

## 2.1 Crystalline silicon solar cells

According to Fraunhofer ISE Photovoltaibs Report (2016), crystalline silicon solar cells presently dominate the photovoltaic market contributing to more than 90% of the solar cells produced. This market domination is largely due to a drop in silicon costs that have made this technology very affordable and competitive. Some of the major solar companies in the world, namely SunPower, Yingli, Trina and Jinko, use crystalline silicon based technology to manufacture solar cells. Cell efficiency using technology is in the range of 14-24% depending on the type of crystalline silicon being used.

The two basic types of crystalline silicon cells are monocrystalline silicon (n and p type) crystal and polycrystalline silicon (n and p type) crystal. Both of these crystals have ordered lattice structure that can be grown by adding on more elements. Crystallization is a process of adding more molten elements to a crystal and cooling, so that new elements take an orderly structure around as they solidify. The most natural example of crystallization is freezing of water. In the same way, molten silicon can be solidified to manufacture silicon crystals. If the silicon crystallization happens around a single seed crystal (as in Czochralski process), it is called *monocrystalline silicon*. If crystallization starts at multiple locations, it is called *polycrystalline silicon*. When crystals grow from different locations towards each other, they will form a boundary region where the crystal structure is not perfect, which is a disadvantage of polycrystalline silicon. One should expect slightly better properties (such as efficiency) from monocrystalline silicon than polycrystalline silicon. But monocrystalline silicon growth takes more time and effort to produce so the monocrystalline silicon is more expensive.

Besides type of silicon crystal used in a device, efficiency is also dependent on the type of technology used to build a solar cell. While all solar technology primarily relies on the formation of p-n junction, efficiency improvements can be made by improving the quality of the p-n junction and improvements in materials engineering and design. According to Tyagi et al. (2013), Interdigited back contact (IBC), Heterojunction solar technology (HIT) and Passivated Emitter Rear Cell (PERC) are some of the technologies being currently used to manufacture high efficiency n-type monocrystalline solar cells. Figure 4 highlights the type of technology in actual production. It is observed that crystalline Si dominates actually production, with about 90% of all PV production being that of c-Si (Crystalline Silicon).

## 2.2 Thin film solar cells

Thin film solar cell technology primarily relies on thin films that constitute the p-n junction of the solar cells. Copper Indium Gallium Selenide (CIGS), Cadmium Telluride (CdTe), amorphous silicon thin films and Gallium Arsenide thin film devices constitute the majority of the materials used in thin film solar cell devices. R&D with some success is also being carried out in the organic photovoltaic thin films and organic-inorganic hybrid materials. Thin films generally have lower efficiencies than crystalline silicon

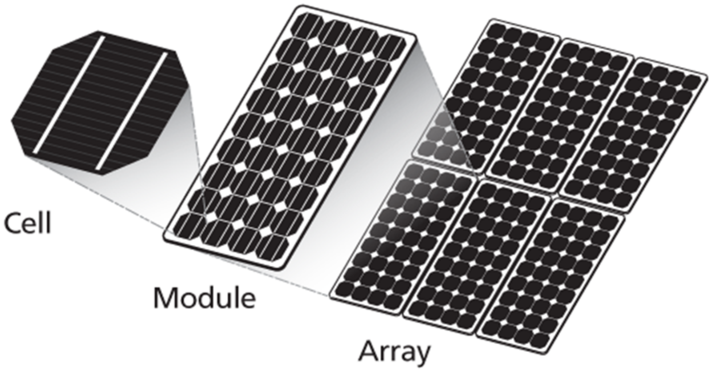


Figure 5: Photovoltaics: Cells to Modules to Arrays

based solar cells, however thin film technologies like CdTe and CIGS tend to be quite cost competitive.

First Solar is a thin film based company and it made CdTe innovations over the years (Green 2015). While it is true that the thin film market share in solar production is less than 10%, innovation and improvements in technology will largely come from the thin film space as present crystalline silicon technology is nearing its theoretical efficiency limitations.

### 2.3 Multi-junction (Tandem) solar cells

Multi-junction or Tandem photovoltaic cell technologies rely on a combination of multiple p-n junctions to build high efficiency solar cells. Usually materials with different bandgaps are used in the tandem cell. Each p-n junction absorbs light from certain wavelengths of the solar spectrum and converts solar energy to electrical energy. However, in a tandem junction individual p-n junctions should only allow wavelength specific absorption. All other wavelengths of light should be transmitted through the p-n junction to the next layer of junction in order for each p-n junction of the tandem cell to behave as a photovoltaic device. Ensuring minimum reflectivity, maximum transmission of other wavelengths and maximum photo-electric conversion is crucial to build a good tandem junction.

Tandem junctions are usually more expensive than traditional crystalline silicon and thin films based solar cells. While tandem junctions have much higher efficiencies than the best performing crystalline solar cells, this technology comes at a much higher cost. Tandem cells are usually used in space exploration applications and Concentrated Photo-Voltaic (CPV) applications. With record efficiencies of 44%, tandem cells still are the forefront of R&D. Innovations to reduce costs and improve efficiencies are key to improving affordability of solar photo-voltaic technology.

## 3 Power from a Solar Array

A solar module is a connected assembly of one or more solar cells, packaged together for applications that require long life and high reliability. In turn, modules are connected together to produce solar arrays; see Figure 5. Solar cells by themselves degrade under extreme environments and it is essential to build robust solar modules for residential, commercial and utility application. Solar modules in turn are strung together to form arrays.

Solar modules made from crystalline silicon generally comprise 60 (6x10) cells or 72 (6x12) cells strung together in series determining the output of the solar module. Series connection of cells have implications on the design of the cell:

• The total voltage generated by the module is the sum of the voltages generated by cells in the module. On the other hand, essentially the same amount of current flows through each module when they are connected in series. To keep the total voltage not too high and current not too low over a module, each cell in the module is designed to generate a low voltage with a relatively high current.

• The amount of current that flows through a module is dictated by the bottleneck cell which allows the lowest current flow among the others. To the extent that the bottleneck cell is different from the others, it becomes a more limiting factor. Said differently, to minimize the adverse effect of a bottleneck cell, cells need to be designed and produced with lowest amount of variability possible.

The implications of series cell connections in a module are amplified when modules are connected in series in an array. That is, the lowest performing (bottleneck) cell in an array dictates the performance of an entire array of solar panels.

Before estimating the power from a solar array, it is important to understand how solar modules are rated. The solar industry is dependent on the universally accepted Standard Testing Conditions (STC) to determine the performance of a solar module. STC states that the power rating of module is the output power produced by a solar module in laboratory conditions at a particular temperature and irradiance. Irradiance is the radiant power received by a per unit area on the surface of the world and its units are generally Watt per squaremetre ($W/m^2$).

### 3.1 Input Power

Irradiance depends on the (solar zenith) angle at the location of the module. This angle is between the extrapolation of the line connecting the location to the center of the earth and the line connecting the module to the sun. The location does not receive any sun light when the zenith angle is 90° or more. For our purposes, the zenith angle is always less than 90°, so it is an acute angle whose cosine is between 0 and 1. As the zenith angle increases from 0° to 90°, the sun falls more directly and eventually perpendicularly at the location. Since the cosine decreases as the angle increases, we use the reciprocal of the cosine to capture the extent to which the sun falls perpendicularly. This reciprocal is named as Air Mass ($AM$) index. $AM\ x$ corresponds to a zenith angle whose cosine is $1/x$ for $x \geq 1$:

$$AM\ x = AM\ \frac{1}{1/x} \quad \text{and} \quad \frac{1}{x} = \cos(\text{zenith angle}).$$

Example What is the AM for zenith angle of 0? What zenith angles lead to AM of 1.5, 2 and 2.5?
ANSWER Zenith angle of 0 has cosine of 1, so $x = 1$ it leads to AM 1. For AM 1.5, $x = 1.5$ and $1/1.5 = \cos(48°)$. For AM 2, $x = 2$ and $1/2 = \cos(60°)$. For AM 2.5, $x = 2.5$ and $1/2.5 = \cos(66.5°)$. ◇

We can summarize the results of the last example in Table 1 by appending two special cases of AM 0 and AM ∞. AM 0 corresponds to the irradiance outside the atmosphere and AM ∞ gives the irradiance after the sunset.

Table 1: AM values versus solar zenith angle.

| AM Values | Solar Zenith Angle | Irradiance $E$ in $W/m^2$ |
|---|---|---|
| AM 0.0 | Irradiance outside the atmosphere | 1360 |
| AM 1.0 | Sun rays perpendicular to the surface, 0.0° | - |
| AM 1.5 | Industry standard, 48.2° | 1000 |
| AM 2.0 | 60.0° | - |
| AM 2.5 | 66.5° | - |
| AM ∞ | After sunset, ≥ 90.0° | 0 |

As the table shows, the irradiance is the highest outside the atmosphere. As sun rays go through the atmosphere, they can be reflected by air molecules so the irradiance drops with the distance travelled by the rays through the atmosphere. From the definition of the zenith angle, we approximately have

$$\begin{aligned}\text{Distance travelled in the atmosphere} &\approx \frac{1}{\cos(\text{zenith angle})}\text{ Thickness of atmosphere}\\ &= [x \text{ of } AM\ x] \text{ Thickness of atmosphere.}\end{aligned}$$

Hence, higher zenith angle implies longer distance for sun rays to cover in the atmosphere and in turn less of irradiance on the surface. More exact calculations of irradiance require the elevation of the location and the thickness of the atmosphere at that particular location.

Complicating the calculations more is the tilt of earth's rotational axis. The earth's rotational axis is parallel to the normal of the elliptic surface it covers annually around the sun only on equinox days, approximately March 21 and September 23. Only on equinox days, the zenith angle at equator is $0^\circ$ at noon. Latitude $23.5^\circ$ north on June solstice and latitude $23.5^\circ$ south on on December solstice have the zenith angle of $0^\circ$ at noon. On December solstice, the northern hemisphere receives the least irradiance while the southern hemisphere receives the most over the year. The opposite happens on June solstice as the northern hemisphere receives the most irradiance while the southern hemisphere receives the most over the year. Furthermore, the tilt of earth is not constant and varies between $22.1^\circ$ and $24.5^\circ$ over 41,000 years.

To avoid calculational complexities of irradiance, the Standard Testing Conditions (STC) are specified as AM 1.5 with irradiance of $E = 1000$ W/m$^2$ under 25 Celcius. For a given irradiance level $E$ and a module, we can compute the solar (input) power falling on to the module:

$$P_{in} = [\text{Area of the Module}]\ E.$$

This is only the input power received by a solar module and hence it is an upper bound on what the module can generate.

### 3.2 Output Power

In general, the power output of a device is the product of the voltage and current on that product. For $V$ and $I$ respectively denoting voltage in Volts and current in Amperes, the power generated by a device with resistance $R$ in ohms is

$$P = VI = I^2R = \frac{V^2}{R}.$$

In particular for a solar module,

$$P_{max} = V_{mp}I_{mp}.$$

$V_{mp}$ and $I_{mp}$ correspond to the maximum voltage and current at maximum power $P_{max}$. These voltage and current values can be determined empirically when rating a module. Hence, the pair $(V_{mp}, I_{mp})$ can be obtained from the device simultaneously.

One can also seek a theoretical upper bound on the empirically measured $P_{max}$ by separately constructing bounds on the current and voltage. The current in a circuit is maximized when the current faces no resistance:

$$\max_I (I = \sqrt{P/R}) \equiv \min_R R \;\Rightarrow\; R \downarrow 0 \text{ and } I_{sc}.$$

A circuit without resistance is called *short circuit* and the associated current is denoted by $I_{sc}$. Similarly, the voltage in a circuit is maximized when the current faces infinite resistance:

$$\max_V (V = \sqrt{PR}) \equiv \max_R R \;\Rightarrow\; R \uparrow \infty \text{ and } V_{oc}.$$

A circuit with infinite resistance is called *open circuit* and the associated voltage is denoted by $V_{oc}$. $(V_{oc}, I_{sc})$ cannot be obtained from the device simultaneously, so this pair is only fictitious.

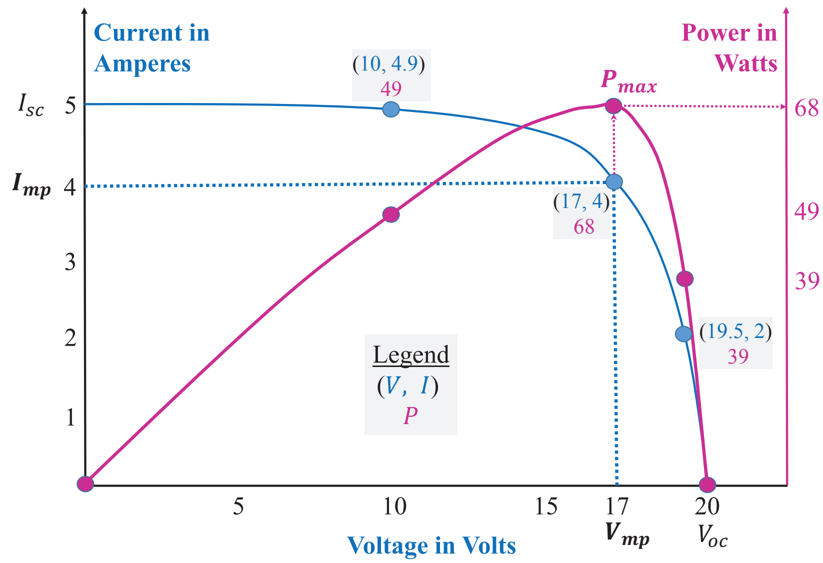


Figure 6: Current ($I$) - Voltage ($V$) pairs constitute the characteristic curve. Power is obtained from $P = VI$.

By construction, the empirically measured power is bounded by the theoretical power based on short and open circuit arguments: $P_{max} < V_{oc}I_{sc}$. The gap between left-hand side and right-hand side can be measured by the Fill factor (FF) for a device:

$$FF = \frac{P_{max}}{V_{oc}I_{sc}} = \frac{V_{mp}I_{mp}}{V_{oc}I_{sc}}.$$

Example The Fill Factor ($FF$), open circuit voltage ($V_{oc}$) and short circuit current ($I_{sc}$) can be determined from the I-V characteristic curve of the solar module as shown in Figure 6. The figure can be supplied by the manufacturer of panels. Find $FF$ for the panel in the figure.
*ANSWER* Reading from the figure, $V_{mp} = 17$ volts, $V_{oc} = 20$ volts, $I_{mp} = 4$ amperes, $I_{sc} = 5$ amperes. Hence, $P_{max} = V_{mp}I_{mp} = 17(6) = 68$ W. Note that the panel can operate at $(V_{mp} = 17, I_{mp} = 4)$ but not at $(V_{oc} = 20, I_{sc} = 5)$. Finally,

$$FF = \frac{P_{max}}{V_{oc}I_{sc}} = \frac{68}{20(5)} = 68\%.$$

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### 3.3 Panel Efficiency

The efficiency $\eta$ of the module is the ratio of output power to input power:

$$\eta = \frac{P_{max}}{P_{in}}.$$

Example A solar module with area of 1.64 m$^2$ has $V_{oc} = 39.4$ volts, $I_{sc} = 10$ ampere, $V_{mp} = 34.5$ volts and $I_{mp} = 8.7$ ampere under STC. Calculate the maximum power output $P_{max}$, fill factor $FF$ and efficiency $\eta$ of the module.
*ANSWER* We have

$$\begin{aligned} P_{max} &= V_{mp}I_{mp} = (34.5)(8.7) = 300.15\ W; \\ FF &= \frac{V_{mp}I_{mp}}{V_{sc}I_{sc}} = \frac{(34.5)(8.7)}{(39.4)(10.0)} = 0.7618; \\ P_{in} &= [\text{Area of the Module}]\ E = (1.64)(1000) = 1640\ W, \end{aligned}$$

where we take 1,000 W under STC. Therefore, the efficiency is

$$\eta = \frac{P_{max}}{P_{in}} = \frac{300.15}{1640} = 0.183 = 18.3\%. \quad \diamond$$

The last example holds for a module under STC. However STC are laboratory conditions, which rarely occurs in reality. Let us now reconsider this example with more realistic conditions, where irradiance and temperature are not 1000 W/m$^2$ or 25 Celcius. Then, the thermal co-efficient of the module plays a role in predicting the linear drop in output power with increase in module temperature. The drop in power output is modelled by

$$P(T) = (1 + \tau(T - 25))P_{max},$$

where $P(T)$ is the maximum power as a function of temperature $T$. When the temperature $T = 25$ Celsius, the formula yields the identity $P_{max}(T = 25) = P_{max}$. The thermal coefficient is often negative and captures the drop in the power output for $T > 25$.

Example Consider the output power $P_{max} = 300.15$ W for a module under STC. However in operation, the actual irradiance and module temperature measured are 800 W/m$^2$ and 30 Celcius. Calculate the change in $P(T = 30)$ under these conditions if the thermal co-efficient of the panels is $\tau = -0.004$ Celcius.

*ANSWER* First we compute the power output under 1000 W/m$^2$ and 30 Celcius:

$$P_{max}(T = 30) = (1+\tau(T-25))P_{max}(25) = (1-0.004(30-25))300.15 = 294.147\ W \quad \text{with } E = 1000\ W/m^2.$$

This power is obtained under 1000 W/m$^2$ and 20% less power is obtained under 800 W/m$^2$

$$P_{max}(T = 30) = 294.147\frac{800}{1000} = 253.32\ W \quad \text{with } E = 800\ W/m^2.$$

Power drops from 300.15 W to 253.32 W because of less irradiance and higher temperatures than those under STC. $\diamond$

It is evident from the last example that the module performance in reality is different from that under STC. Moving from one module to an array of modules also affects the output power. This is especially true as there exists a mismatch in the performance of the modules in an array strung in series. It is to be noted that when considering an entire array of modules the module with the lowest current $I_{mp}$ will dominate the string. Let us consider an array of $n$ modules indexed as $1, 2, \dots, n$. Module $i \in \{1, 2, \dots, n\}$ delivers voltage $V^i_{mp}$ and current $I^i_{mp}$. Then

$$\text{Total power output of an n-module array} \;=\; \min_{1\le i\le n}\{I^i_{mp}\}\sum_{i=1}^{n} V^i_{mp}.$$

This is illustrated in the next example.

Example Consider an array of 10 similar modules strung together in series. Each module generates $V_{mp} = 34.5$ volts and $I_{mp} = 8.7$ ampere.

a. Calculate the total output power of the array of modules at STC.

b. Calculate the total output power of the array of modules at STC when Module 4 $I_{mp}$ drops by 20% and Module 8 $I_{mp}$ drops by 40%.

*ANSWER* a. Under STC, irradiance and temperature are $E$= 1000 W/m$^2$ and $T$=25 Celius. Total output power of the array = (10) (34.5) (8.7) = 3001.5 W.

b. The current on module 8 drops the most so $\min_{1\le i\le n}\{I^i_{mp}\} = I^8_{mp} = (0.6)8.7 = 5.22$. The total power output then is as follows.

$$\text{Total power output of an 10-module array} \;=\; \min_{1\le i\le 10}\{I^i_{mp}\}\sum_{i=1}^{10} V^i_{mp} = 5.22(10)34.5 = 1800.9\ W. \quad \diamond$$

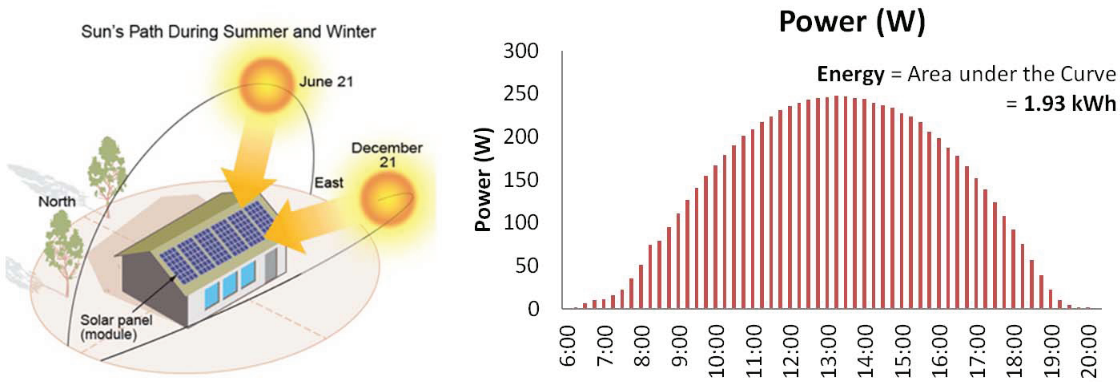


Figure 7: Sun path and hourly power curve for 300 W panels in San Jose, Ca on June 6, 2016

The performance of an array is not just affected by module performance mismatch and actual irradiance-temperature effects. Several other factors also determine the performance of a PV system. Panel orientation, panel tilt, dust on the panels, aging of the panels, installation location, electrical losses, soiling losses, inverter efficiency, transmission losses, and module degradation are some of the many factors that determine the performance of a PV system. For more information, see Chapter 2 of Krauter (2006).

## 4 Energy and Revenue

The fundamental difference between power and energy is that power is instantaneous and energy is power averaged across time. In photovoltaics this period of time is usually 60 mins = 1 hour. Hence, the unit of energy is Watt-hr (Wh).

Energy is more relevant than instantaneous power in photovoltaics as the sun's irradiation is not constant through the day. The suns irradiation in the morning is very different from that at noon. This variation in the sun's irradiation determines the performance of the solar panel.

Figure 7 depicts the change of instantaneous power through a day for a 300 W module installed at 30 degrees (with horizontal) facing south. It is evident as the irradiation continuously changing over the day, the power produced by the panel changes. This change in power is averaged hourly and then summed across 24 hours to determine the daily energy production in kWh. It should be noted here that the sun's path is not constant through the year. The winter path is different from the summer path. The sunlight falls onto panels in the northern hemisphere close to perpendicular angles at noon times during summer. A consequential change in intensity of sunlight falling on a solar module is evident as the sun's path changes through the year. As a result, energy produced by a panel varies over a year. To maximize performance of the PV modules, location, orientation and angle of tilt are crucial. The performance of a PV system can be simulated with historical irradiation data.

Example For a 3 kW system in San Francisco, Ca and using calculations at http://pvwatts.nrel.gov, we can estimate the actual energy output of the system when the modules are oriented south tilted at 20 degrees. When considering a 19% efficient panel with 14% system losses, the annual energy production of the 3 kW system is 4,700.75 kWh. This estimation is done with historical irradiance from a weather station nearby. PV professionals rely on such estimations to estimate the economic benefits of photovoltaics when designing PV systems. Figure 8 gives the breakdown of the energy across the 12 months of the year. ⋄

Example To replicate the above example for a residential polar system installed on a roof, go to http://pvwatts.nrel.gov and use UT-Dallas campus address 800 West Campbell Road, Richardson 75080 TX. Select the weather data for this location. The closest data location is TMY3 Dallas/Addison. Then you have to enter PV system information. Consider a 4 kW system with standard (polycrystalline silicon) module type. If modules are for residential use, they are most likely to be polycrystalline silicon. A 4 kW system is likely to have 16 solar panels. With panels of the standard 1.6 m by 1 m size, we need about 30

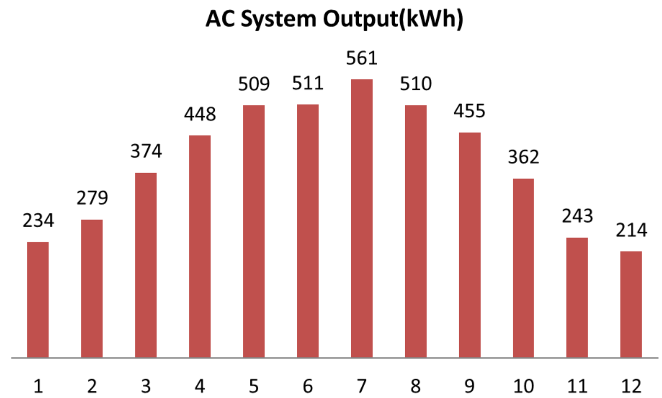


Figure 8: Monthly Energy Production for a 3 kW PV System

squaremetre of roof space; see Figure 9. Most residential panels are fixed (as opposed to one-axis or two-axis sun tracking) on the roof. If your panel is facing south directly, the azimuth angle is 180 degrees. The tilt angle is the (slope of the roof for fixed panels) angle panel surface makes with the horizontal line, you can put 20 degrees for the tilt angle in Dallas where the roofs are not too steep. You can keep the default system loss of 14% for taking into account shading, soiling, wiring, etc. Moreover, let us suppose that the electricity cost is $0.1 per kWh.

Figure 9: 4kW capacity from 16 standard solar panels of size 1 metre $\times$ 1.6 metre.

With the parameters above http://pvwatts.nrel.gov reports columns A and E in Table 2. Then we obtain the other columns as follows: B=30 A; D=B*C; F=E/D. This system yields 5,216 kWh of energy annually, which worths $512.6. It is also worth noting that the efficiency in column F is minimum in the summer months and maximum during winter months; this can be due to loss of power due to high temperatures.

The last example illustrates that energy produced by a PV system varies through the year. Because of this reason, it is a good practice to compute annual revenue. The annual revenue of 4 kW fixed roof-top panels installed at UT Dallas turned out to be $512.6, which can later be compared with the cost of the system.

It is inevitable that energy production is higher in the summer months and lower in the winter months. However the same cannot be said for consumption. As more and more renewables are entering into the grid, the impact of this problem is more evident. Besides variability across the year, variability in energy consumption also exists in a day. According to Swanick (2016), the "Duck curve" forecast in California is a direct result of a mismatch of energy production and consumption in a day. The Duck curve in Figure 10 forecasts the net consumer load based on historical data for a particular day in the year. It was observed that when forecasting the energy demand (net load) into the future, there would be a strong demand for

Table 2: Solar input and output from 4 kW fixed roof-top panels installed at UT Dallas.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| | kWh/($m^2$*day) | kWh/day | Days | Input kWh | Output kWh | |
| | Solar | Solar | per | Solar | AC | % |
| Months | radiation | radiation | month | radiation | Energy | Efficiency |
| Jan | 2.76 | 82.8 | 31 | 2,566.8 | 284 | 11.06 |
| Feb | 4.08 | 122.4 | 28 | 3,427.2 | 366 | 10.68 |
| Mar | 5.04 | 151.2 | 31 | 4,687.2 | 500 | 10.67 |
| Apr | 4.55 | 136.5 | 30 | 4,095.0 | 433 | 10.57 |
| May | 4.27 | 128.1 | 31 | 3,971.1 | 411 | 10.35 |
| Jun | 6.11 | 183.3 | 30 | 5,499.0 | 535 | 9.73 |
| Jul | 6.69 | 200.7 | 31 | 6,221.7 | 591 | 9.50 |
| Aug | 6.36 | 190.8 | 31 | 5,914.8 | 574 | 9.70 |
| Sep | 6.08 | 182.4 | 30 | 5,472.0 | 532 | 9.72 |
| Oct | 3.96 | 118.8 | 31 | 3,682.8 | 379 | 10.29 |
| Nov | 2.96 | 88.8 | 30 | 2,664.0 | 288 | 10.81 |
| Dec | 3.13 | 93.9 | 31 | 2,910.9 | 323 | 11.10 |
| Total | 55.59 | 1,679.7 | 365 | 51,112.5 | 5,216 | |

energy production between 3 pm and 9 pm as a result of increased PV energy. This is because as energy consumption peaks late in the evening, PV systems continue to produce peak energy closer to noon. As a result of this energy storage and other renewables might be required to close the gap between supply and demand.

While these potential future challenges exist, traditionally Power Purchase Agreements (PPA) are signed by consumers based on annual energy production numbers. This practice is gradually changing. However in order to explain revenue and profitability in solar let us assume that the consumer is paying a single price independent of when the energy is being consumed.

## 5 System Costs

Before understanding business models that have made solar profitable, it is important to know the other cost contributing components of PV systems. Besides PV modules, all other components of a PV system are called balance of system (BOS). These components comprise inverters, electrical wiring, racking, labor etc. These components also contribute to the costs involved in installing PV system and are often more than 50% of the costs in a PV System.

In order to better understand PV System costs, cost information from a 1 MW system installed in Jan 2016 in Los Angeles California has been shared in Table 3. As observed in Table 3, pricing for a PV installation is done on a \$/Watt basis. Therefore, to calculate the total costs of installing a PV system, the size of the system is multiplied with the \$/Watt. For the 1 MW system given in Table 3, the total costs of system is \$1.6 million. It is important to note that installing a PV system has economies of scale; cost of the system per Watt decreases in the capacity of the system. Residential installations which are typically less than 10 kW are generally more expensive (\$2.5-3 /Watt in 2016). However Utility scale solar farms that are in the MW scale are generally less expensive. Besides systems installation costs, PV systems also have Operations & Maintenance (O&M) costs. O&M costs vary dependent on location and local conditions, and are accounted for when estimating system profitability.

The Levelized Cost of Energy (LCOE) metric is widely used to compare and identify the best value offered by competing PV technologies in the solar market. From the lifetime energy production of a PV System, the LCOE calculates the net present value of unit-cost of electricity.

$$LCOE = \frac{\text{Total PV system costs}}{\text{Energy produced through PV system life}}.$$

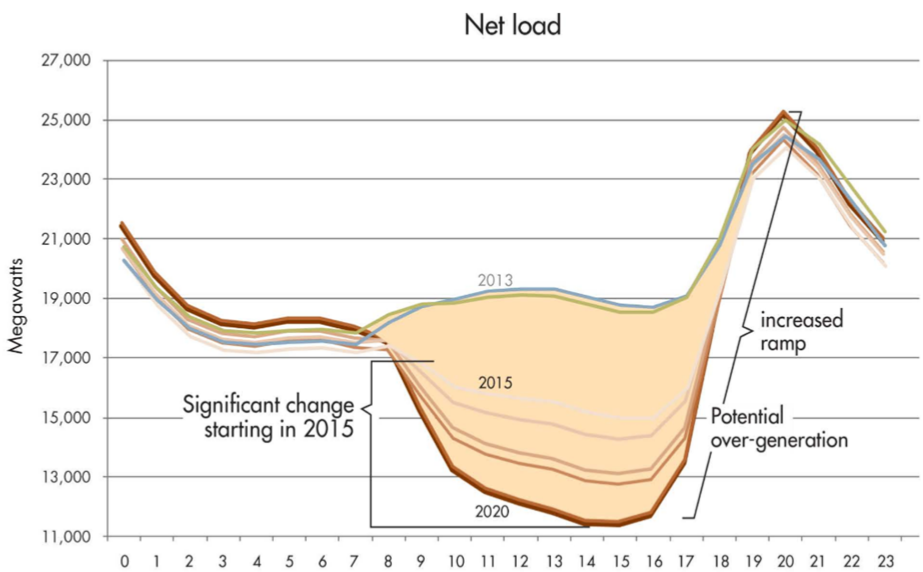


Figure 10: Duck curve highlights a drop in the demand over 9-15 coupled with its steep rise over 15-21.

Table 3: PV Components and costs.

| PV Components | Costs ($/Watt) |
|---|---|
| Module | 0.48 |
| Racking | 0.10 |
| Electrical | 0.40 |
| Inverter | 0.06 |
| Labor | 0.10 |
| Miscellaneous | 0.02 |
| Total | 1.16 |

In this formula, the life of the PV system is usually about 25 years and annual energy production of a system can be calculated based on 20-30 year averaged irradiance data. The cost of installing a system is available upon installation, which makes it easy to calculate LCOE. A lower LCOE stands for a better value proposition. It is important to note that in reality, some amount of annual performance degradation (aging) is inevitable and PV system designers generally account for such variability when determining LCOE.

Example Consider two different PV Systems A and B, that use 2 different PV technologies. For a 10 kW system, the total annual energy produced by system A is 18,000 kWh and the total annual energy produced by system B is 16,000 kWh. Total $/Watt for system A is $2.75/Watt and $2.5/Watt for system B. Determine LCOE for both systems for 25 years and identify the PV system that has the best value proposition.

*ANSWER*

$$\begin{aligned}\text{LCOE for System A} &= \frac{(10,000)(2.75)}{(25)(18,000)} = 0.0611 = 6.11\ c/Watt.\\ \text{LCOE for System B} &= \frac{(10,000)(2.50)}{(25)(16,000)} = 0.0625 = 6.25\ c/Watt\end{aligned}$$

While System A is more expensive, its ability to produce more energy reduces its LCOE. This makes System A a better value proposition than System B. $\diamond$

# 6 Profitability of a solar farm

Solar installations are capital intensive projects. Be it a small residential system or a utility scale system, it is usually impossible for the end consumer to bear the complete costs of a solar installation upfront. This has provided an opportunity for numerous business models to sell and profit from solar. While business innovation has enabled customer adoption, the financial fundamentals remain the same. A large capital investment is made and the returns are observed over the life of the system. Profit is made both by the investors who invest in the system for the customers, and by the consumers who benefit from a lower cost of power over a period of time. These business models that have moved the high capital cost from the consumer to an "investing entity" has enabled consumers and investors, to benefit from the long term benefits of solar.

Example Let us estimate the ROI (Return on Investment) on a 4 kW residential system when the price of energy is \$0.12/kWh. This system at full capacity generates 35,040 = 365(24)4 kWh annually.
a. Due to irradiance variations, the annual energy production of the system on average is estimated to be 6,218 kWh and the total cost of the system is 2.5 \$/Watt. Find ROI under these parameters.
b. Revise your ROI computation when the annual energy generation of the 4 kW system is 5,216 kWh and the price of electricity is \$0.08/kWh.
*ANSWER* a. The ROI for this system can be calculated as,

$$\begin{aligned} ROI &= \frac{\text{(Revenue generated from the PV installation over its life)} - \text{(Total PV system costs)}}{\text{Total PV system costs}} \\ &= \frac{6,218(0.12)25 - 2.5(4,000)}{2.5(4,000)} = \frac{18,656 - 10,000}{10,000} = 0.86 = 86\%. \end{aligned}$$

An ROI of 86% is high and this makes solar a worthwhile investment proposition for both investors and end consumers.

b. When the annually generated energy drops slightly to 5,216 kWh and the price drops to \$0.08, the ROI drops significantly

$$ROI = \frac{5,216(0.08)25 - 2.5(4,000)}{2.5(4,000)} = \frac{10,432 - 10,000}{10,000} = 0.0432 = 4.32\%$$

An ROI of 86%. ◇

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