Aggregate Planning
Capacity & Production Planning

Prof. Metin Çakanyıldırım used various resources to prepare this document for teaching/training. To use this in your own course/training, please obtain permission from Prof. Çakanyıldırım. If you find any inaccuracies, please contact metin@utdallas.edu for corrections.
Aggregate Planning (Ag-gregate: Past part. of Ad-gregare: Totaled)

- If the actual is different than the plan, why bother sweating over detailed plans

- Aggregate planning: General plan for our frequency decomposition
  - Combined products = aggregate product
    - Short and long sleeve shirts = shirt
      - Single product
    - AC and Heating unit pipes = pipes at Lennox Iowa plant
  - Pooled capacities = aggregated capacity
    - Dedicated machine and general machine = machine
      - Single capacity
        - E.g. SOM has 100 instructors
  - Time periods = time buckets
    - Consider all the demand and production of a given month together
      - When does the demand or production take place in a time bucket?
      - Increase the number of time buckets; decrease the bucket length.
Fundamental Tradeoffs

Capacity: Regular time, Over time, Subcontract?
Inventory: Backlog / lost sales, combination: Customer patience?

Basic Strategies

- **Chase (the demand) strategy;** produce at the instantaneous demand rate
  - fast food restaurants

- **Level strategy;** produce at the rate of long run average demand
  - swim wear

- **Time flexibility;** high levels of workforce or capacity
  - machining shops, army

- **Deliver late strategy**
  - spare parts for your Jaguar
Matching the Demand

- Which is which?
  Level
  Deliver late
  Chase
  Time flexibility

Adjust the capacity to match the demand

Use capacity

Use inventory

Use delivery time
Capacity Demand Matching
Inventory/Capacity Tradeoff

- **Level strategy:** Leveling capacity forces inventory to build up in anticipation of seasonal variation in demand

- **Chase strategy:** Carrying low levels of inventory requires capacity to vary with seasonal variation in demand or enough capacity to cover peak demand during season
Case Study:
Aggregate planning at Red Tomato

- **Farm tools:**
  - Shovels
  - Spades
  - Forks

Are they the same characteristics?

Generic tool, call it Shovel

Aggregate by similar characteristics
Demand at Red Tomato Tools

Initial availabilities:
80 workers are available on Jan 1.
1,000 shovels available on Jan 1.
Finish June with at least 500 shovels

<table>
<thead>
<tr>
<th>Month</th>
<th>Demand Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1,600</td>
</tr>
<tr>
<td>February</td>
<td>3,000</td>
</tr>
<tr>
<td>March</td>
<td>3,200</td>
</tr>
<tr>
<td>April</td>
<td>3,800</td>
</tr>
<tr>
<td>May</td>
<td>2,200</td>
</tr>
<tr>
<td>June</td>
<td>2,200</td>
</tr>
<tr>
<td>Total</td>
<td>16,000</td>
</tr>
</tbody>
</table>
Aggregate Planning

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>$10/unit</td>
</tr>
<tr>
<td>Inventory holding cost</td>
<td>$2/(unit*month)</td>
</tr>
<tr>
<td>Marginal cost of a backorder</td>
<td>$5/(unit*month)</td>
</tr>
<tr>
<td>Hiring &amp; training costs</td>
<td>$300/worker</td>
</tr>
<tr>
<td>Layoff cost</td>
<td>$500/worker</td>
</tr>
<tr>
<td>Labor hours required</td>
<td>4 hours/unit</td>
</tr>
<tr>
<td>Regular time cost</td>
<td>$4/hour</td>
</tr>
<tr>
<td>Overtime cost</td>
<td>$6/hour</td>
</tr>
<tr>
<td>Max overtime hrs per employee per month</td>
<td>10 hours</td>
</tr>
<tr>
<td>Cost of subcontracting</td>
<td>$30/unit</td>
</tr>
<tr>
<td>Revenue</td>
<td>$40/unit</td>
</tr>
</tbody>
</table>

What is the cost of production per tool? That is materials plus labor. Overtime production is more expensive than subcontracting. What is the saving achieved by producing a tool in house rather than subcontracting?
1. Decision Variables

For month index $t \in \{1, 2, \ldots, 6\}$

$W_t =$ Number of employees in month $t$

$H_t =$ Number of employees hired at the beginning of month $t$

$L_t =$ Number of employees laid off at the beginning of month $t$

$P_t =$ Production in units of shovels in month $t$

$I_t =$ Inventory at the end of month $t$

$S_t =$ Number of units backordered at the end of month $t$

$C_t =$ Number of units subcontracted for month $t$

$O_t =$ Number of overtime hours worked in month $t$

Did we aggregate production capacity?
2. Objective Function:

Min $4 \times 8 \times 20 \sum_{t=1}^{6} W_t + 300 \sum_{t=1}^{6} H_t + 500 \sum_{t=1}^{6} L_t + 6 \sum_{t=1}^{6} O_t + 2 \sum_{t=1}^{6} I_t + 5 \sum_{t=1}^{6} S_t + 10 \sum_{t=1}^{6} P_t + 30 \sum_{t=1}^{6} C_t$

4*8*20: Monthly labor cost of a worker; (300, 500): (Hiring, Firing) cost; 6: Overtime labor rate 2: Inventory holding cost; 5: Backordering cost; 10: Material cost; 30: Subcontracting cost

3. Constraints

- **Capacity Constraint:** Production (in hrs) cannot exceed capacity (in hrs) in month $t$
  
  $4P_t \leq 8 \times 20W_t + O_t$
  
  or $160W_t + O_t - 4P_t \geq 0$.

- **Limited Overtime Constraint:** For each month $t$, $O_t \leq 10W_t$ or $10W_t - O_t \geq 0$.

- **Workforce Balance Constraint:** For each month $t$, with hiring and layoffs
  
  $W_0 = 80$.
  
  $W_t = W_{t-1} + H_t - L_t$. 
3. Constraints

- Inventory balance for each month $t$

$$\begin{align*}
I_{t-1} + P_t + C_t + S_t &= D_t + S_{t-1} + I_t \\
\text{or} \\
(I_{t-1} - S_{t-1}) + P_t + C_t - D_t - (I_t - S_t) &= 0
\end{align*}$$

$I_t - S_t$; Net inventory

$I_0 = 1,000; S_0 = 0; I_6 \geq 500.$
Complete Formulation

Min $4 \times 8 \times 20 \sum_{t=1}^{6} W_t + 300 \sum_{t=1}^{6} H_t + 500 \sum_{t=1}^{6} L_t + 6 \sum_{t=1}^{6} O_t + 2 \sum_{t=1}^{6} I_t + 5 \sum_{t=1}^{6} S_t + 10 \sum_{t=1}^{6} P_t + 30 \sum_{t=1}^{6} C_t$

Workforce Balance Constraint: $W_0 = 80$;$W_t = W_{t-1} + H_t - L_t$ for $t \in \{1, \ldots, 6\}$.

Inventory Balance Constraint: $I_0 = 1,000$; $S_0 = 0$; $I_6 \geq 500$; \((I_t - S_t) = (I_{t-1} - S_{t-1}) + P_t + C_t - D_t$ for $t \in \{1, \ldots, 6\}$.

Capacity Constraint: \[160W_t + O_t - 4P_t \geq 0 \text{ for } t \in \{1, \ldots, 6\}.

Limited Overtime Constraint: \[10W_t - O_t \geq 0 \text{ for } t \in \{1, \ldots, 6\}.

Nonnegativity Constraint: \[W_t, H_t, L_t, O_t, I_t, S_t, P_t, C_t \geq 0 \text{ for } t \in \{1, \ldots, 6\}.

Execution

- Solve the formulation, see `redTomato.xlsx`
  - Total cost=$422,276 K, total revenue=$640K

<table>
<thead>
<tr>
<th></th>
<th>Dec</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
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</thead>
<tbody>
<tr>
<td>Employees</td>
<td>80</td>
<td>64.58</td>
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<tr>
<td>Hiring</td>
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<td>15.42</td>
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<td>Overtime</td>
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<tr>
<td>Production</td>
<td>2,583.3</td>
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<td>2,583.3</td>
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<td>2,583.3</td>
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<td>Subcontracting</td>
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<tr>
<td>Inventory</td>
<td>1,000</td>
<td>1,983.3</td>
<td>1,566.6</td>
<td>949.9</td>
<td>116.7</td>
<td>500</td>
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<tr>
<td>Backorder</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>266.6</td>
</tr>
</tbody>
</table>

- Apply the first month of the plan
- Delay applying the remaining part of the plan until the next month
- Rerun the model with new data next month
- This is called rolling horizon execution
## Effect of Demand Fluctuations

<table>
<thead>
<tr>
<th>Month</th>
<th>Original Demand Forecast</th>
<th>Fluctuating Demand Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1,600 = 1,000 + 600</td>
<td>1,000</td>
</tr>
<tr>
<td>February</td>
<td>3,000</td>
<td>3,000</td>
</tr>
<tr>
<td>March</td>
<td>3,200</td>
<td>3,200 + 600 = 3,800</td>
</tr>
<tr>
<td>April</td>
<td>3,800</td>
<td>3,800 + 200 + 800 = 4,800</td>
</tr>
<tr>
<td>May</td>
<td>2,200 = 2,000 + 200</td>
<td>2,000</td>
</tr>
<tr>
<td>June</td>
<td>2,200 = 1,400 + 800</td>
<td>1,400</td>
</tr>
<tr>
<td>Total</td>
<td>16,000</td>
<td>16,000</td>
</tr>
</tbody>
</table>

See `<redTomato.xlsx>`

- Revenue = 640,000
- Cost = 422,276
- Profit = 217,724

Cost = 432,858
Summary

- Qualitative strategies of matching demand and supply
- Quantitative methods
Material Requirements Planning

- Master Production Schedule (MPS)
- Bill of Materials (BOM)
- MRP explosion

Advantages
- Disciplined database
- Component commonality

Shortcomings
- Rigid lead times
- No capacity consideration
Optimized Production Technology

- Focus on bottleneck resources to simplify planning
- Product mix defines the bottleneck(s)?
- Provide plenty of non-bottleneck resources.
- Shifting bottlenecks
Just in Time production

- Focus on timing
- Advocates pull system, use Kanban
- Design improvements encouraged
- Lower inventories / set up time / cycle time
- Quality improvements
- Supplier relations, fewer closer suppliers, Toyota city

- JIT philosophically different than OPT or MRP, it is not only a planning tool but a continuous improvement scheme