A WTP-Choice Model: Empirical Validation & Competitive Pricing

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Outline

3 Steps to Pricing

- **Model** demand function according to the consumer behavior.

- **Estimate** parameters of demand function and forecast future demand.

- **Optimize** monopolist price and analytically find equilibrium prices for duopoly.
Introduction

How consumers choose between products / retailers?

Medical Brain examination

Statistical Data examination

Choice Data
Classical Choice Process

Clasical Model
Assumes: Knowledge about all product prices and quality of the product to make a choice.
Reality: Customers do not check the product prices and quality at every retail store and not very frequently (Seiler 2010).

Assumes: Always maximize surplus = utility - price.
Reality: Consumers do not always maximize their surplus and instead are satisfied with just nonnegative surplus - ‘Bounded Rationality’ (Gigerenzer and Selten 2001).
Motivation: Consumer Preferences

Consumers have a preference between products (and retailers)

Established habits, weekly/daily routine, convenience of shopping in patterns or sequences, …

To disclose preference, we can ask if you would

First consider buying fat free or healthy heart yogurt?

dark chocolate or light chocolate?

wheat bread or honey-oat bread?

from WalMart or Target? … FedEx or UPS?

Preference does not necessarily indicate purchase

buy the alternative if the preferred is expensive

Choice process starts with preferences and continues with surplus evaluation.
WTP Model Captures Sequential Decision Making

\[ \Phi \]

\[ p^1: price \]

\[ w^1: \text{wtp for Walmart} \]

\[ w^2: \text{wtp for Target} \]

\[ p^1 > w^1 \]

\[ p^1 \leq w^1 \]

\[ p^2 > w^2 \]

\[ p^2 \leq w^2 \]

Do not buy if \( p^1 > w^1 \) and \( p^2 > w^2 \)

Do not buy if \( p^1 > w^1 \) and \( p^2 > w^2 \)

BUY from Walmart !!

BUY from Target !!
WTP-choice Model
Decision Tree

Preferences
(\(\phi, \delta\))

Perception / Beliefs; Prices
(\(w_1, w_2; p_1, p_2\))

Choice
(\(y_1, y_2\))

\[ W^i : \text{Distribution of wtp at } i \]

Choice (1,0):
\[ \rho^1(p_1, p_2) = \delta(1 - W^1(p_1))\{(1 - \phi)W^2(p_2) + \phi\}, \]

Choice (0,1):
\[ \rho^2(p_1, p_2) = \delta(1 - W^2(p_2))\{\phi W^1(p_1) + (1 - \phi)\} \]
# Multinomial Logit vs. WTP-choice Model

<table>
<thead>
<tr>
<th></th>
<th>Logit Model</th>
<th>WTP-choice Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Decision Process</strong></td>
<td>At Once</td>
<td>Sequential</td>
</tr>
<tr>
<td><strong>Information Required</strong></td>
<td>More</td>
<td>Less</td>
</tr>
<tr>
<td><strong>Customer Preferences</strong></td>
<td>Not modeled explicitly</td>
<td>Modeled explicitly as $\Phi$</td>
</tr>
<tr>
<td><strong>Independence of Irrelevant Alternatives</strong></td>
<td>Yields IIA property</td>
<td>No IIA property</td>
</tr>
<tr>
<td><strong>Choice probability</strong></td>
<td>By a ratio [ \frac{\exp{-\alpha_1 + \beta p^1}}{\sum_{j=0}^{2} \exp{-\alpha_j + \beta p^j}} ]</td>
<td>By a product [ \delta (1 - W^1(p^1)) { (1 - \phi) W^2(p^2) + \phi } ]</td>
</tr>
</tbody>
</table>
Roadmap

3 Steps to Pricing

**Model** demand function according to the **consumer behavior**.

**Estimate** parameters of demand function and **forecast** future demand.

**Optimize** monopolist **price** and analytically find **equilibrium prices** for duopoly.
Empirical Validation: Data, Parameterization, Estimation

Parameterization of WTPs: Uniform, Triangular or (Shifted) Exponential distributions.
Parameterization of Utility in Logit: Gumbel distribution.

Log-likelihood maximization for estimating preference and WTP parameters in the WTP-choice

$$LL_{wtp}(\delta, \phi, W^1, W^2; D) = \sum_{n=1}^{N} \sum_{i=0}^{2} y^i_n \log p^i_n(\delta, \phi, W^1, W^2; p^1_n, p^2_n)$$

utility parameters in the multinomial logit

$$LL_{logit}(\alpha_1, \alpha_2, \beta; D) = \sum_{n=1}^{N} \sum_{i=0}^{2} y^i_n \log \lambda^i(\alpha_1, \alpha_2, \beta; p^1_n, p^2_n)$$

Choice probabilities

Standard optimization procedures for multinomial logit; new procedures for WTP-choice in R software.

Candy-melt data obtained from a local retailer. Others: Ketchup, Yogurt and Tuna data.
Comparison with Prediction Testing

1. **Split** data into 2 parts.
2. One part to estimate the model parameters.
3. Use these estimated parameters to **Predict** sales in the other part.
4. Accuracy of sales forecast by **Mean Percent Error in Predicting Actual Sales**.

<table>
<thead>
<tr>
<th>Data</th>
<th>Logit</th>
<th>WTP-Choice Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pure</td>
<td>Mixed</td>
</tr>
<tr>
<td>Yogurt</td>
<td>2.52</td>
<td>2.31</td>
</tr>
<tr>
<td>Ketchup</td>
<td>8.84</td>
<td>8.66</td>
</tr>
<tr>
<td>Candy melt</td>
<td>2.30</td>
<td>2.29</td>
</tr>
<tr>
<td>Tuna</td>
<td>5.06</td>
<td>5.06</td>
</tr>
</tbody>
</table>

To beat logit models, we take the best of all WTP parameterizations.
Roadmap

3 Steps to Pricing

Model — demand function according to the consumer behavior.

Estimate — parameters of demand function and forecast future demand.

Optimize — monopolist price and analytically find equilibrium prices for duopoly.
## Competitive Pricing without Stockouts

- **Logit Model**

\[
\lambda^1(p^1, p^2) = \frac{\exp\{-(\alpha_1 + \beta p^1)\}}{\sum_{j=0}^{2} \exp\{-(\alpha_j + \beta p^j)\}}
\]

\[
\max_{p^1} (p^1 - c_1) \lambda^1(p^1, p^2)
\]

- **WTP-choice Model**

\[
\rho^1(p^1, p^2) = \delta(1 - W^1(p^1))\{(1 - \phi)W^2(p^2) + \phi\},
\]

\[
\max_{p^1} (p^1 - c_1) \rho^1(p^1, p^2);
\]

- **Price response**

\[
p^1(p^2) = c_1 - \frac{1}{\beta \{1 - \lambda^1(p^1, p^2)\}}.
\]

- **Equilibrium price has no a simple expression. In particular, depends on the other’s price.**

- **Equilibrium price depends only on the WTP and cost of the retailer’s own product.**

- **Profit depends on the other’s price.**

- **Separate prices but couple profits “Loose coupling of retailers”**

- **Local retailer sets prices independent of competitors**

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“each firm … can ignore its impact on, and hence reactions from, other firms” … monopolistic competition.
Summary and Conclusions

S Proposed a new choice model based on WTP of customers
  
  incorporates bounded rationality and sequential decision making.
  
  compare qualitatively with the logit model.

C Empirically validation the new choice model using real-life data

S The new choice model is versatile to incorporate WTP dependence and yields price cycles as equilibrium.

C Price cycle equilibrium

S The new choice model is versatile to study competitive pricing under stockouts with 3 cases – lost sales, backorders with retailer preferring customers and backorders with availability favoring customers.

C Loose coupling of retailers
Pricing with Dependent WTPs

- **Multinomial Logit** assumes identically distributed and independent (iid) Gumbel variables.

- If WTPs are distinctly/identically distributed and independent, loose coupling is valid.
- Loose coupling fails if WTPs are dependent:
  - identical, i.e., $W_1(\omega) = W_2(\omega)$;
  - identically distributed but dependent.

- With independent WTPs, a pair of prices is an equilibrium.
- With dependent WTPs, a price cycle can be an equilibrium!
  - Table below shows $P(W_1 = p^1, W_2 = p^2)$; Profit$^1(p^1, p^2)$; Profit$^2(p^1, p^2)$.

<table>
<thead>
<tr>
<th>((p^1, p^2))</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.00; (0.40, 0.60)</td>
<td>0.05; (0.58, 0.84)</td>
<td>0.25; (0.73, 0.81)</td>
</tr>
<tr>
<td>2</td>
<td>0.25; (0.56, 0.72)</td>
<td>0.10; (0.92, 1.08)</td>
<td>0.10; (1.16, 1.11)</td>
</tr>
<tr>
<td>7</td>
<td>0.05; (0.70, 0.90)</td>
<td>0.10; (0.91, 1.24)</td>
<td>0.10; (1.33, 1.23)</td>
</tr>
</tbody>
</table>

- Even with multiple retailers, loose coupling is valid.
- Even when stockouts affect the sales, loose coupling is valid.
Pricing with Stockouts

**Retailer 1** and **2** have stockout rates $\nu_1$ and $\nu_2$.

If the product is **stocked out then the customer**: 

**Does Not Buy:** *Lost Sales*.

**Backorders**

Wait and Backorder from Preferred Retailer: *Retailer Preferring*.
Check Inventory at the other Retailer to Buy: *Availability Preferring*.

Loss of goodwill cost to a retailer is $d_i$. 

Competitive Pricing under Stockouts

Lost Sales
Sales Probability: \[ \rho^1(p^1, p^2) = (p - c_i)(1 - W^i(p))(1 - \nu_i)[\phi_i + \phi_{-i}((1 - \nu_{-i})W^{-i}(p^{-i}) + \nu_{-i})] \]. 
\[ \max_p \left\{ (1 - \nu_1) \left[ p(1 - W_1(p)) (\phi + (1 - \phi)(W_2(p^2) + \nu_2)) - c_1 \right] \right\}. \]

Backorders with Retailer Preferring customers
\[ \max_p (p - c_1 - \nu_1 d_1)[1 - W^1(p)][\phi + (1 - \phi)W^2(p^2)]. \]

Backorders with Availability Preferring customers
\[ \max_p \left\{ (p - c_1 - \nu_1 d_1) \left[ 1 - W^1(p) \right] \right\} \left\{ (1 - \nu_1) \left[ \phi + (1 - \phi) \left\{ (1 - \nu_2)W^2(p^2) + \nu_2 \right\} \right] \right\} + \nu_1 \left[ \phi \left\{ \nu_2 + (1 - \nu_2)W^2(p^2) \right\} + (1 - \phi)W^2(p^2) \right\} \}. \]

Every profit expression has three terms multiplied.
First two structurally same as before while the third is independent of own price.

“Loose coupling of retailers” is preserved under Stockouts!