Profit Maximization in Air Cargo Overbooking

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Interior of a Boeing 747 Cargo Plane
Outline

- Significance and Introduction
  - Introduction
  - Pricing Structure
  - Literature
  - Contribution and Implementation

- Detailed Revenue

- Detailed, Aggregate and Upper Bound Profits

- Results

- Performance Analysis
Significance

- Cargo:
  - generates substantial revenues.
  - is expected to grow at a rate of 6.2%.
  - is one of the fastest growing segment in the U.S. economy.
Overbooking

- Cargo booking process:

  - Shippers make reservations, usually on the internet.
  - Airlines *accept a booking* if total cargo \( \leq \) overbooking limits.
  - *Spoilage* happens when *many* cargo bookings do not show up.
  - *Overbook* cargo planes to reduce spoilage.
  - *Offload* happens when *many* of the overbooked bookings show up.
  - Spoilage and Offload reduce the profits.
  - Airlines make revenue from the *loaded cargo*. 
A Two-Dimensional Pricing Structure

- B2B Contracts: Revenue collected when the cargo is loaded.
- Airlines charge the shippers depending by volume and weight.
  
  - Chargeable weight computed using an "inverse density $d^{-1}$".
  - Inverse density allows us to find **the weight equivalent of volume**

\[
\text{Revenue}(\text{Volume, Weight}) = a \max\{\frac{\text{Volume}}{d^{-1}}, \text{Weight}\}
\]

\[
\text{Offload cost}(\text{Volume, Weight}) = b \text{ Chargeable Weight}
\]

- $a, b, d$ are known constants. Are they really?
- Choose the volume units so that $d^{-1} = 1$. 
Literature

Passenger overbooking

- Static Models:
  Shlifer and Vardi (1975), Bodily and Pfeifer (1992)
  McGill and van Ryzin (1999); Talluri and van Ryzin (2004)

- Dynamic Models:

Cargo Papers

- Overbooking Papers:
  Kasilingam (1996), Luo et al. (2005)

- Other Papers:
  Popescu et al. (2005), Cooper et al. (2005)
Contribution and Implementation

- Two-dimensional nonlinear chargeable weight overbooking problem.
- **Detailed** formulation, considering cargos individually
  - Difficult to construct and solve.
  - “So many airlines don’t have adequate data to implement a [cargo] RM [revenue management] system”, (Talluri and van Ryzin, 2004).
- **Parsimonious** formulation needed.
  - Parsimonious (=Aggregate) formulation has the sum of volume and weight of cargos.
    - Easy to set up: Aggregate model is **independent** of the booking requests distribution.
    - Easy to solve: Optimal overbooking **curve** is easy to find.
    - Easy to implement: The optimal overbooking curve turns out to be a **box**.
- An upper bound formulation used to measure the difference between aggregate and detailed models.
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Notation

- Inputs:
  - $k_v, k_w$: the volume and weight capacity of the cargo aircraft.
  - $B$: estimate of the total booking requests.
  - $\theta$: is the physical cargo density.
  - $\xi$: is the show up rate.

- Decision Variable:
  - $r(\cdot)$: the overbooking limit curve in the volume x weight plane.

- Consequential variables:
  - $N$: the number of cargos that show up at departure.
  - $N'$: the number of cargos that is loaded at departure.

- The profit $\Pi = \text{Revenue from loaded cargos } - \text{Offload cost.}$
Detailed Revenue

- For the $i$th cargo, we denote by $w_i$ and $v_i$ the weight and volume.

$$\text{Detailed Revenue} = a \sum_{i=1}^{N'} \max\{v_i, w_i\}$$

sum of the chargeable weights for loaded cargos

- Requires estimation of $N'$, $v_i$, $w_i$: Extensive data needs.

  - $N'$ is a consequence of the booking requests, which are not captured in databases when not accepted.
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Detailed, Aggregate and Upper Bound Formulations

Detailed Profit $= \Psi^d(r) = \mathbb{E} \left[ a \sum_{i=1}^{N'(r)} \max\{v_i, w_i\} - b \max \left\{ \sum_{i=N'(r)+1}^{N(r)} v_i, \sum_{i=N'(r)+1}^{N(r)} w_i \right\} \right]$. 

Aggregate Profit $= \Psi(r) = \mathbb{E} \left[ a \max \left\{ \sum_{i=1}^{N'(r)} v_i, \sum_{i=1}^{N'(r)} w_i \right\} - \text{Offload Cost} \right]$. 

Aggregate Profit $\leq$ Detailed Profit $\leq$ Upper Bound Profit

$\Psi(r) \leq \Psi^d(r) \leq \Psi^u(r)$
**Assumptions**

- **Same density assumption**: Loaded cargo has the same density $\theta$ as the showing up cargo.

- **Divisible cargo assumption**: Approximates the volume and weight loaded on the aircraft.
The cargo is really divisible
Rewriting Aggregate and Upper Bound Profits Using Show-Ups

- Reparametrization of show ups

\[
\text{volume of show-ups} = S_v(r, \theta) := \min \{ r(\theta), B(\theta) \} \frac{1}{\sqrt{1+\theta^2}} \xi = \sum_{i=1}^{N(r,\theta)} v_i
\]

\[
\text{weight of show-ups} = S_w(r, \theta) := \min \{ r(\theta), B(\theta) \} \frac{\theta}{\sqrt{1+\theta^2}} \xi = \sum_{i=1}^{N(r,\theta)} w_i
\]

- We write profits as functions of \( S_v(r, \theta), S_w(r, \theta), k_v \), and \( k_w \) using the assumptions.

- If no offload

  volume loaded = volume of the show-ups

  weight loaded = weight of the show-ups

- If offload due to lack of volume capacity

  volume loaded = volume capacity \( k_v \)

  weight loaded = \( k_v \) multiplied by the density \( \frac{S_w}{S_v} \)

- If offload due to lack of weight capacity

  weight loaded = weight capacity \( k_w \)

  volume loaded = \( k_w \frac{S_v}{S_w} \)
Cases for Computing Aggregate Revenue and Offload Cost

Case III: Weight capacity caused offload
\[ \frac{S_w}{S_v} \geq \frac{k_w}{k_v}, \]
\[ S_w \geq k_w \]

Case II: Volume capacity caused offload
\[ \frac{S_w}{S_v} \leq \frac{k_w}{k_v}, \]
\[ S_v \geq k_v \]

Case I: No offload
\[ S_v \leq k_v \]
\[ S_w \leq k_w \]
Steel pipes in 100 ton capacity Antonov 124
Profits in Detail

- **Aggregate Profit:**

\[
\Pi(r, \theta) = \begin{cases} 
\text{revenue} & - \text{Offload Cost} \\
\max \{S_v, S_w\} & 0 \\
\max \{k_v S_w / S_v\} - b \max \{S_w - S_w k_v S_v, S_v - k_v\} & \text{if no offload,} \\
\max \{k_w S_v / S_w, k_w\} - b \max \{S_w - k_w, S_v - S_v k_w S_w\} & \text{if offload is because of volume,} \\
\max \{k_w S_v / S_w, k_w\} - b \max \{S_w - k_w, S_v - S_v k_w S_w\} & \text{if offload is because of weight}
\end{cases}
\]

- **Upper Bound Profit:**

\[
\Pi^u(r, \theta) = \begin{cases} 
\alpha S_v + \beta S_w & - 0 \\
\alpha k_v S_w / S_v + \beta k_v S_w & - b \max \{S_w - S_w k_v S_v, S_v - k_v\} & \text{if no offload,} \\
\alpha k_w S_v / S_w + \beta k_w S_w & - b \max \{S_w - k_w, S_v - S_v k_w S_w\} & \text{if offload is because of volume,} \\
\end{cases}
\]

where \(\alpha = \left(\frac{1-\theta}{\theta-\theta}\right)\) and \(\beta = \left(\frac{\theta-1}{\theta-\theta}\right)\), depend on the support of the density.
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- **Easy to solve:** Optimization separates over densities

\[
\max_{r(.)} \Psi(r) = \max_{r(.)} \int_0^{\infty} \Pi(r(\theta), \theta) dH(\theta) = \int_0^{\infty} \max_r \Pi(r, \theta) dH(\theta)
\]

- **Easy to set up:** Distribution of \(B\) unnecessary because limiting \([B \to \infty]\) yields the optimal curve solution

\[
\arg \max_r \Pi(r, \theta) = \arg \max_r \Pi^\infty(r, \theta)
\]

- **Easy to implement:** Optimality of a box: \(\{\arg \max_r \Pi(r, \theta) : 0 \leq \theta < \infty\}\) is a box characterized by \((l_v, l_w)\)

\[
k_w/l_w^* = k_v/l_v^*, \text{ where } l_v \text{ solves } \frac{b}{a + b} = \frac{1}{\text{E}(\xi)} \int_0^{k_v/l_v^*} x dG(x)
\]
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Performance Analysis of the Aggregate Model

- Parameters:
  - Show up rate estimated with Beta distribution: $\xi \sim \beta(\eta = 3.59, \zeta = 2.23)$.
  - Weight and volume in ton and $5 \text{ m}^3$.
  - $a=\$5K$, $b = \$7K$. $k_v = 125 \text{ m}^3$ and $k_w = 10 \text{ tons}$.
  - Booking requests are not available, but suppose it is normal with standard deviation 16, and means 40, 50 and 60.
  - Density has a triangular distribution with support $[\underline{\theta}, \bar{\theta}]$, which depends on the flight.
Differences between the Aggregate and Upper bound objectives

<table>
<thead>
<tr>
<th>Support of the physical density $(\theta, \overline{\theta})$</th>
<th>% Difference</th>
<th>( B \to \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Expected booking requests</td>
<td></td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>(0.70, 1.10)</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>(0.65, 1.15)</td>
<td>0.3</td>
<td>2.1</td>
</tr>
<tr>
<td>(0.60, 1.20)</td>
<td>2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>(0.55, 1.25)</td>
<td>3.6</td>
<td>3.8</td>
</tr>
<tr>
<td>(0.50, 1.30)</td>
<td>3.7</td>
<td>4.2</td>
</tr>
<tr>
<td>(0.45, 1.35)</td>
<td>4.0</td>
<td>6.0</td>
</tr>
<tr>
<td>(0.40, 1.40)</td>
<td>5.2</td>
<td>6.8</td>
</tr>
<tr>
<td>(0.35, 1.45)</td>
<td>6.3</td>
<td>7.1</td>
</tr>
<tr>
<td>(0.30, 1.50)</td>
<td>7.3</td>
<td>8.6</td>
</tr>
<tr>
<td>(0.25, 1.55)</td>
<td>7.8</td>
<td>9.0</td>
</tr>
<tr>
<td>(0.20, 1.60)</td>
<td>8.5</td>
<td>9.3</td>
</tr>
<tr>
<td>(0.00, 1.80)</td>
<td>13.8</td>
<td>14.4</td>
</tr>
</tbody>
</table>
Differences between the Aggregate and Upper bound objectives

- Difference increases as $\mathbb{E}(B)$ does so, because the effect of the overbooking curve becomes more pronounced.
- Difference increases as $\overline{\theta} - \underline{\theta}$ does so, because the upper bound revenue increases in $\overline{\theta} - \underline{\theta}$.

- Difference $\leq 5\%$ good enough?
The actual Difference between the detailed and the aggregate profits is less.
Aggregate formulation does not need the information $B$, $v_i$ and $w_i$ that the detailed formulation requires.
- Sacrifice less than 5\% for an easy to set up, to solve and to implement formulation?

- The support $[\underline{\theta}, \overline{\theta}]$ is narrow because the problem is solved for a certain flight.
Same/similar type of cargo is sent between an origin and a destination pair.
### Our Model: A Good Approximation for 60% of Real-Life Instances

<table>
<thead>
<tr>
<th>First column</th>
<th>Number of instances with density range $\overline{\theta} - \underline{\theta} &lt; $ first column</th>
<th>Percentage of instances over all RDs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reading Days (RDs) 2 5 7 10 14 21 28</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>3 2 0 2 3 3 3</td>
<td>28.6%</td>
</tr>
<tr>
<td>0.7</td>
<td>4 3 3 3 4 6 3</td>
<td>46.4%</td>
</tr>
<tr>
<td>0.9</td>
<td>4 3 4 6 5 7 4</td>
<td>58.9%</td>
</tr>
<tr>
<td>1.0</td>
<td>4 5 4 6 6 7 5</td>
<td>66.1%</td>
</tr>
<tr>
<td>1.1</td>
<td>5 5 5 6 6 7 5</td>
<td>69.6%</td>
</tr>
<tr>
<td>1.2</td>
<td>5 5 6 6 6 7 6</td>
<td>73.2%</td>
</tr>
<tr>
<td>1.3</td>
<td>5 6 6 6 6 7 6</td>
<td>75.0%</td>
</tr>
<tr>
<td>1.4</td>
<td>5 6 7 7 7 7 6</td>
<td>80.4%</td>
</tr>
<tr>
<td>1.6</td>
<td>7 7 8 7 7 7 6</td>
<td>87.5%</td>
</tr>
<tr>
<td>1.8</td>
<td>8 8 8 8 8 8 7</td>
<td>98.2%</td>
</tr>
</tbody>
</table>
Conclusions

- Aggregate formulation is appropriate for 60% of the cases found in our real-life data.

- Aggregate formulation is
  - easy to set up,
  - easy to solve,
  - easy to implement,

- Future Research: Solution of the detailed formulation or a dynamic multi-period formulation.