The Passivity Paradigm in the Control of Bipedal Robots

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This work was performed in collaboration with Gagandeep Bhatia, Andrew Block, and Francesco Bullo. Partial support was provided by the National Science Foundation grant ECS-0122412.

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The Passivity Paradigm

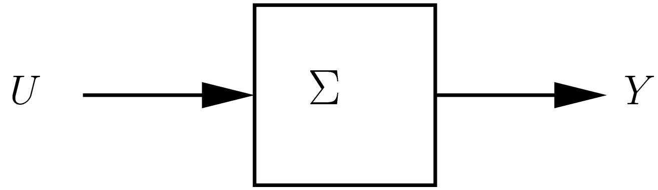
Consider a dynamical system represented by the state model

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x,u) \end{aligned}$$

where f is locally Lipschitz, h is continuous, $f(0,0) = 0$, $h(0,0) = 0$ and the system has the same number of inputs and outputs.

The system is said to be *Passive* if there exists a C^1 positive semidefinite scalar function $S : R^n \to R$ called the *Storage Function* such that

$$\dot{S} \leq y^T u \text{ for all } (x,u) \in R^n \times R^p$$



Such a passive system Σ is stabilizable by output feedback

$$u = -ky$$

since then we have

$$\dot{S} \leq -ky^T y = -k||y||^2 \leq 0$$

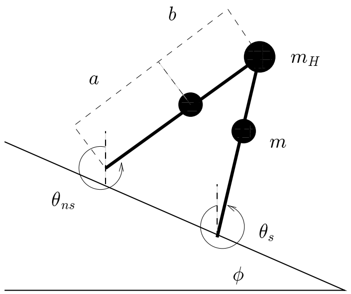
Under some additional (detectability) conditions asymptotic stability follows.

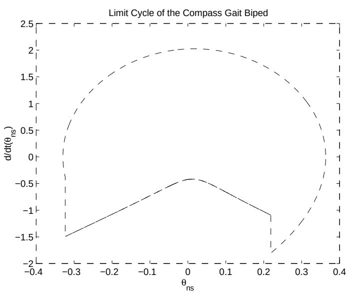
- Parallel and feedback interconnections of passive systems are passive
- Generally the convergence is to a manifold (LaSalle's Invariance Principle)

Passive Walking and Passivity Based Control

Stable passive limit cycles for bipedal robots, when they exist, are typically extremely sensitive to changes in ground slope, have small basins of attraction, and slow convergence. In this talk we will show:

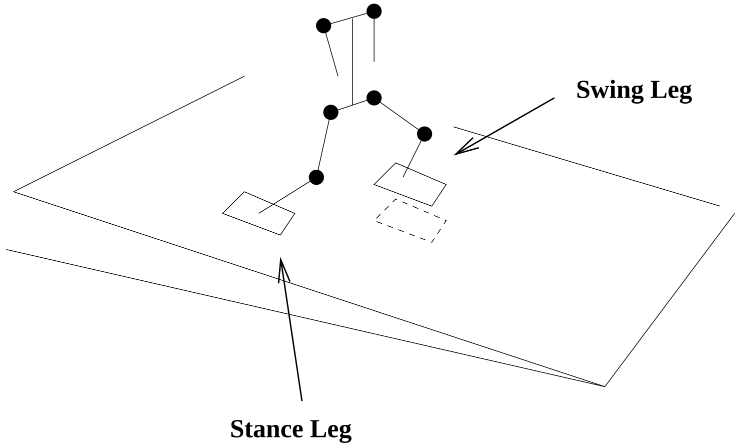
- a fundamental connection between passive walking and passivity based control
- how to completely remove the sensitivity to ground slope
- How to increase the basin of attraction and speed of convergence to the limit cycles





- Our results rely on some ideas of symmetry under group actions in Lagrangian systems as well as passivity-based control of hybrid systems.
- Furthermore, our results hold for the most general case of an n-link biped in 3-D.

Let Q represent the configuration space of the robot and let $h(q) = 0$ represent the constraint mapping (e.g. foot/ground contact under the usual assumptions of perfectly inelastic impact, no slipping, and instantaneous transfer of support).



The change in velocity at impact is given as a projection onto $\{v \in T_qQ | dh_i(q) \cdot v = 0\}$

$$\dot{q}(t^+) = P_q(\dot{q}(t^-))$$

The dynamics of a general biped can be therefore be written as a hybrid nonlinear system

$$
\begin{array}{rcll}
L(t,q,\dot q) & = & u, & \text{for } h(q(t^-)) \neq 0 \\
q(t^+) & = & q(t^-) & \\
\dot q(t^+) & = & P_q(\dot q(t^-)) & \text{for } h(q(t^-)) = 0
\end{array}
$$

where the operator $L(t,q,\dot q) = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot q} - \frac{\partial \mathcal{L}}{\partial q}$ and $\mathcal{L}$ is the Lagrangian (Kinetic minus Potential energy)

In local coordinates we may write the more familiar expression

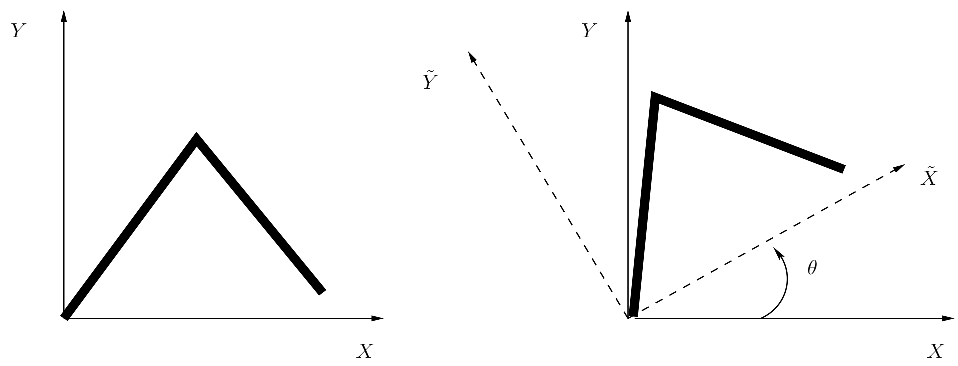
$$
\begin{array}{rcll}
M(q)\ddot q + C(q,\dot q)\dot q + g(q) & = & u, & \text{for } h(q(t^-)) \neq 0 \\
q(t^+) & = & q(t^-) & \\
\dot q(t^+) & = & P_q(\dot q(t^-)) & \text{for } h(q(t^-)) = 0
\end{array}
$$

Slope Changing Action

A passive limit cycle results from a delicate balance among kinetic energy, potential energy, and impacts.

The key observation to address the sensitivity of the limit cycle to the ground slope is to recognize that the act of changing the ground slope at the stance leg can be represented by a Group Action, Φ, of the rotation group $SO(3)$ on the configuration space of the robot. For $A \in SO(3)$, $\Phi_A : Q \to Q$.

Pictorially, the action is shown below



Invariance Under Group Actions

One can show (For details see Spong, Bullo, IEEE TAC 2004, under review)

1. the kinetic energy $\mathcal{K}\colon TQ \to \mathbb{R}$ is invariant under the slope changing action, that is, for all $A \in \mathrm{SO}(3)$, $\mathcal{K}(q, \dot{q}) = \mathcal{K}(\Phi_A(q), T_q\Phi_A(\dot{q}))$.
2. the impact projection map P is equivariant with respect to $SO(3)$, i.e.

$$T\Phi_A\left(P_q(v)\right) = P_{\Phi_A(q)}\left(T\Phi_A(v)\right),$$

for all $v \in T_qQ$.

Remark: If the Potential Energy were invariant under this group action then flows (i.e. solutions of the equations of motion would also be invariant) The implication is that a passive limit cycle would exist on any slope if it existed on one particular slope!

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A Symmetry in a mechanical system arises when the Lagrangian is invariant under a group action Φ, i.e.

$$\mathcal{L}(q, \dot{q}) = \mathcal{L}(\Phi_A(q), T_q\Phi_A(\dot{q})) \quad \text{for all } A \in G \text{ (a Lie Group)}$$

Symmetries give rise to conserved quantities, for example, translational symmetry gives rise to conservation of momentum, etc.

Definition *Controlled Symmetry*
We say that an Euler-Lagrange system has a Controlled Symmetry with respect to a group action Φ if, for every $A \in G$, there exists an admissible control input $u_A(t)$ such that

$$L(t, q, \dot{q}) - u_A(t) = L(t, \Phi_A(q), T_q\Phi_A(\dot{q}))$$

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Let $E(q,\dot{q}) = \mathcal{K} + \mathcal{V}$ be the total energy of the robot and E_{ref} a reference energy (for example, the energy along a limit cycle trajectory).

For $A \in SO(3)$ define the Storage Function

$$S = \frac{1}{2}(E \circ \Phi_A - E_{ref})^2$$

Then

$$\dot{S} = (E \circ \Phi_A - E_{ref})\dot{q}^T \left[u - \frac{\partial}{\partial q}\Big(\mathcal{V}(q) - \mathcal{V} \circ \Phi_A(q)\Big)\right]$$

If we define the control input u as $u = u_A + \tilde{u}$, where

$$u_A = \frac{\partial}{\partial q}\Big(\mathcal{V}(q) - \mathcal{V} \circ \Phi_A(q)\Big)$$

Then

$$\dot{S} = (E \circ \Phi_A - E_{ref})\dot{q}^T \tilde{u} = y^T \tilde{u}$$

where

$$y = \dot{q}(E \circ \Phi_A - E_{ref})$$

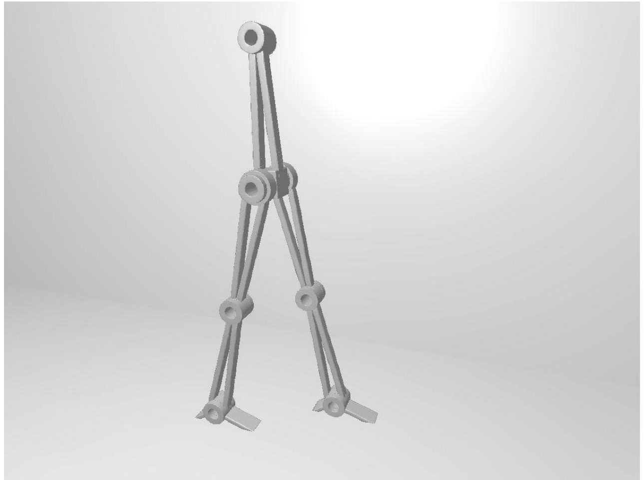
defines a passive output.

It can be shown (Spong, Bullo, 2004) that for $\tilde{u} = 0$, the control u_A defines a Controlled Symmetry and E_{ref} is an invariant manifold.

Main Result #1

Theorem: Suppose there exists a passive gait on one ground slope, represented by $A_0 \in SO(3)$, and let $A \in SO(3)$ represent any other slope. Then the control input $u_{A^T A_0}$ generates a walking gait on slope A. Moreover the basin of attraction of the passive gait is mapped to the basin of attraction of the controlled gait.

This video shows a biped with a torso walking on level ground



- Having made the passive limit cycles slope invariant via potential energy shaping we now investigate total energy shaping for robustness.
- Improving the rate of convergence to the limit cycle and increasing the basin of attraction are needed for robustness to external disturbances, changes in ground slope, etc.
- We now consider the design of the control input $\tilde{u}$ in

$$u = u_A + \bar{u}$$

- In other words we consider the system

$$L(t, \Phi_A(q), T_q\Phi_A(\dot{q})) = \bar{u}$$

With the Storage Function $\mathcal{S}$ as before

$$\mathcal{S} = \frac{1}{2}(E \circ \Phi_A - E_{ref})^2$$

We saw that

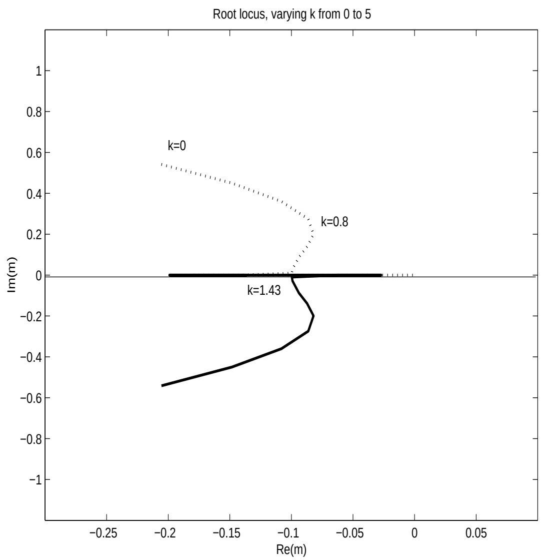
$$\dot{S} = (E \circ \Phi_A - E_{ref})\dot{q}^T \tilde{u} = y^T \tilde{u}$$

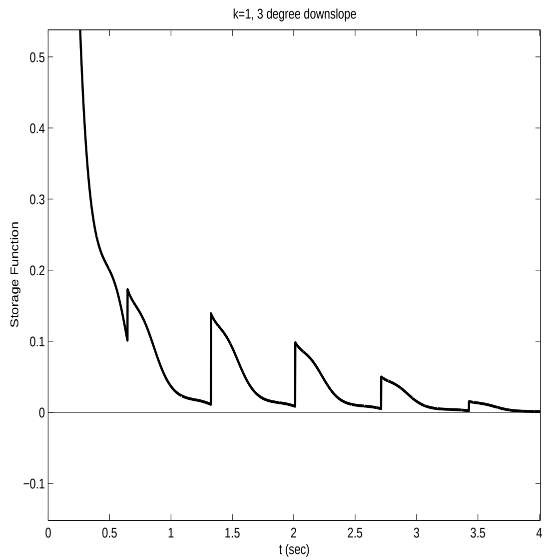
if we choose the additional control $\bar{u}$ according to

$$\bar{u} = -ky = -k\dot{q}(E \circ \Phi_A - E_{ref})$$

we obtain

$$\dot{\mathcal{S}} \quad = \quad -ky^T y = -k||\dot{q}||^2 S$$

- Thus $S(t)$ converges exponentially toward zero during each step[a]
- At impacts, S will experience a jump discontinuity. If the value of S at impact $k+1$ is less than it's value at impact k it follows that $E(t)$ converges to E_{ref}.
- 



[a] assuming $\dot{q}$ is bounded away from zero

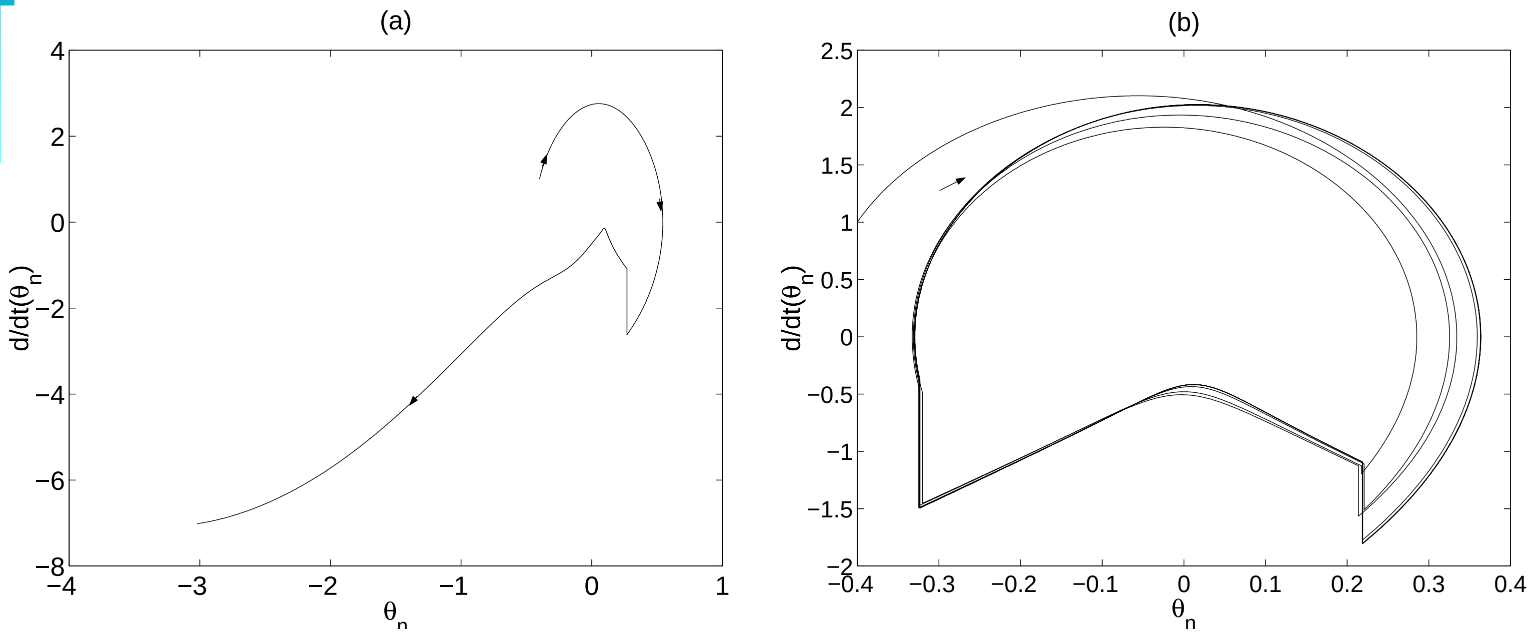
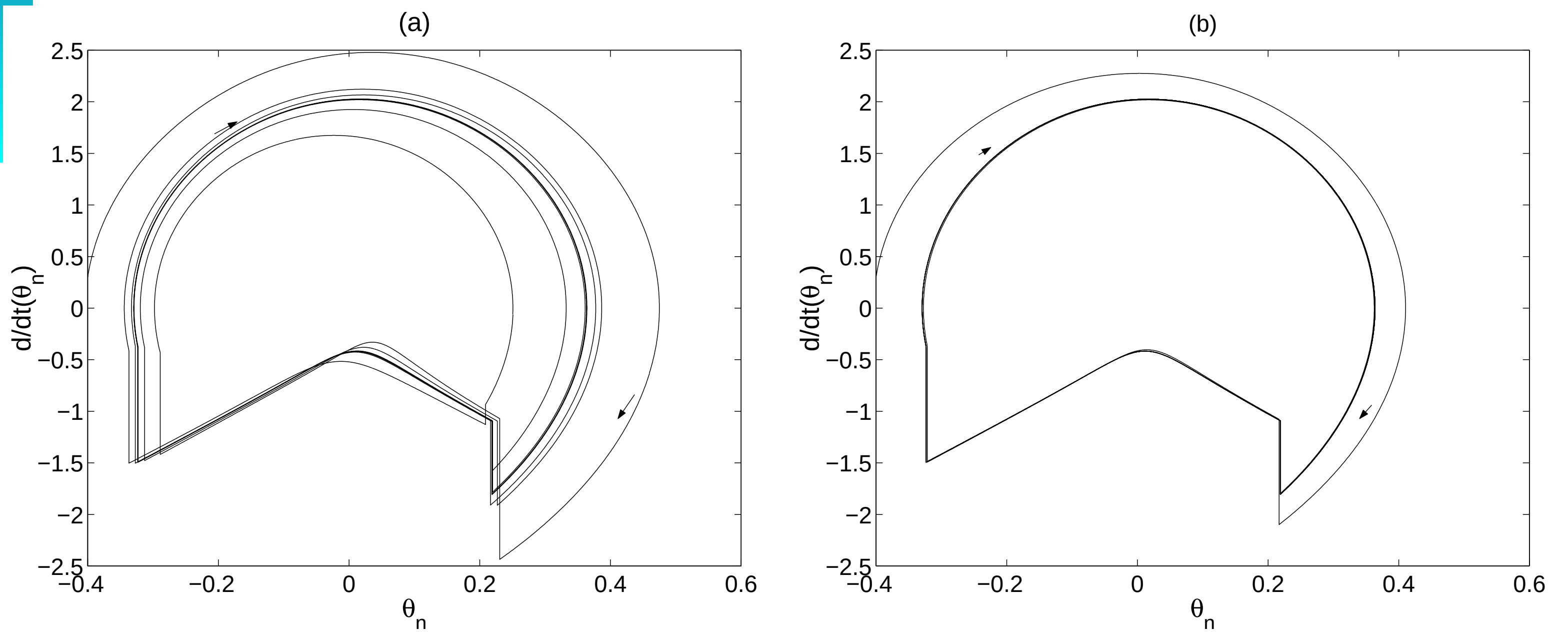


Illustration of increased basin of attraction (a) without total energy control (b) with total energy control.



Convergence to the limit cycle (a) without total energy control (b) with total energy control.

- Simulation: Compass Gait Biped Walking on a Varying Slope