What's Passivity Got To Do With It?

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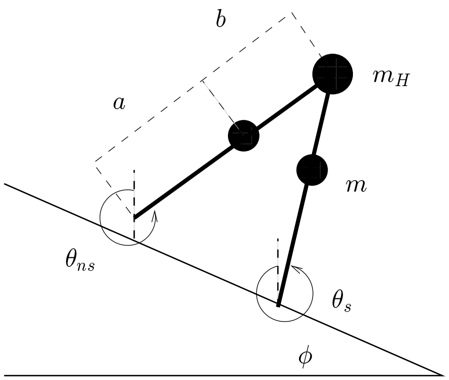
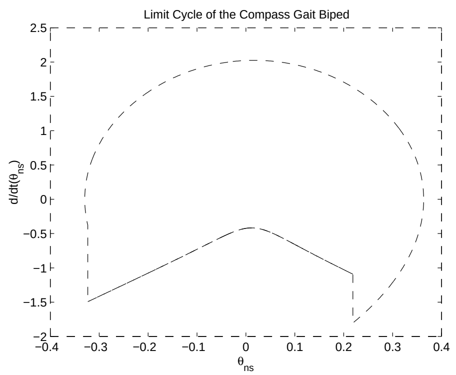
OUTLINE

- Why is walking easy? Why is it difficult?
- What tools are available from control theory?
- What new tools must be developed?

Why is Walking Easy?

- passive gaits can be found on shallow slopes without control.
- rigorous analysis can be carried out at least in the planar case.
- ‘Simple’ actuation can mimic these trajectories on level ground.





Why is Walking Difficult?

- Complex dynamics — impacts, friction, underactuation
- The control problems are inherently hybrid and nonlinear
- Most “success stories” have been limited to
  1. planar walking
  2. level terrain
  3. few degrees-of-freedom
  4. slow or low performance
  5. heuristic methods

What Tools Are Available?

- Hybrid and Switching Control
- Geometric Nonlinear Control
  1. Feedback Linearization
  2. Hybrid Zero Dynamics
- Lagrangian and Hamiltonian Methods
  1. Symmetry
  2. Reduction
  3. Passivity-Based Control
  4. Synchronization

What Tools Must Be Developed?

- To develop a rigorous theory of hybrid systems treating:
  1. Impacts
  2. Underactuation
  3. 3-D Motion
  4. Gait Transitions
  5. Limited Control Effort
  6. . . .

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Some Examples

An n-link biped can be modeled as a hybrid Euler-Lagrangian system subject to unilateral (holonomic) constraints due to impacts:

$$\begin{aligned} L(t, q, \dot{q}) &= u && \text{for } h(q(t^-)) \neq 0 \\ q(t^+) &= q(t^-) && \text{for } h(q(t^-)) = 0 \\ \dot{q}(t^+) &= P_q(\dot{q}(t^-)) && \end{aligned}$$

where the operator $L(t, q, \dot{q}) = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q}$

Let $\Phi : G \times Q \to Q$ be a group action of a Lie Group G on the configuration space Q of an n-link biped.

A Symmetry in a mechanical system arises when the Lagrangian is invariant under such a group action, i.e.

$$\mathcal{L}(q, \dot{q}) = \mathcal{L}(\Phi_A(q), T_q\Phi_A(\dot{q})) \quad \text{for all } A \in G$$

where $\mathcal{L}$ is the Lagrangian (Kinetic minus Potential energy)

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Controlled Symmetry

Definition

We say that an Euler-Lagrange system has a Controlled Symmetry with respect to a group action Φ if, for every $A \in G$, there exists an admissible control input $u_A(t)$ such that

$$L(t, q, \dot{q}) - u_A(t) = L(t, \Phi_A(q), T_q\Phi_A(\dot{q}))$$

where the operator $L(t, q, \dot{q}) = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q}$

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Let $\mathcal{V}$ be the potential energy of the robot. For $A \in SO(3)$ define the control input

$$u_A = \frac{\partial}{\partial q}\Big(\mathcal{V}(q) - \mathcal{V} \circ \Phi_A(q)\Big)$$

Theorem:

1. u_A defines a Controlled Symmetry
2. Suppose there exists a passive gait on one ground slope, represented by $A_0 \in SO(3)$, and let $A \in SO(3)$ represent any other slope. Then the control input $u_{A^T A_0}$ generates a walking gait on slope A.

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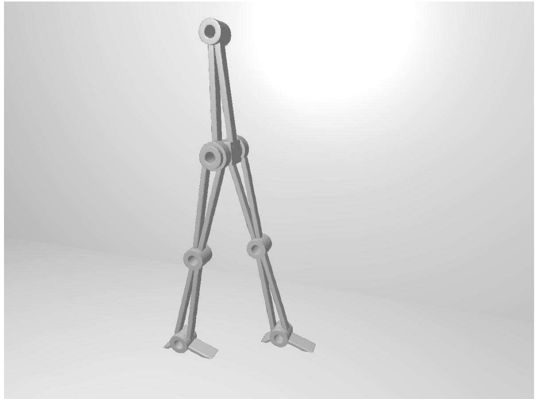
[Ref: Spong, M.W., and Bullo, F., "Controlled Symmetries and Passive Walking," IEEE Transactions on Automatic Control, Vol. 50, No. 7, pp: 1025-1031, July, 2005]

This video shows a biped with a torso walking on level ground using the above energy shaping control.



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Passivity Based Control

Definition: A system with input u and output y is *Passive* if there exists a nonnegative definition scalar function $S : X \to R$, called a Storage Function, from the state space X to R such that

$$S(x(t)) - S(x(0)) \leq \int_0^t u^T(\sigma) y(\sigma) d\sigma$$

If S is differentiable, then

$$\dot{S}(x(t)) \leq u^T(t) y(t)$$

A passive system can be stabilized by output feedback

$$u = -ky$$

which yields

$$\dot{S}(x(t)) \leq -ky^T(t)y(t) \leq 0$$

Under a zero-state-detectability assumption, the system is asymptotically stable. We can use this idea to “robustify” passive limit cycles.

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Consider the system

$$L(t, \Phi_A(q), T_q\Phi_A(\dot{q})) = \bar{u}$$

resulting from the control input

$$u = u_A + \bar{u}$$

The term u_A renders a passive limit cycle slope invariant and $\bar{u}$ is an additional control to be designed using the above notion of passivity.

Let $E = \mathcal{K} + \mathcal{V}$ be the Total Energy (Kinetic plus Potential) of the biped and define a Storage Function

$$\mathcal{S} = \frac{1}{2}(E \circ \Phi_A - E_{ref})^2$$

where E_{ref} is a reference energy. One can show that

$$\dot{S} = (E \circ \Phi_A - E_{ref})\dot{q}\bar{u} = y^T\bar{u}$$

Then $\bar{u} = -ky = -k\dot{q}(E \circ \Phi_A - E_{ref})$ yields

$$\dot{\mathcal{S}} = -ky^2 = -k||\dot{q}||^2 S$$

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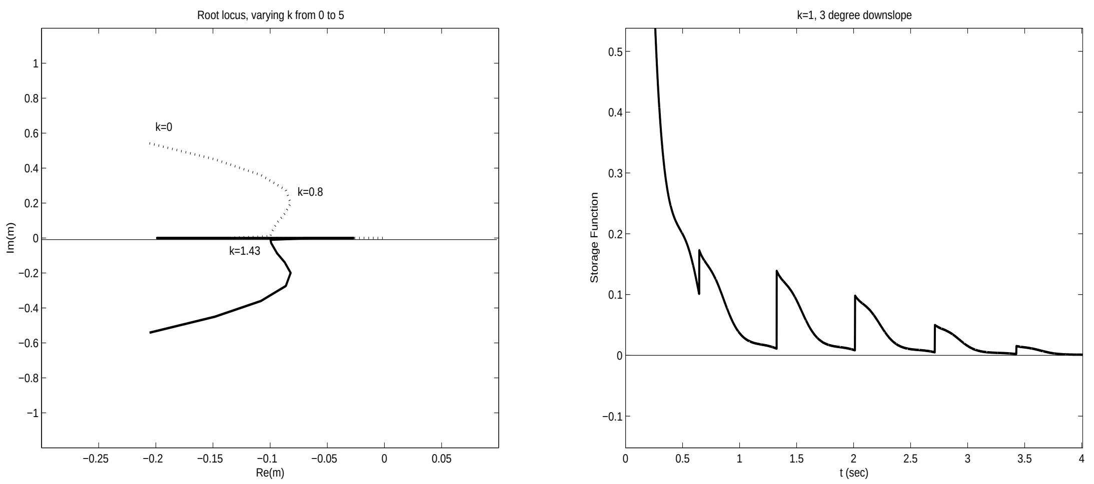
- Thus $S(t)$ converges exponentially toward zero during each step.
- If the value of S at impact $k+1$ is less than it's value at impact k it follows that $E(t)$ converges to E_{ref}. [Ref: G. Bhatia and M.W. Spong, IROS 2003]
- 

- Simulation: Walking on a Varying Slope

Reduction

- Lagrangian systems with cyclic variables can be "reduced" to lower dimensional systems.
- For example, 2-D walking can be exploited to achieve 3-D walking by suitable "dividing out" the lateral dynamics. The details are known as Routhian reduction.

[Ref: Ames and Sastry, "Towards the Geometric Reduction of Controlled Three-Dimensional Bipedal Robotic Walkers," IFAC 3rd Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control, Nagoya, Japan, July, 2006.]

Synchronization is a fascinating phenomenon arising in many natural and man-made systems:

- Synchronously flashing fireflies
- Schooling of fish and flocking of birds
- Superconducting Josephson junction arrays
- Kuramoto Oscillators

Example: Synchronization of Metronomes

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Synchronization

Consider N coupled passive systems

$$\begin{aligned} \dot{x}_i &= f_1(x_i) + g(x_i)u_i \\ y_i &= h(x_i)\ ;\ i = 1, \ldots, n \end{aligned}$$

The coupling control inputs $u_i = K \sum_{j \in \mathcal{N}_i} (y_j - y_i)$ results in output synchronization of the entire system.

[Ref: N. Chopra and M.W. Spong, “Output Synchronization of Networked Passive Systems, submitted, 2006]

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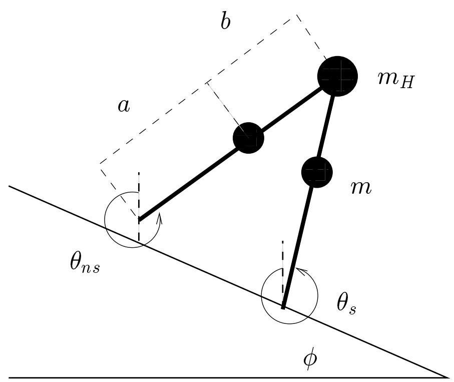
Synchronization



Consider a compass-gait biped with only a hip torque as a system of two-coupled pendula. Let the hip torque control be given as

$$u_H = K_1 + K_2(\dot{\theta}_s - \dot{\theta}_{ns})$$

The result is that the legs synchronize to a stable gait. This is provably correct via Poincaré analysis.

Conclusions

“. . . le souci du beau nous conduit aux mêmes choix que celui de l’utile.”— Henri Poincaré

- Heuristics can neither guarantee nor quantify stability, robustness, and performance
- Advanced control can lead to provably correct, computationally tractable algorithms

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