Some New Results in Passivity Based Control of Robots

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The Passivity Paradigm

In this talk we will discuss some uses of passivity-based control to two classes of systems that are receiving considerable current attention

1. hybrid control systems and
2. networked control systems

We will discuss applications of these ideas to

1. Bipedal Locomotion and
2. Teleoperation over Communication Networks

Passivity concepts have been well documented and so will not be repeated here due to time constraints. We just give one brief observation to start.

Consider a dynamical system represented by the state model

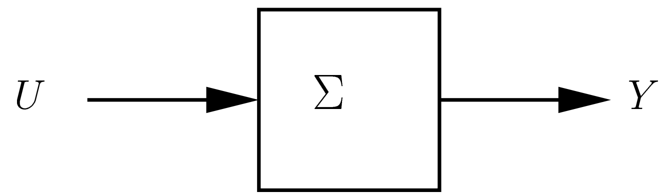
$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x,u) \end{aligned}$$

where f is locally Lipschitz, h is continuous, $f(0,0) = 0$, $h(0,0) = 0$ and the system has the same number of inputs and outputs.

The system is said to be *Passive* if there exists a C^1 positive semidefinite scalar function $S : R^n \to R$ called the *Storage Function* such that

$$\dot{S} \leq y^T u \text{ for all } (x,u) \in R^n \times R^p$$

Passivity Based Control



Such a passive system Σ is stabilizable by output feedback

$$u = -ky$$

since then we have

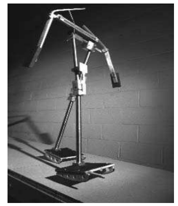
$$\dot{S} \leq -ky^T y = -k||y||^2 \leq 0$$

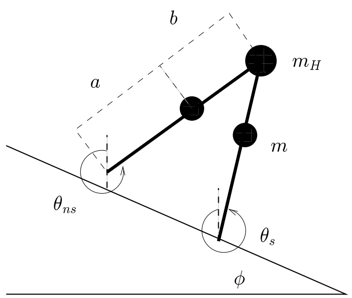
Under some additional (detectability) conditions asymptotic stability follows.

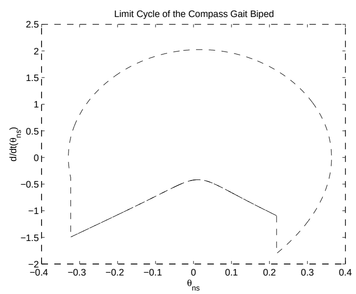
- Parallel and feedback interconnections of passive systems are passive
- Generally the convergence is to a manifold (LaSalle's Invariance Principle)

Passive Walking and Passivity Based Control

We first turn the attention of the Passivity Paradigm to the control of bipedal locomotion – a particular class of Hybrid Nonlinear Systems.

- It is well known that locomotion of mechanisms is achievable passively $\longrightarrow$ i.e., without actuation
- 3D Walker of Collins, Wisse, & Ruina 
- video courtesy of Martijn Wisse, Delft University of Technology, http://mms.tudelft.nl/Dbl/
- Impacts (foot/ground, knee strike) cause jumps in velocity $\longrightarrow$ a loss of Kinetic Energy. A passive limit cycle results when the loss of kinetic energy equals the change in potential energy during the step.





Let Q represent the configuration space of the robot and let $h(q) = 0$ represent the constraint surface (e.g. ground surface). The change in velocity at impact is given as a projection

$$\dot{q}(t^+) = P_q(\dot{q}(t^-))$$

onto $\{v \in T_qQ | dh(q) \cdot v = 0\}$. The dynamics of a general biped can be therefore be written as a hybrid nonlinear system

$$\begin{aligned} L(t, q, \dot{q}) &= u, & &\text{for } h(q(t^-)) \neq 0 \\ q(t^+) &= q(t^-) & &\text{for } h(q(t^-)) = 0 \\ \dot{q}(t^+) &= P_q(\dot{q}(t^-)) & & \end{aligned}$$

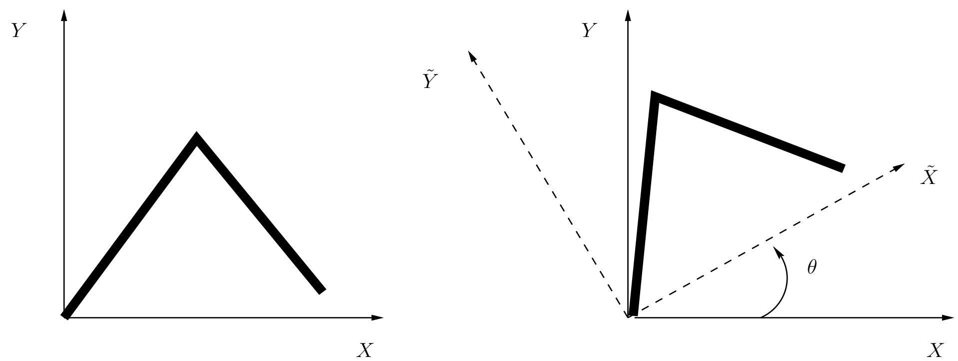
where the operator $L(t, q, \dot{q}) = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q}$ and $\mathcal{L}$ is the Lagrangian (Kinetic minus Potential energy)

Slope Changing Action

- A passive limit cycle, when it exists, is extremely sensitive to the ground slope and has a narrow basin of attraction (and of course little disturbance rejection capability since there is no control)
- In order to walk on level ground (or other slopes) and increase the basin of attraction, etc. active control is required

The key observation to address the sensitivity of the limit cycle to the ground slope is to recognize that the act of changing the ground slope at the stance leg can be represented by a Group Action, Φ, of the rotation group $SO(3)$ on the configuration space of the robot. For $A \in SO(3)$, $\Phi_A : Q \to Q$.

Pictorally, the action is shown below (For details see Spong, Bullo, IEEE TAC 2004, under review)



A Symmetry in a mechanical system arises when the Lagrangian is invariant under a group action Φ, i.e.

$$\mathcal{L}(q,\dot{q}) = \mathcal{L}(\Phi_A(q), T_q\Phi_A(\dot{q})) \quad \text{for all } A \in G \text{ (a Lie Group)}$$

Symmetries give rise to conserved quantities, for example, translational symmetry gives rise to conservation of momentum, etc.

Definition *Controlled Symmetry*

We say that an Euler-Lagrange system has a Controlled Symmetry with respect to a group action Φ if, for every $A \in G$, there exists an admissible control input $u_A(t)$ such that

$$L(t,q,\dot{q}) - u_A(t) = L(t, \Phi_A(q), T_q\Phi_A(\dot{q}))$$

Passivity Based Control

Let $E(q,\dot{q}) = \mathcal{K} + \mathcal{V}$ be the total energy of the robot and E_{ref} a reference energy (for example, the energy along a limit cycle trajectory).

For $A \in SO(3)$ define the Storage Function

$$S = \frac{1}{2}(E \circ \Phi_A - E_{ref})^2$$

Then

$$\dot{S} = (E \circ \Phi_A - E_{ref})\dot{q}^T \left[u - \frac{\partial}{\partial q}\Big(\mathcal{V}(q) - \mathcal{V} \circ \Phi_A(q)\Big)\right]$$

If we define the control input u as $u = u_A + \tilde{u}$, where

$$u_A = \frac{\partial}{\partial q}\Big(\mathcal{V}(q) - \mathcal{V} \circ \Phi_A(q)\Big)$$

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Then

$$\dot{S} = (E \circ \Phi_A - E_{ref})\dot{q}^T \tilde{u} = y^T \tilde{u}$$

where

$$y = \dot{q}(E \circ \Phi_A - E_{ref})$$

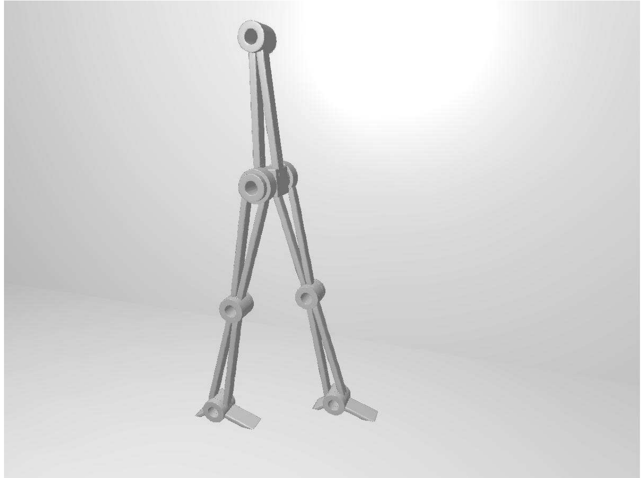
defines a passive output.

It can be shown (Spong, Bullo, 2004) that for $\tilde{u} = 0$, the control u_A defines a Controlled Symmetry and E_{ref} is an invariant manifold.

Main Result #1

Theorem: Suppose there exists a passive gait on one ground slope, represented by $A_0 \in SO(3)$, and let $A \in SO(3)$ represent any other slope. Then the control input $u_{A^T A_0}$ generates a walking gait on slope A. Moreover the basin of attraction of the passive gait is mapped to the basin of attraction of the controlled gait.

This video shows a biped with a torso walking on level ground



- Having made the passive limit cycles slope invariant via potential energy shaping we now investigate total energy shaping for robustness.
- Improving the rate of convergence to the limit cycle and increasing the basin of attraction are needed for robustness to external disturbances, changes in ground slope, etc.
- We now consider the design of the control input $\tilde{u}$ in

$$u = u_A + \bar{u}$$

- In other words we consider the system

$$L(t, \Phi_A(q), T_q\Phi_A(\dot{q})) = \bar{u}$$

With the Storage Function $\mathcal{S}$ as before

$$\mathcal{S} = \frac{1}{2}(E \circ \Phi_A - E_{ref})^2$$

We saw that

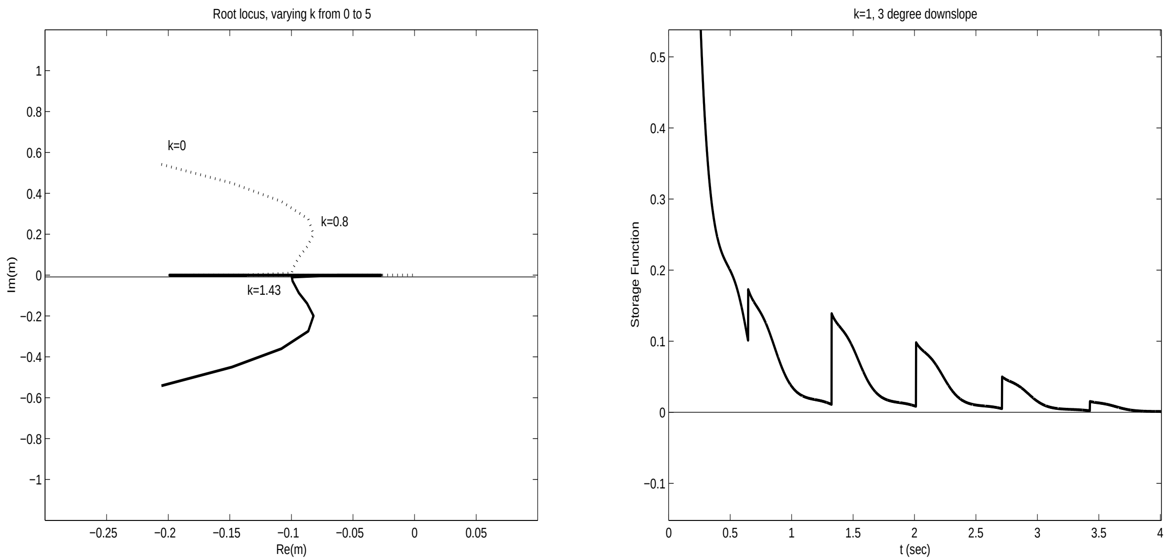
$$\dot{S} = (E \circ \Phi_A - E_{ref})\dot{q}\tilde{u} = y^T\tilde{u}$$

if we choose the additional control $\bar{u}$ according to

$$\bar{u} = -ky = -k\dot{q}(E \circ \Phi_A - E_{ref})$$

we obtain

$$\dot{\mathcal{S}} \quad = \quad -ky^2 = -k||\dot{q}||^2 S$$

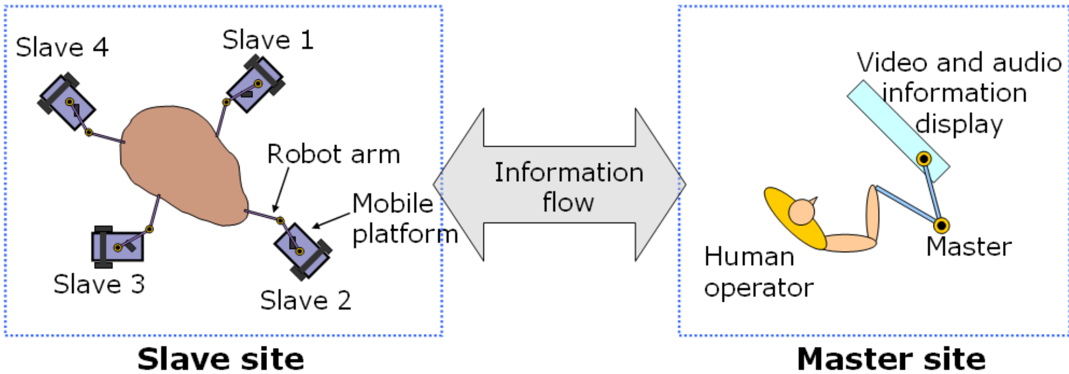
- Thus $S(t)$ converges exponentially toward zero during each step[a]
- At impacts, S will experience a jump discontinuity. If the value of S at impact $k+1$ is less than it's value at impact k it follows that $E(t)$ converges to E_{ref}.
- 

- Simulation: Walking on a Varying Slope

[a] assuming $\dot{q}$ is bounded away from zero

Networked Control Systems

Networked Control Systems (NCS) are currently of great interest for many applications. We present here a new passivity based architecture for a particular class of Networked Control Systems, namely Bilateral Teleoperators.



Our new architecture overcomes several difficulties associated with previous approaches, such as

- Lack of position tracking (drift)
- Poor transparency
- Sensitivity to parameter uncertainty

The Passivity Paradigm

Within the Passivity framework for networked systems one may incorporate:

- Time Varying Delay [cf: Lozano, R., Chopra, N., and Spong, M.W., “Passivation of Force Reflecting Bilateral Teleoperators with Time Varying Delay,” Mechatronics’02, Entschede, Netherlands, June 24-26, 2002]
- Discrete-Time Formulation, Packet Loss, Data Interpolation [cf: Berestesky, P., Chopra, N., and Spong, M.W., “Discrete Time Passivity in Bilateral Teleoperation over the Internet”, IEEE International Conference on Robotics and Automation, New Orleans, LA, April 26-May 1, 2004]
- Multi-Agent Coordination and Manipulation [cf: Lee, D., and Spong, M.W., “Passive Bilateral Teleoperation of Multiple Cooperative Robots with Delayed Communication,” in preparation]

Due to time constraints I will only discuss the new passivity architecture in:
Chopra, N., Spong, M.W., and Lozano, R., “Adaptive Coordination Control of Bilateral Teleoperators with Time Delay,” in preparation.

which addresses issues of transparency, position drift, and parameter uncertainty.

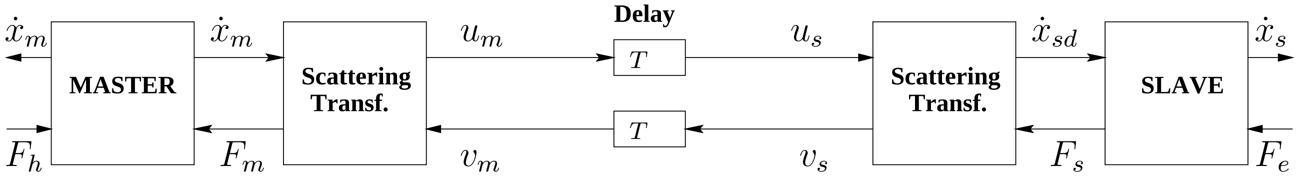
The Traditional Architecture

A bilateral teleoperator can be modeled as an interconnection of n-port networks. By designing control laws which impose the passivity property on each of the network blocks, passivity of the interconnection may be guaranteed.

The communication subsystem introduces a time delay, T, and is made passive by the well-known scattering transformation approach [cf: Anderson and Spong, 1989] where the scattering variables

$$u_m = \frac{1}{\sqrt{2b}}(F_{md} + \dot{x}_{md}) \qquad v_m = \frac{1}{\sqrt{2b}}(F_{md} - \dot{x}_{md})$$
$$u_s = \frac{1}{\sqrt{2b}}(F_{sd} + \dot{x}_{sd}) \qquad v_s = \frac{1}{\sqrt{2b}}(F_{sd} - \dot{x}_{sd})$$

are transmitted across the delay line instead of the original velocities and forces.



Passivity of Master and Slave Robots

The master and the slave robots are Lagrangian systems and are modeled as

$$\begin{aligned} M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) &= \tau_m \\ M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) &= \tau_s \end{aligned}$$

The well-known passivity property of the robot dynamics follows from the choice of storage functions, for $i = m, s$,

$$S_i(q_i, \dot{q}_i) = \frac{1}{2}\dot{q}_i^T M_i(q_i)\dot{q}_i + G_i(q_i)$$

where G_i is the potential energy. It is easy to verify, using the skew-symmetry property of robot dynamics that

$$\dot{S}_i = \dot{q}_i^T \tau_i$$

and hence the master dynamics are passive with $\tau_i, \dot{q}_i$ as input-output pairs.

The Control Algorithm

In order to achieve these design objectives, the master and slave torques are given, for $i = m, s$ as

$$\tau_i = -\hat{M}_i(q_i)\lambda\dot{q}_i - \hat{C}_i(q_i, \dot{q}_i)\lambda q_i + \hat{g}_i(q_i) + \bar{\tau}_i$$

where

- $\bar{\tau}_m$, $\bar{\tau}_s$ are the additional motor torques required for coordination control,
- "hats" represented estimated quantities and
- λ is a constant positive definite diagonal matrix.

It is easy to verify using linearity in the parameters that the master and slave dynamics reduce to

$$
\begin{aligned}
M_m \dot{r}_m + C_m r_m &= Y_m \tilde{\theta}_m + F_h + \bar{\tau}_m \\
M_s \dot{r}_s + C_s r_s &= Y_s \tilde{\theta}_s + \bar{\tau}_s - F_e
\end{aligned}
$$

where $\tilde{\theta}_m = \theta_m - \hat{\theta}_m$, $\tilde{\theta}_s = \theta_s - \hat{\theta}_s$ are the parameter estimation errors and

$$
\begin{aligned}
r_m &= \dot{q}_m + \lambda q_m \\
r_s &= \dot{q}_s + \lambda q_s
\end{aligned}
$$

The new master and slave dynamics are passive with $(\tilde{\tau}_m, r_m)$ and $(\tilde{\tau}_s, r_s)$ as the input-output pairs.

Define the coordinating torques F_{sd} and F_{md} as

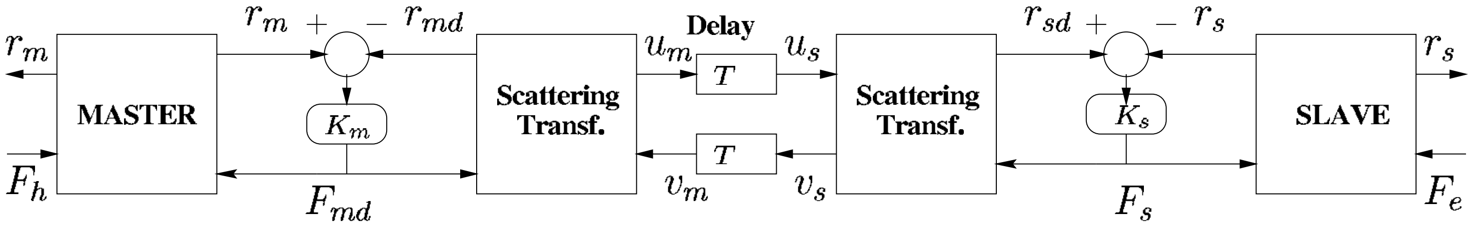
$$F_{sd} = K_s(r_{sd} - r_s) = \bar{\tau}_s$$
$$F_{md} = K_m(r_m - r_{md}) = -\bar{\tau}_m$$

where the gains K_s, K_m are constant positive definite diagonal matrices. Define the coordination errors between the master and slave robots as

$$e_m(t) = q_m(t - T) - q_s(t)$$
$$e_s(t) = q_s(t - T) - q_m(t)$$

Driving the coordination errors to the origin ensures that the master/slave system behave as a *kinematically locked* system.

We now concentrate on the communications and again use the scattering or the wave-variable transformation to passify the communication block.
The proposed architecture is



Within this architecture, we use the scatttering transformation as follows

$$u_m = \frac{1}{\sqrt{2b}}(F_{md} + br_{md}) \qquad v_m = \frac{1}{\sqrt{2b}}(F_{md} - br_{md})$$
$$u_s = \frac{1}{\sqrt{2b}}(F_{sd} + br_{sd}) \qquad v_s = \frac{1}{\sqrt{2b}}(F_{sd} - br_{sd})$$

Main Result

We choose parameter update laws according to

$$\begin{aligned}\dot{\hat{\theta}}_m &= \Gamma Y_m^T r_m \\ \dot{\hat{\theta}}_s &= \Lambda Y_s^T r_s\end{aligned}$$

where Γ and Λ are constant positive definite matrices.

Theorem 1 *Consider the nonlinear bilateral teleoperation system described above. Then all signals are bounded and the master and slave coordination errors are globally convergent to zero. Furthermore, in steady state with $\dot{e}_i$, $e_i = 0$ ($i = m, s$), the master and the slave velocities converge to the origin.*

Proof: The proof relies on the Storage/Lyapunov function

$$\begin{aligned} S &= \frac{1}{2}r_m^T M_m r_m + \frac{1}{2}r_s^T M_s r_s + e_m^T K_1 e_m + e_s^T K_2 e_s \\ &\quad + \tilde{\theta}_m^T \Gamma^{-1} \tilde{\theta}_m + \tilde{\theta}_s^T \Gamma^{-1} \tilde{\theta}_s + \int_0^t (F_e^T r_s - F_h^T r_m) d\tau \\ &\quad + \int_0^t (F_{md}^T r_{rd} - F_{sd}^T r_{sd}) d\tau \end{aligned}$$

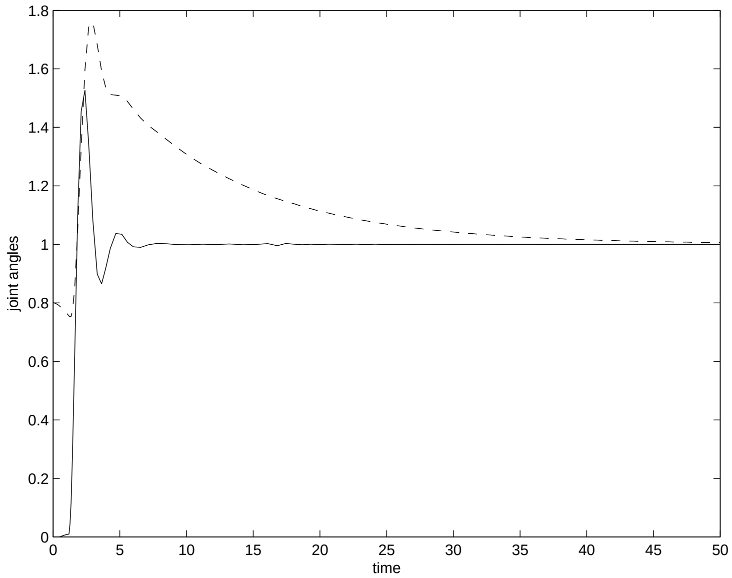
The term $\int_0^t (F_{md}^T r_{rd} - F_{sd}^T r_{sd}) d\tau$ is positive as a result of the scattering transformation. The term $\int_0^t (F_e^T r_s - F_h^T r_m) d\tau$ as long as the human/environment subsystems are passive.

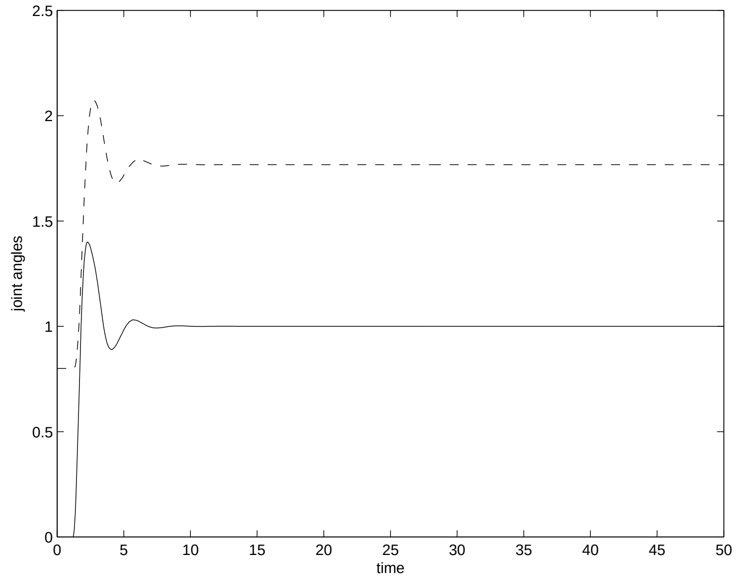
We simulated the schemes on a single-degree of freedom bilateral teleoperator, with the master and slave dynamics given as

$$\begin{aligned} M_m \ddot{q}_m &= F_h + \tau_m \\ M_s \ddot{q}_s &= \tau_s - F_e \end{aligned}$$

The master robot was commanded to execute a step position change. The delay was 0.4s RTT and there was an initial error of 0.8 units between the master and the slave robot.

Position Tracking



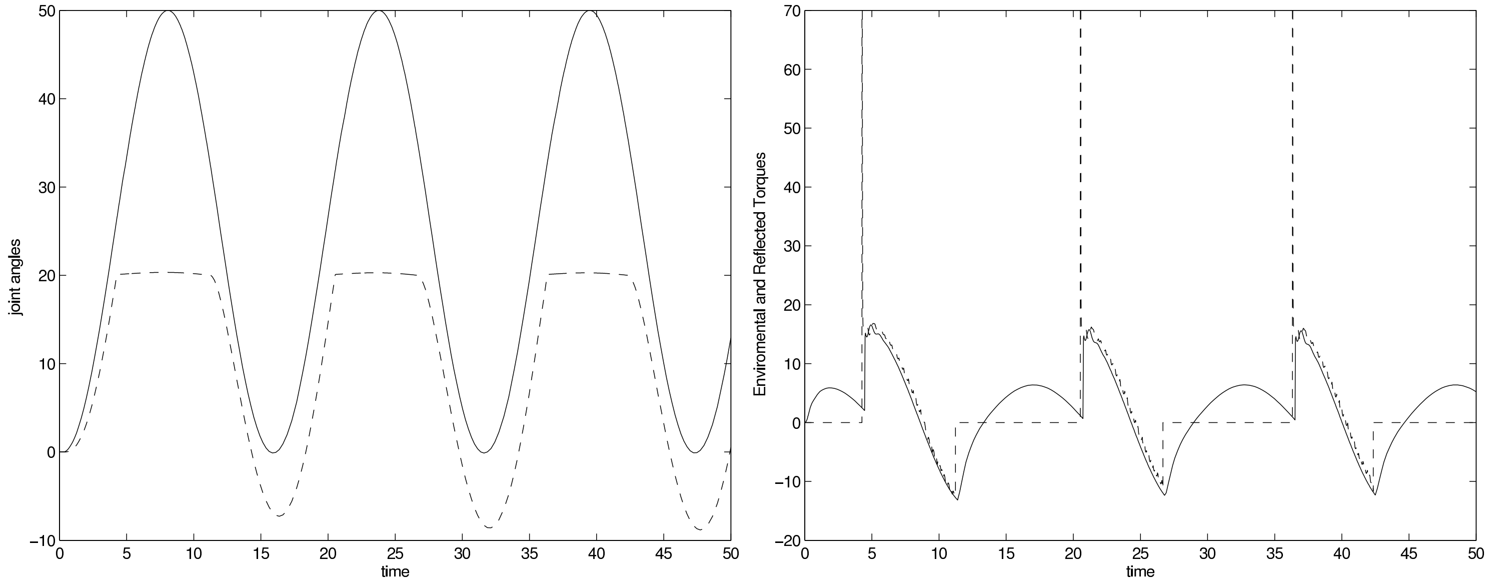


The new architecture (left) ensures convergence of the tracking error to zero even after an initial offset.

The traditional architecture (right) cannot ensure position tracking after an initial position offset

Force Tracking

Next we investigate the force tracking (transparency) of the new architecture.



The master and the slave joint positions during contact with the environment(left).

The reflected torque to the master F_{md} tracks the environmental torque F_e(right)