

Synchronization of Networked Passive Systems with Time Delay

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OUTLINE

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INTRODUCTION

- In this work we study output synchronization of networked agents whose dynamics are input/output passive
- Collective synchronization phenomena have been observed in many biological, chemical, physical and social systems
 - Coronary pacemaker cells
 - Circadian rhythm
 - Synchronously flashing fireflies
 - schooling of fish
 - Superconducting Josephson junctions
 - Arrays of lasers
- Synchronization is a key concept in understanding the phenomenon of self-organization occurring in systems of the dissipative type

- Consider N agents with the following dynamics

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i$$

$$y_i = h_i(x_i) \quad i = 1, 2, \dots, N$$

where $x_i \in R^n$, $u_i \in R^m$, $y_i \in R^m$, and f_i, g_i, h_i are smooth of appropriate dimensions with $f(0) = 0$, $h(0) = 0$.

- The above nonlinear system is said to output synchronize if

$$y_1 = y_2 = \dots = y_N \quad \text{as } t \rightarrow \infty$$

NONLINEAR PR CONDITION

Recall the following (Moylan, IEEE TAC, 1974)

Theorem 1 Consider a nonlinear system as above. Then TFAE:

1. There exists a C^1 scalar function $V(x) \geq 0$, $V(0) = 0$ and a function $S(x) \geq 0$ such that for all $t \geq 0$:

$$V(x(t)) - V(x(0)) = \int_0^t u^T(s)y(s)ds - \int_0^t S(x(s))ds$$

Such a system is said to be **strictly passive** for $S(x) > 0$, **passive** for $S(x) \geq 0$ and **lossless** for $S(x) = 0$.

2. There exists a C^1 scalar function $V(x) \geq 0$, $V(0) = 0$, such that

$$L_f V(x) = -S(x)$$

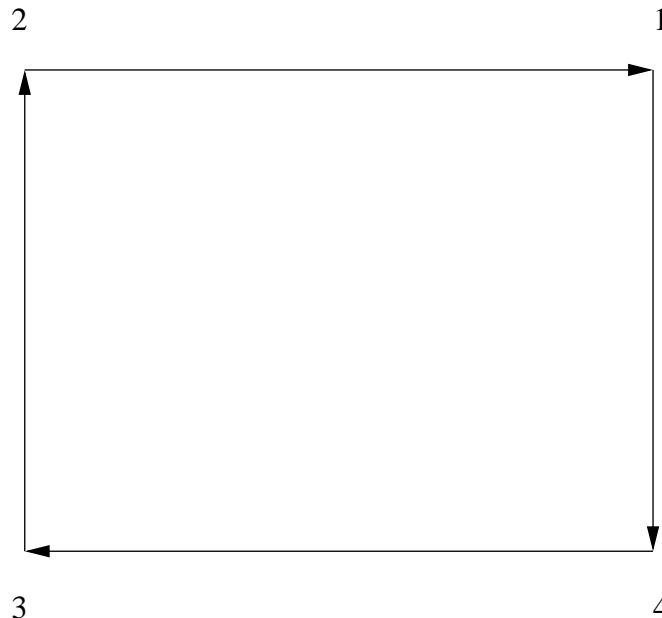
$$L_g V(x) = h^T(x)$$

where $L_f V(x) = \frac{\partial V}{\partial x} f(x)$ and $L_g V(x) = \frac{\partial V}{\partial x} g(x)$

NETWORK TOPOLOGY

The network topology refers to the way the agents are interconnected, i.e., how the information exchange flows between agents. We make the following assumptions:

- The agents form an *m regular* connected graph with respect to information exchange. This means that each agent has the same number (m) of neighbors.
- Every agent influences m agents and is in turn influenced by m other agents.
- We note that the agents from which the i – th agent receives information may be different from the agents to which the i – th agent sends information.
- An example of such a topology with 4 agents and $m=1$ is shown below.



MAIN RESULT

Let the agents be coupled together by the following control law

$$u_i = \sum_{j \in \mathcal{N}_i} K(y_j(t - T) - y_i) \quad i = 1, \dots, N$$

where K is a positive constant, \mathcal{N}_i is the set of m agents which are transmitting their outputs to the i - *th* agent, and T is the constant time-delay in the network.

Theorem 2 *Consider the N passive systems coupled together using the above control law. Then for arbitrary initial conditions, all signals in the system are bounded and the agents output synchronize.*

OUTLINE OF THE PROOF

Define a Lyapunov function candidate for the system as

$$V = 2(V_1 + \dots + V_N) + mK \int_{t-T}^t (y_1^T y_1 + \dots + y_N^T y_N) d\tau$$

The derivative of this Lyapunov function along trajectories of the system is given as

$$\dot{V} = 2 \sum_{i=1}^N (L_{f_i} V_i + L_{g_i} V_i u_i) + mK \sum_{i=1}^N (y_i^T y_i - y_i(t-T)^T y_i(t-T))$$

Using **Moylan's result** using the above control law for u yields

$$\begin{aligned} \dot{V} &= 2 \sum_{i=1}^N (y_i^T u_i - S_i(x_i)) + mK \sum_{i=1}^N (y_i^T y_i - y_i(t-T)^T y_i(t-T)) \\ &= 2 \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} y_i^T K(y_j(t-T) - y_i) + mK \sum_{i=1}^N (y_i^T y_i - y_i(t-T)^T y_i(t-T)) - 2 \sum_{i=1}^N S_i(x_i) \end{aligned}$$

Using the fact that

$$mK \sum_{i=1}^N y_i^T y_i = K \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} y_i^T y_i$$
$$-mK \sum_{i=1}^N y_i(t-T)^T y_i(t-T) = -K \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} y_i(t-T)^T y_i(t-T)$$

The derivative of the Lyapunov function can be written as

$$\dot{V} = -K \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} (y_j(t-T) - y_i)^T (y_j(t-T) - y_i) - 2 \sum_{i=1}^N S_i(x_i)$$

Using an extension of Lasalle's Theorem for time delay systems, we can conclude that output of every $i - th$ agent asymptotically converges to that of its neighbors belonging to \mathcal{N}_i . Connectivity of the network then implies output synchronization.

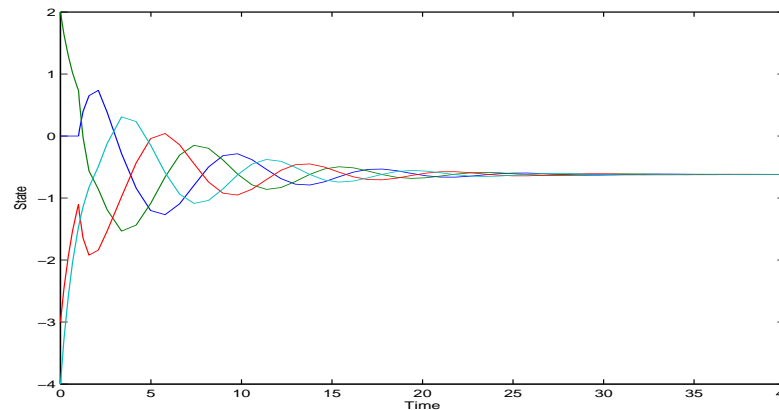
EXAMPLE

Consider the previous example of four agents coupled via a ring topology with integrator dynamics. Suppose the dynamics are given as

$$\dot{x}_i = u_i \quad y_i = x_i \quad i = 1, 2, 3, 4.$$

Let the agents be coupled using control which is given as

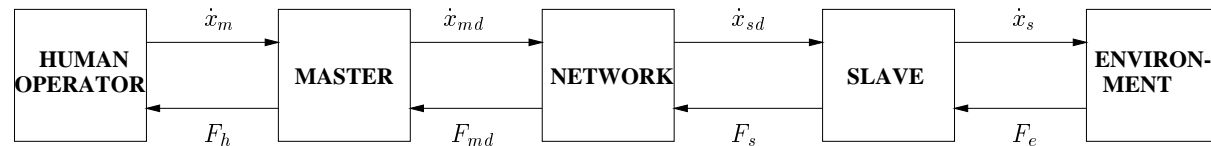
$$\begin{aligned} \dot{x}_1 &= K(x_2(t-T) - x_1) & \dot{x}_2 &= K(x_3(t-T) - x_2) \\ \dot{x}_3 &= K(x_4(t-T) - x_3) & \dot{x}_4 &= K(x_1(t-T) - x_4) \end{aligned}$$



The output (full state) of the four agents converges asymptotically.

APPLICATION TO BILATERAL TELEOPERATION

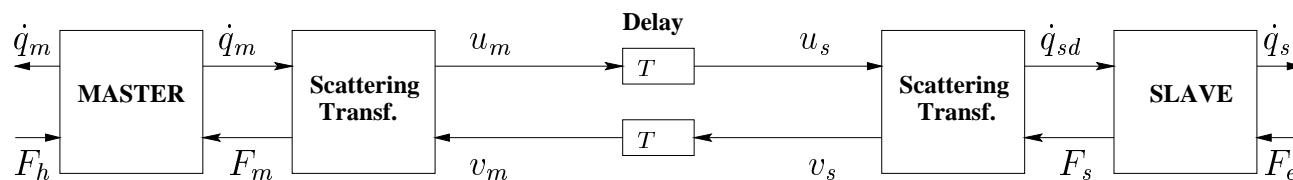
A bilateral teleoperator can be modeled as an interconnection of n -port networks. By designing control laws which impose the passivity property on each of the network blocks, passivity of the interconnection may be guaranteed.



The communication subsystem introduces a time delay, T , and can be made passive by the well-known scattering transformation approach [cf: Anderson and Spong, 1989] where the scattering variables

$$\begin{aligned} u_m &= \frac{1}{\sqrt{2b}} (F_m + b\dot{q}_m) ; & v_m &= \frac{1}{\sqrt{2b}} (F_m - b\dot{q}_m) \\ u_s &= \frac{1}{\sqrt{2b}} (F_s + b\dot{q}_{sd}) ; & v_s &= \frac{1}{\sqrt{2b}} (F_s - b\dot{q}_{sd}) \end{aligned} \quad (1)$$

are transmitted across the delay line instead of the original velocities and forces.



DRAWBACKS OF THE TRADITIONAL ARCHITECTURE

Although the traditional scattering-based architecture guarantees stability for all constant delays in the network, there are innate limitations on the transparency of the system

- The objective is to control the position of the remote slave, but the traditional passivity based design of the bilateral teleoperator necessitates that only velocities can be encoded in the scattering variables.
- Therefore potential data losses in the unreliable communication channel can lead to drift in the master and the slave manipulators.
- The transient tracking performance may also degrade with increase in network delay.
- The use of scattering transformation can lead to wave-reflections.

THE PROPOSED ARCHITECTURE

In order to develop an effective coordination strategy within the passivity framework, the following goals need to be accomplished

- A feedback control law for the master and the slave manipulator that renders the manipulator dynamics passive with respect to an output containing both position and velocity information
- A passive coordination control law which uses this output from the master and the slave to *kinematically lock* the motion of the two mechanical systems
- An adaptation mechanism to account for unknown parameters

All of these objectives may be achieved within the framework of passivity-based control.

THE CONTROL ALGORITHM

The master and the slave robots are Lagrangian systems and are modeled as

$$\begin{aligned}M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) &= \tau_m + J_m^T F_h \\M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) &= \tau_s - J_s^T F_e\end{aligned}$$

where q_m, q_s are the joint displacements, τ_m, τ_s are the applied torques, F_h, F_e are the human and environment forces, J_m, J_s are the master and slave Jacobians, $M_m(q), M_s(q)$ are the inertia matrices, $C_m(q, \dot{q}), C_s(q, \dot{q})$ are Centripetal and Coriolis matrices, $g_m(q), g_s(q)$ the gravitational torques.

In order to achieve the design objectives, the master and slave torques are given, for $i = m, s$ as

$$\tau_i = -M_i(q_i)\lambda\dot{q}_i - C_i(q_i, \dot{q}_i)\lambda q_i + g_i(q_i) + \bar{\tau}_i$$

$\bar{\tau}_m, \bar{\tau}_s$ are the additional motor torques required for coordination control

It is easy to verify that the master and slave dynamics reduce to

$$\begin{aligned}M_m \dot{r}_m + C_m r_m &= \bar{\tau}_m + J_M^T F_h \\M_s \dot{r}_s + C_s r_s &= \bar{\tau}_s - J_s^T F_e\end{aligned}$$

where

$$\begin{aligned}r_m &= \dot{q}_m + \lambda q_m \\r_s &= \dot{q}_s + \lambda q_s\end{aligned}$$

The new master and slave dynamics are passive with $(\bar{\tau}_m, r_m)$ and $(\bar{\tau}_s, r_s)$ as the input-output pairs.

PASSIVITY OF THE MASTER/SLAVE DYNAMICS

The feedback interconnection of the master robot along with the human operator together can shown to be passive with respect to the storage function

$$V_m = \frac{1}{2} \left(r_m^T M_m r_m \right) - \int_0^t r_m^T J_m^T F_h ds$$

The interconnection of the slave robot along with environment can shown to be passive with respect to the storage function

$$V_s = \frac{1}{2} \left(r_s^T M_s r_s \right) + \int_0^t r_s^T J_s^T F_e ds$$

Let these two systems be coupled using the control law

$$\bar{\tau}_m = K(r_s(t - T) - r_m)$$

$$\bar{\tau}_s = K(r_m(t - T) - r_s)$$

Thus we have two passive systems which are coupled using their outputs (r_m, r_s) .

Using the output synchronization result, the derivative of the Lyapunov function

$$V = 2(V_m + V_s) + K \int_{t-T}^t (r_m^T r_m + r_s^T r_s) ds$$

is given by

$$\begin{aligned} \dot{V} &= -K(r_m(t-T) - r_s)^T (r_m(t-T) - r_s) - K(r_s(t-T) - r_m)^T (r_s(t-T) - r_m) \\ &= -\bar{\tau}_m^T K^{-1} \bar{\tau}_m - \bar{\tau}_s^T K^{-1} \bar{\tau}_s \end{aligned}$$

Thus all signals in the system are bounded and $\bar{\tau}_m, \bar{\tau}_s \in \mathcal{L}_2$. Also,

$$\begin{aligned} \bar{\tau}_m &= r_m(t-T) - r_s = \dot{e}_m + \lambda e_m && \text{where } e_m(t) = q_m(t-T) - q_s(t) \\ \bar{\tau}_s &= r_s(t-T) - r_m = \dot{e}_s + \lambda e_s && \text{where } e_m(t) = q_s(t-T) - q_m(t) \end{aligned}$$

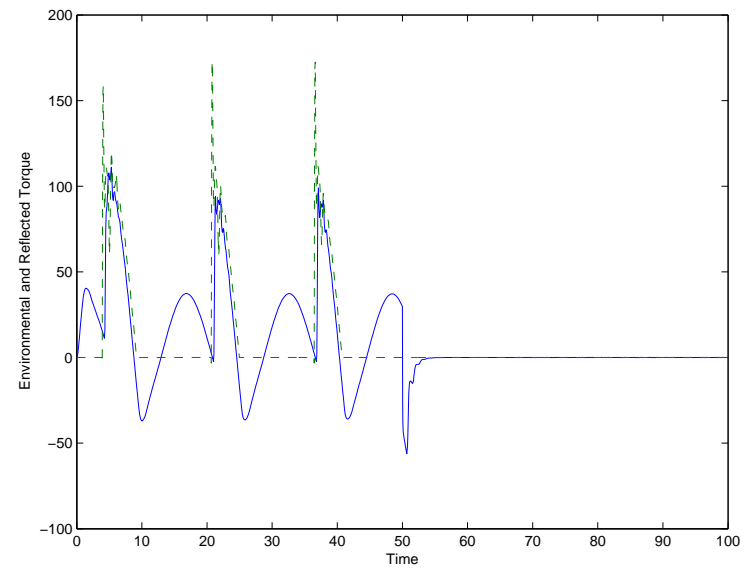
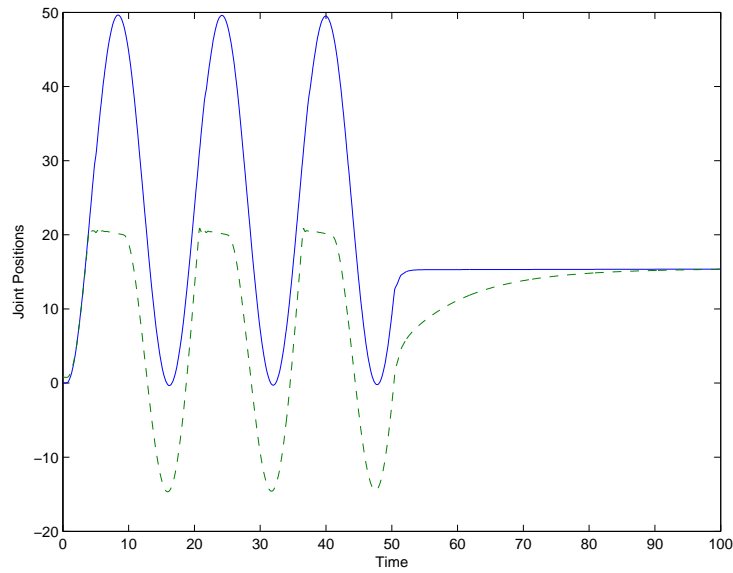
SIMULATIONS

Thus we have exponentially stable linear systems with state e_m and e_s driven by \mathcal{L}_2 inputs $\bar{\tau}_m$ and $\bar{\tau}_s$ respectively. Thus the coordination errors e_m, e_s exponentially converge to the origin.

Simulations were performed on single-degree of freedom bilateral teleoperator, with the master and slave dynamics given as

$$\begin{aligned}M_m \ddot{q}_m &= F_h + \tau_m \\M_s \ddot{q}_s &= \tau_s - F_e\end{aligned}$$

The master robot was commanded to follow a sinusoidal trajectory till time $t=50s$ and the human operator command was shut down after this time. To obstruct the motion of the slave, a virtual wall (spring-damper system) was also constructed



Blue shows the master position (left) and force (right)
 Green shows the slave position (left) and force (right)

CONCLUSIONS

- It was shown that agents with passive dynamics, and a regular information graph imposed on them, when coupled together using a proportional control strategy, output synchronize in the presence of delays in the network.
- An important application to the problem of bilateral teleoperation was also demonstrated.
- The result guarantees delay independent exponential stability of the position and force tracking errors without using scattering theory.
- A passivity-based adaptive version of the result is easily derived.