

The Passivity Paradigm in Robot Control

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The Passivity Paradigm

Passivity is one of the most physically appealing concepts in systems theory and has been used as a fundamental tool in the development of control tools for linear and nonlinear systems.

For robots and other mechanical systems, passivity is related to **energy dissipation** and so leads to natural results on stability and tracking.

In this talk we will discuss some uses of passivity-based control theory to two classes of systems that are receiving considerable current attention

1. **networked control systems and**
2. **hybrid control systems**

We will discuss applications of these ideas to

1. **Teleoperation over the Internet and**
2. **Bipedal Locomotion**

Background

Passivity concepts have been well documented and so will not be repeated here. We just give a few basic definitions to start. Consider a dynamical system represented by the state model

$$\begin{aligned}\dot{x} &= f(x, u) \\ y &= h(x, u)\end{aligned}$$

where f is locally Lipschitz, h is continuous, $f(0, 0) = 0$, $h(0, 0) = 0$ and the system has the same number of inputs and outputs. The above system is said to be

passive if $\dot{S} \leq u^T y$ for all $(x, u) \in R^n \times R^p$

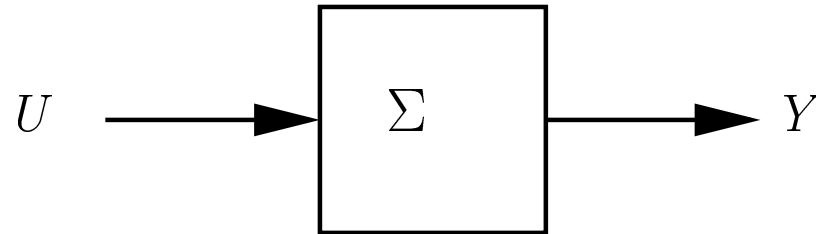
strictly passive if $\dot{S} \leq u^T y - \varphi(x)$ for $\varphi > 0$

input strictly passive if $\dot{S} \leq u^T y - u^T \psi(u)$ for $u^T \psi(u) > 0$

output strictly passive if $\dot{S} \leq u^T y - y^T \rho(y)$ for $y^T \rho(y) > 0$

for some C^1 semidefinite scalar function $S : R^n \rightarrow R$ called the **Storage Function**.

Passivity Based Control



For example, a passive system Σ is stabilized by output feedback

$$u = -ky$$

since then we have

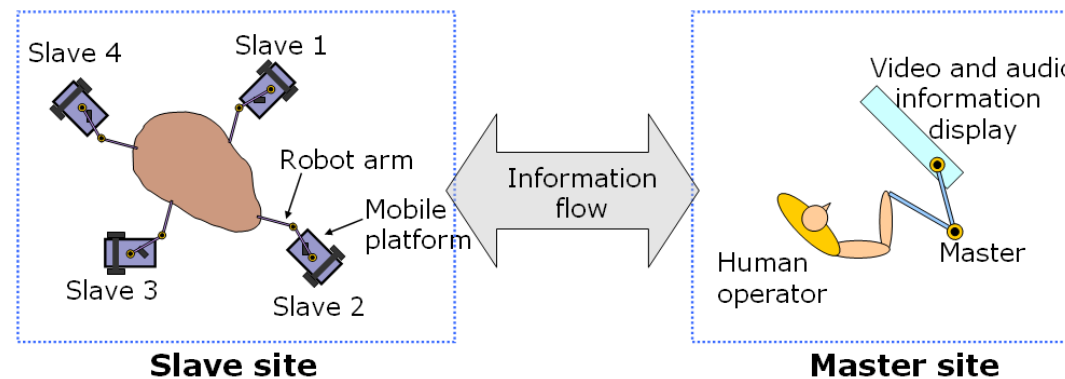
$$\dot{S} \leq -ky^T y = -k\|y\|^2 \leq 0$$

Under some additional (detectability-like) conditions asymptotic stability follows (LaSalle's Invariance Principle).

- Generally the convergence is to an invariant manifold
- Parallel and feedback interconnections of passive systems are passive

Networked Control Systems

Networked Control Systems (NCS) are currently of great interest for many applications. We present here a new passivity based architecture for a particular class of Networked Control Systems, namely Bilateral Teleoperators.



Our new architecture overcomes several difficulties associated with previous approaches, such as

- Lack of position tracking (drift)
- Poor transparency
- Sensitivity to parameter uncertainty

The Passivity Paradigm

Within the Passivity framework for networked systems one may incorporate:

- **Time Varying Delay** [cf: Lozano, R., Chopra, N., and Spong, M.W., “Passivation of Force Reflecting Bilateral Teleoperators with Time Varying Delay,” Mechatronics’02, Enschede, Netherlands, June 24-26, 2002]
- **Discrete-Time Formulation, Packet Loss, Data Interpolation** [cf: Berestesky, P., Chopra, N., and Spong, M.W., “Discrete Time Passivity in Bilateral Teleoperation over the Internet”, IEEE International Conference on Robotics and Automation, New Orleans, LA, April 26-May 1, 2004]
- **Multi-Agent Coordination and Manipulation** [cf: Lee, D., and Spong, M.W., “Passive Bilateral Teleoperation of Multiple Cooperative Robots with Delayed Communication,” in preparation]

Due to time constraints I will only discuss the new passivity architecture in:

Chopra, N., Spong, M.W., and Lozano, R., “Adaptive Coordination Control of Bilateral Teleoperators with Time Delay,” in preparation.

which addresses issues of transparency, position drift, and parameter uncertainty.

The Traditional Architecture

A bilateral teleoperator can be modeled as an interconnection of n -port networks. By designing control laws which impose the passivity property on each of the network blocks, passivity of the interconnection may be guaranteed.

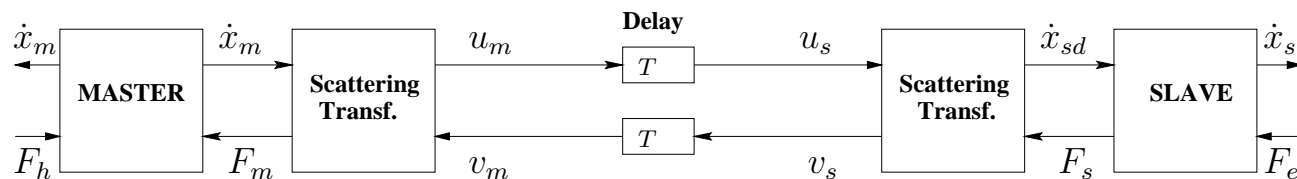


The communication subsystem introduces a time delay, T , and is made passive by the well-known scattering transformation approach [cf: Anderson and Spong, 1989] where the scattering variables

$$u_m = \frac{1}{\sqrt{2b}}(F_{md} + \dot{x}_{md}) \quad v_m = \frac{1}{\sqrt{2b}}(F_{md} - \dot{x}_{md})$$

$$u_s = \frac{1}{\sqrt{2b}}(F_{sd} + \dot{x}_{sd}) \quad v_s = \frac{1}{\sqrt{2b}}(F_{sd} - \dot{x}_{sd})$$

are transmitted across the delay line instead of the original velocities and forces.



Passivity of Master and Slave Robots

The master and the slave robots are Lagrangian systems and are modeled as

$$\begin{aligned}M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) &= \tau_m \\M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) &= \tau_s\end{aligned}$$

The well-known passivity property of the robot dynamics follows from the choice of storage functions, for $i = m, s$,

$$S_i(q_i, \dot{q}_i) = \frac{1}{2} \dot{q}_i^T M_i(q_i) \dot{q}_i + G_i(q_i)$$

where G_i is the potential energy. It is easy to verify, using the skew-symmetry property of robot dynamics that

$$\dot{S}_i = \dot{q}_i^T \tau_i$$

and hence the master dynamics are passive with τ_i, \dot{q}_i as input-output pairs.

A New Coordination Architecture

In order to develop an effective coordination strategy within the passivity framework, the following goals need to be accomplished

- A feedback control law for the master and the slave manipulator that renders the manipulator dynamics passive with respect to an output containing both position and velocity information
- A passive coordination control law which uses this output from the master and the slave to *kinematically lock* the motion of the two mechanical systems
- An adaptation mechanism to account for unknown parameters

All of these objectives may be achieved within the framework of passivity-based control.

The Control Algorithm

In order to achieve these design objectives, the master and slave torques are given, for $i = m, s$ as

$$\tau_i = -\hat{M}_i(q_i)\lambda\dot{q}_i - \hat{C}_i(q_i, \dot{q}_i)\lambda q_i + \hat{g}_i(q_i) + \bar{\tau}_i$$

where

- $\bar{\tau}_m, \bar{\tau}_s$ are the additional motor torques required for coordination control,
- “hats” represented estimated quantities and
- λ is a constant positive definite diagonal matrix.

It is easy to verify using linearity in the parameters that the master and slave dynamics reduce to

$$\begin{aligned}M_m \dot{r}_m + C_m r_m &= Y_m \tilde{\theta}_m + F_h + \bar{\tau}_m \\M_s \dot{r}_s + C_s r_s &= Y_s \tilde{\theta}_s + \bar{\tau}_s - F_e\end{aligned}$$

where $\tilde{\theta}_m = \theta_m - \hat{\theta}_m$, $\tilde{\theta}_s = \theta_s - \hat{\theta}_s$ are the parameter estimation errors and

$$\begin{aligned}r_m &= \dot{q}_m + \lambda q_m \\r_s &= \dot{q}_s + \lambda q_s\end{aligned}$$

The new master and slave dynamics are passive with $(\tilde{\tau}_m, r_m)$ and $(\tilde{\tau}_s, r_s)$ as the input-output pairs.

Define the **coordinating torques** F_{sd} and F_{md} as

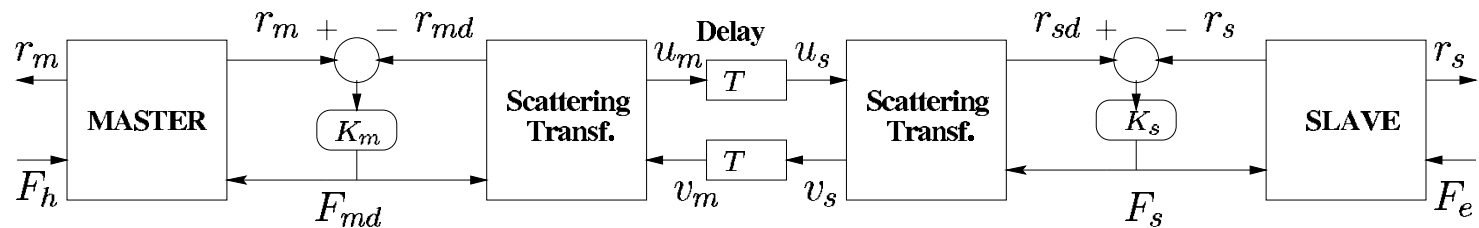
$$\begin{aligned}F_{sd} &= K_s(r_{sd} - r_s) = \bar{\tau}_s \\F_{md} &= K_m(r_m - r_{md}) = -\bar{\tau}_m\end{aligned}$$

where the gains K_s , K_m are constant positive definite diagonal matrices. Define the **coordination errors** between the master and slave robots as

$$\begin{aligned}e_m(t) &= q_m(t - T) - q_s(t) \\e_s(t) &= q_s(t - T) - q_m(t)\end{aligned}$$

Driving the coordination errors to the origin ensures that the master/slave system behave as a **kinematically locked** system.

We now concentrate on the communications and again use the scattering or the wave-variable transformation to passify the communication block.
The proposed architecture is



Within this architecture, we use the scattering transformation as follows

$$\begin{aligned}
 u_m &= \frac{1}{\sqrt{2b}}(F_{md} + br_{md}) & v_m &= \frac{1}{\sqrt{2b}}(F_{md} - br_{md}) \\
 u_s &= \frac{1}{\sqrt{2b}}(F_{sd} + br_{sd}) & v_s &= \frac{1}{\sqrt{2b}}(F_{sd} - br_{sd})
 \end{aligned}$$

Main Result

We choose parameter update laws according to

$$\begin{aligned}\dot{\hat{\theta}}_m &= \Gamma Y_m^T r_m \\ \dot{\hat{\theta}}_s &= \Lambda Y_s^T r_s\end{aligned}$$

where Γ and Λ are constant positive definite matrices.

Theorem 1 *Consider the nonlinear bilateral teleoperation system described above. Then all signals are bounded and the master and slave coordination errors are globally convergent to zero.*

Furthermore, in steady state with \dot{e}_i , $e_i = 0$ ($i = m, s$), the master and the slave velocities converge to the origin.

Proof: The proof relies on the Storage/Lyapunov function

$$\begin{aligned} S &= \frac{1}{2}r_m^T M_m r_m + \frac{1}{2}r_s^T M_s r_s + e_m^T K_1 e_m + e_s^T K_2 e_s \\ &\quad + \tilde{\theta}_m^T \Gamma^{-1} \tilde{\theta}_m + \tilde{\theta}_s^T \Gamma^{-1} \tilde{\theta}_s + \int_0^t (F_e^T r_s - F_h^T r_m) d\tau \\ &\quad + \int_0^t (F_{md}^T r_{rd} - F_{sd}^T r_{sd}) d\tau \end{aligned}$$

The term $\int_0^t (F_{md}^T r_{rd} - F_{sd}^T r_{sd}) d\tau$ is positive as a result of the scattering transformation.
The term $\int_0^t (F_e^T r_s - F_h^T r_m) d\tau$ as long as the human/environment subsystems are passive.

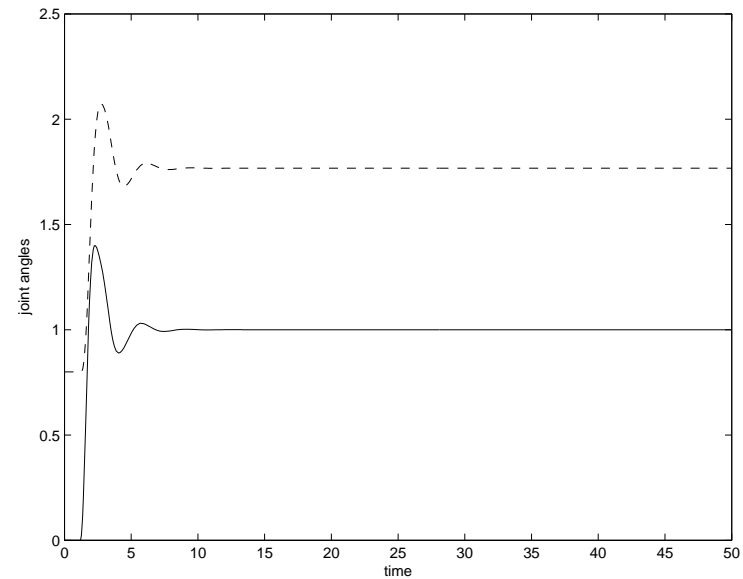
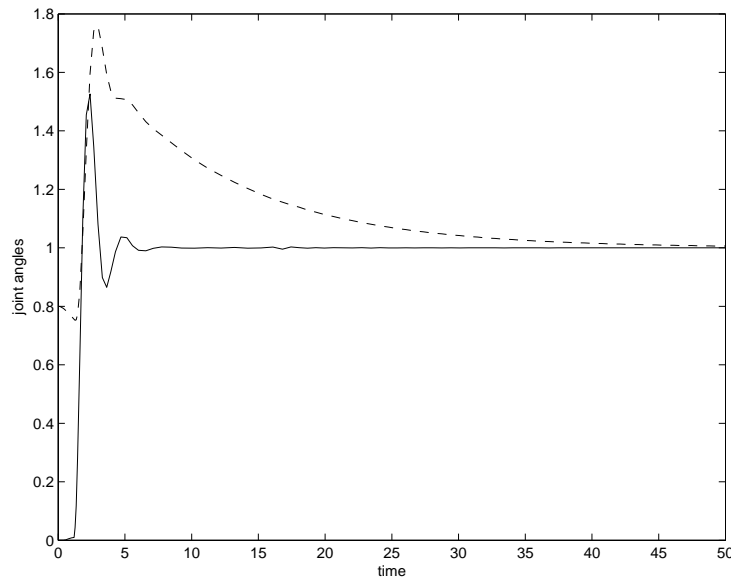
We simulated the schemes on a single-degree of freedom bilateral teleoperator, with the master and slave dynamics given as

$$M_m \ddot{q}_m = F_h + \tau_m$$

$$M_s \ddot{q}_s = \tau_s - F_e$$

The master robot was commanded to execute a step position change. The delay was 0.4s *RTT* and there was an initial error of 0.8 units between the master and the slave robot.

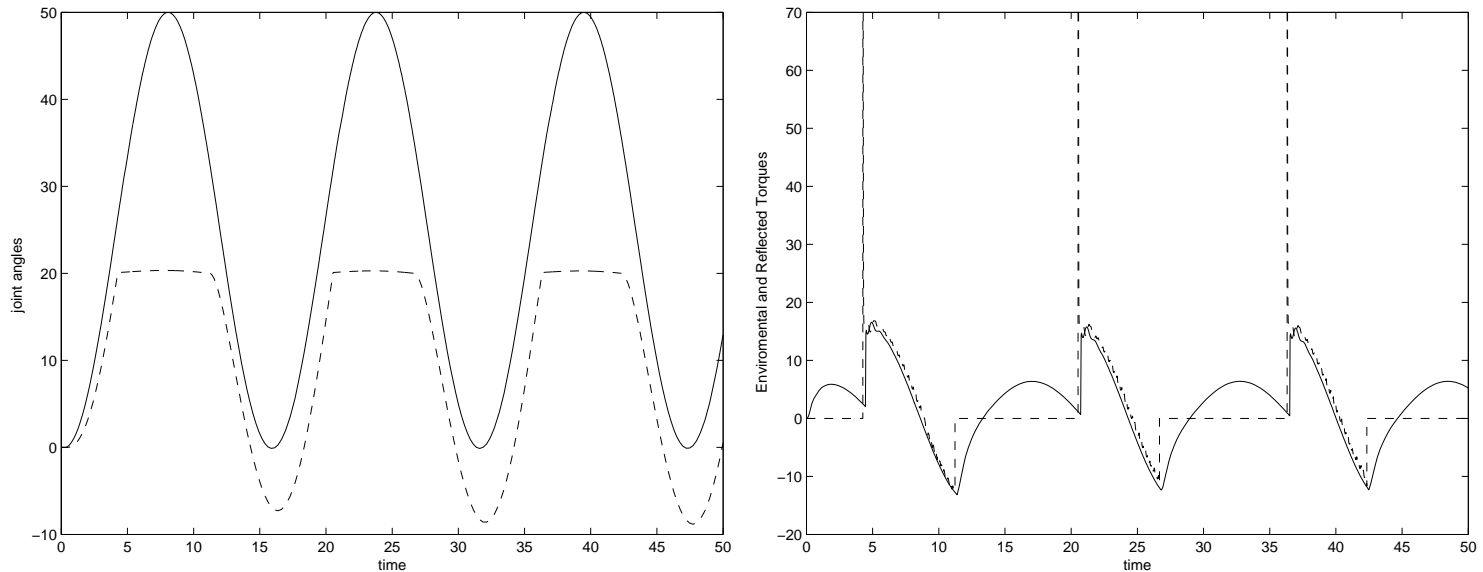
Position Tracking



The new architecture (left) ensures convergence of the tracking error to zero even after an initial offset.

The traditional architecture (right) cannot ensure position tracking after an initial position offset

Next we investigate the force tracking (transparency) of the new architecture.



The master and the slave joint positions during contact with the environment(left).

The reflected torque to the master F_{md} tracks the environmental torque F_e (right)

Passive Walking and Passivity Based Control

We next turn the attention of the Passivity Paradigm to the control of bipedal locomotion – a particular class of **Hybrid Nonlinear Systems**.

- It is well known that locomotion of mechanisms is achievable passively → **i.e., without actuation**
- McGeer, T., “Passive Dynamic Walking,” International Journal of Robotics Research, 1990.
- Goswami, et.al., “A Study of the Passive Gait of a Compass-like Biped Robot: Symmetry and Chaos,” International Journal of Robotics Research, 1998.
- **Such mechanisms can exhibit stable walking down a constant incline, turning gravitational potential energy into kinetic energy of motion.**



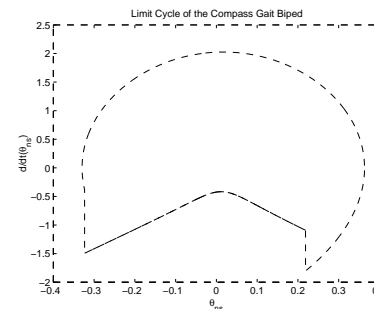
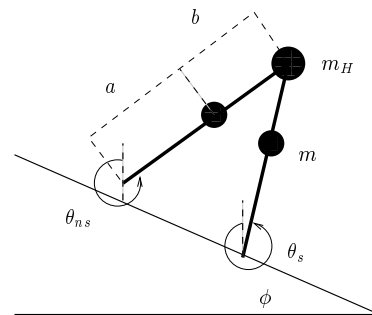
- 3D Walker of Collins, Wisse, & Ruina
- **videos courtesy of Martijn Wisse, Delft University of Technology, <http://mms.tudelft.nl/DbI/>**

How Does This Work?

Walking involves the interaction among

- Kinetic Energy
- Potential Energy
- Impacts

Impacts (foot/ground, knee strike) cause jumps in velocity \rightarrow a loss of Kinetic Energy
A passive limit cycle results when the loss of kinetic energy equals the change in potential energy during the step.



The Compass Gait Biped

Properties of Passive Gaits

In this **Compass Gait** example:

- The limit cycle is extremely sensitive to slope angle
- As the slope is increased period doubling bifurcations leading to eventual chaos occur
- The basin of attraction of the limit cycle is very small

These issues can be addressed by the addition of feedback control.

Let Q represent the configuration space of the robot and let $h(q) = 0$ represent the constraint surface (e.g. ground surface). The change in velocity at impact is given as a projection

$$\dot{q}(t^+) = P_q(\dot{q}(t^-))$$

onto $\{v \in T_q Q \mid dh(q) \cdot v = 0\}$. The dynamics of a general biped can be therefore be written as a hybrid nonlinear system

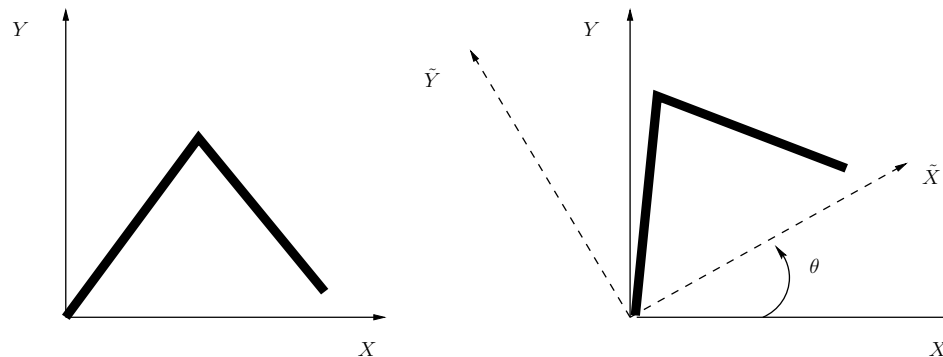
$$\begin{aligned} \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} &= u, & \text{for } h(q(t^-)) \neq 0 \\ q(t^+) &= q(t^-) & \text{for } h(q(t^-)) = 0 \\ \dot{q}(t^+) &= P_q(\dot{q}(t^-)) & \text{for } h(q(t^-)) = 0 \end{aligned}$$

where \mathcal{L} is the Lagrangian (Kinetic minus Potential energy)

Slope Changing Action

The key observation to address the sensitivity of the limit cycle to the ground slope is to recognize that the act of changing the ground slope at the stance leg can be represented by a **Group Action**, Φ , of the rotation group $SO(3)$ on the configuration space of the robot. For $A \in SO(3)$, $\Phi_A : Q \rightarrow Q$.

Pictorially, the action is shown below



Invariance Properties

We have the following (*Spong, Bullo, IEEE Transactions on Automatic Control, submitted, August, 2003*)

Proposition

- The kinetic energy \mathcal{K} is invariant under the slope changing action Φ , i.e., for all $A \in \text{SO}(3)$

$$\mathcal{K}(q, \dot{q}) = \mathcal{K} \circ \Phi_A(q, \dot{q}) = \mathcal{K}(\Phi_A(q), T_q \Phi_A(\dot{q}))$$

- The velocity changes at impacts, $\dot{q}^+ = P_q(\dot{q}^-)$, are invariant under the slope changing action.

Controlled Symmetries

A **Symmetry** in a mechanical system arises when the Lagrangian is invariant under a group action Φ , i.e.

$$\mathcal{L}(q, \dot{q}) = \mathcal{L}(\Phi_A(q), T_q \Phi_A(\dot{q})) \quad \text{for all } A \in SO(3)$$

Symmetries give rise to conserved quantities, for example, translational symmetry gives rise to conservation of momentum, etc.

Definition **Controlled Symmetry**

We say that an Euler-Lagrange system has a Controlled Symmetry with respect to a group action Φ if, for every $A \in SO(3)$ there exists an admissible control input $u_A(t)$ such that

$$L(t, q, \dot{q}) - u_A(t) = L(t, \Phi_A(q), T_q \Phi_A(\dot{q}))$$

Passivity Based Control

Let $E(q, \dot{q}) = \mathcal{K} + \mathcal{V}$ be the total energy of the robot and E_{ref} a (constant) reference energy (for example, the energy along a limit cycle trajectory).

For $A \in SO(3)$ define the Storage Function

$$S = \frac{1}{2}(E \circ \Phi - E_{ref})^2$$

Then

$$\dot{S} = (E \circ \Phi - E_{ref})\dot{q}^T \left[u - \frac{\partial}{\partial q} (\mathcal{V}(q) - \mathcal{V}(\Phi_A(q))) \right]$$

If we define the control input u as $u = u_A + \tilde{u}$, where

$$u_A = \frac{\partial}{\partial q} (\mathcal{V}(q) - \mathcal{V}(\Phi_A(q)))$$

Then

$$\dot{S} = (E \circ \Phi - E_{ref}) \dot{q}^T \tilde{u} = y^T \tilde{u}$$

where

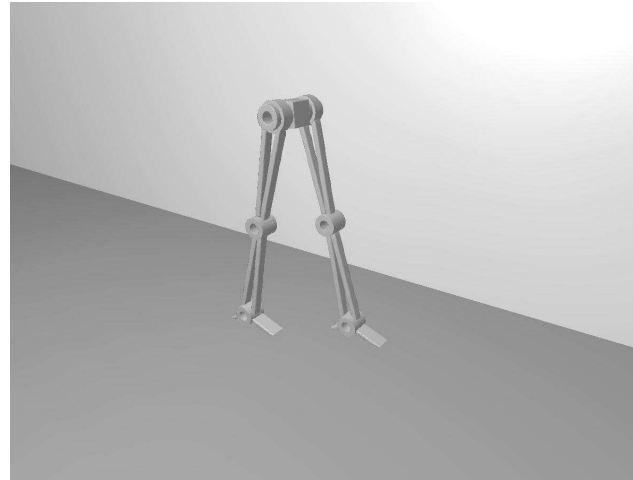
$$y = \dot{q}(E \circ \Phi - E_{ref})$$

defines a passive output.

It follows (Spong, Bullo, 2003) that for $\tilde{u} = 0$, the control u_A defines a **Controlled Symmetry**.

Theorem: Suppose there exists a passive gait on one ground slope, represented by $A_0 \in SO(3)$, and let $A \in SO(3)$ represent any other slope. Then the control input $u_{A^T A_0}$ generates a walking gait on slope A . Moreover the basin of attraction of the passive gait is mapped to the basin of attraction of the controlled gait.

Examples and Simulations



Finding Passive Limit Cycles

Passive Limit Cycles are investigated via the **Poincaré Map**.

$$X_{k+1} = P(X_k)$$

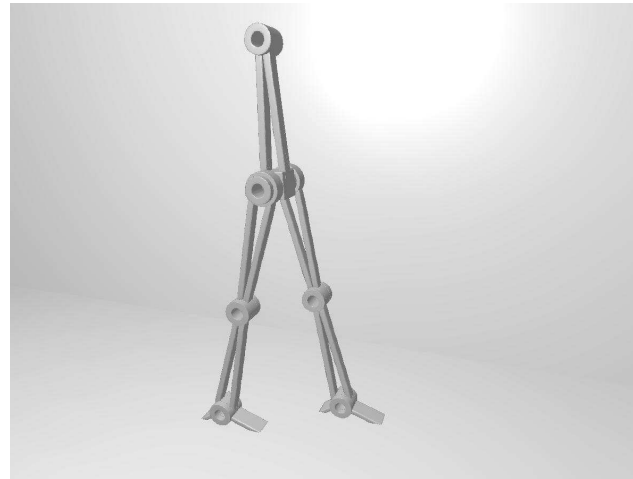
where X_k is the vector of joint positions and velocities at the beginning of step k .

The limit cycle is thus a fixed point of the Poincaré Map. Any initial condition in the basin of attraction of a stable limit cycle for one slope can be used to initialize a controlled gait on any other slope.

The video shows walking on level ground.

Extension to Biped with Torso

The next video shows a biped with a torso walking on level ground



How is this done?

A PD-type control is used to stabilize the torso in the inverted position. The resulting system is analyzed via the Poincaré map to find a stable limit cycle. The details are omitted.

Passivity Based Control

- Having made the passive limit cycles slope invariant via potential energy shaping we now investigate total energy shaping for robustness.
- Improving the rate of convergence to the limit cycle and increasing the basin of attraction are needed for robustness to external disturbances, changes in ground slope, etc.
- We now consider the design of the control input \tilde{u} in

$$u = u_A + \tilde{u}$$

- In other words we consider the system

$$L(t, \Phi_A(q), T_q \Phi_A(\dot{q})) = \tilde{u}$$

With the Storage Function \mathcal{S} as before

$$\mathcal{S} = \frac{1}{2}(E \circ \Phi - E_{ref})^2$$

We saw that

$$\dot{\mathcal{S}} = (E \circ \Phi - E_{ref})\dot{q}\tilde{u} = y^T u$$

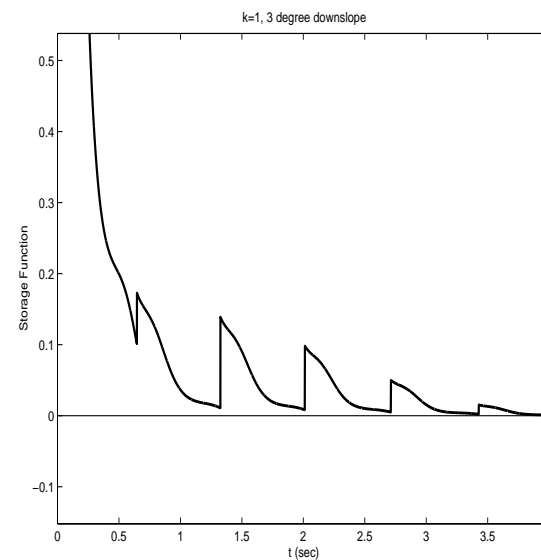
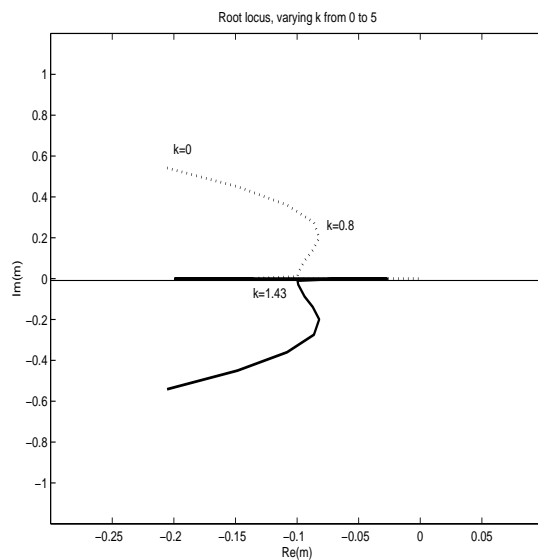
if we choose the additional control \bar{u} according to

$$\bar{u} = -ky = -k\dot{q}(E - E_{ref})$$

we obtain

$$\dot{\mathcal{S}} = -ky^2 = -k\|\dot{q}\|^2 \mathcal{S}$$

- Thus $S(t)$ converges exponentially toward zero during each step^a
- At impacts, S will experience a jump discontinuity. If the value of S at impact $k + 1$ is less than it's value at impact k it follows that $E(t)$ converges to E_{ref} .



- Simulation: Walking on a Varying Slope

^aassuming \dot{q} is bounded away from zero