Stability and Control of Networked Passive Systems

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OUTLINE

- Introduction
- Applications
- Passivity
- Network Topology
- Main Results
- Application to Bilateral Teleoperation
- Conclusions

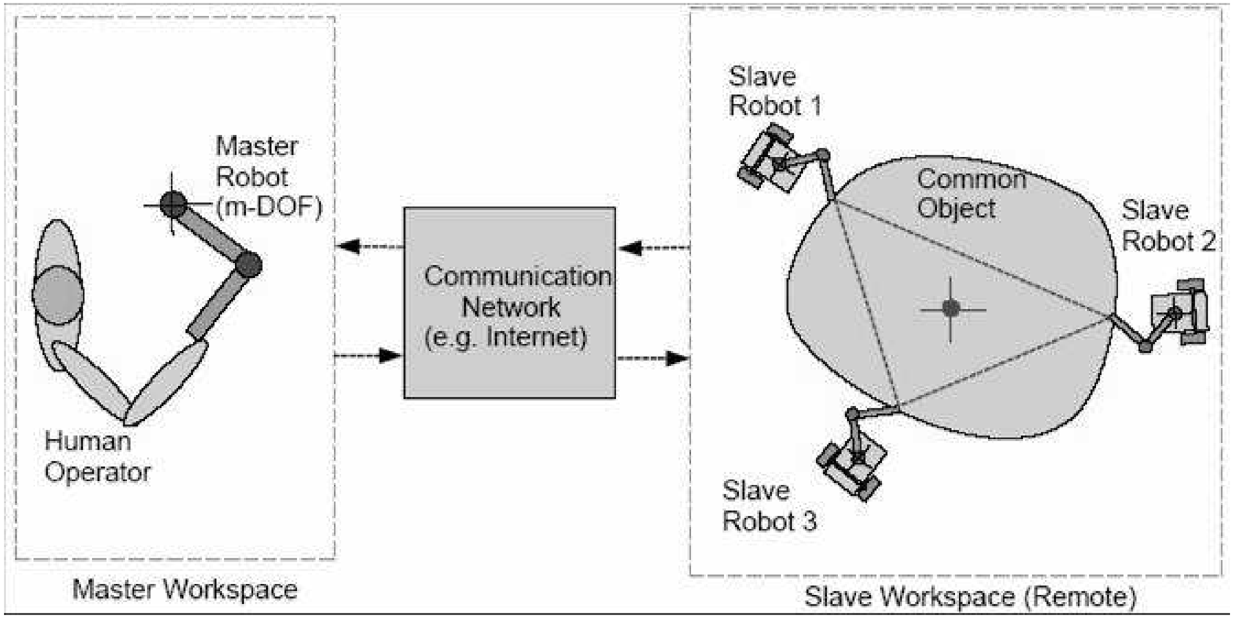
INTRODUCTION

- In this talk we present some stability and control results for classes of networked systems
- In particular we will discuss the problem of output synchronization of networked agents whose dynamics are input/output passive.
- Collective synchronization phenomena have been observed in many biological, chemical, physical and social systems.
    - Coronary pacemaker cells
    - Brain neurons responsible for memory
    - Cirdadian rhythm
    - Synchronously flashing fireflies
    - Schooling of fish
    - Flocking of birds
    - Superconducting Josephson junctions
    - Arrays of lasers

- In engineering systems, coordination and synchronization problems are important to consider for
  - Control of multiple agents
    - UAV’s
    - Helicopters
    - Ground vehicles
    - Mobile robots
  - Bilateral Teleoperation
  - Sensor Networks

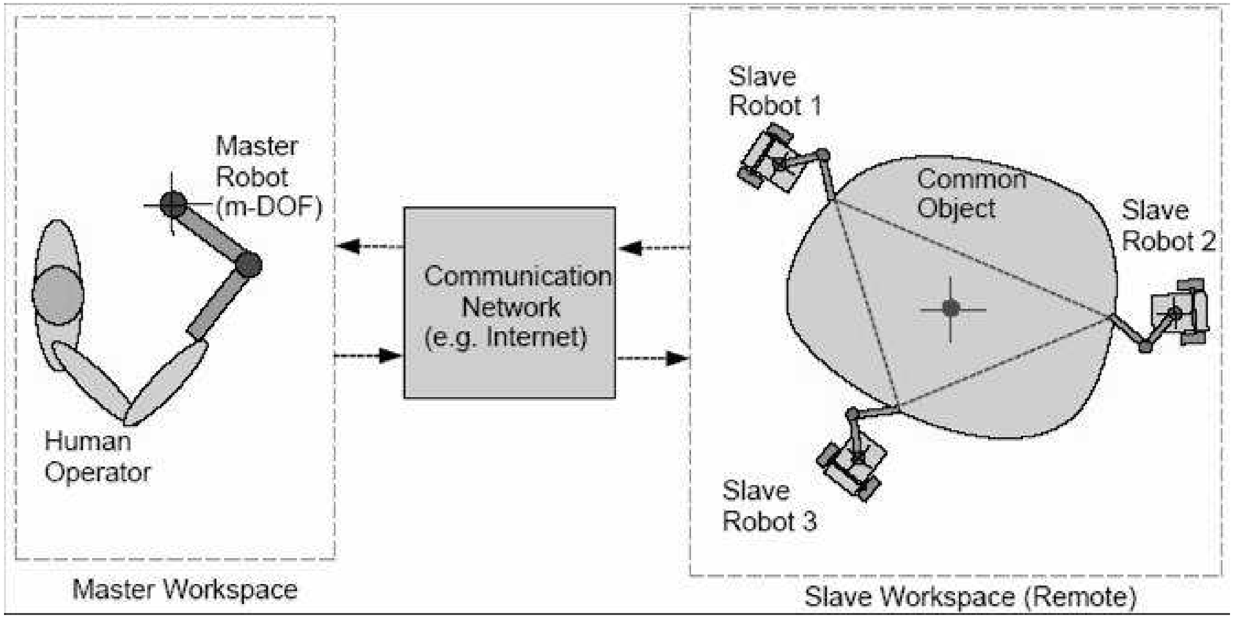
Some Application Examples

A Multi-robot Master/Slave System: In this example, a single operator controls a network of robots. The robots must synchronize their internal formation and follow group commands from the master.

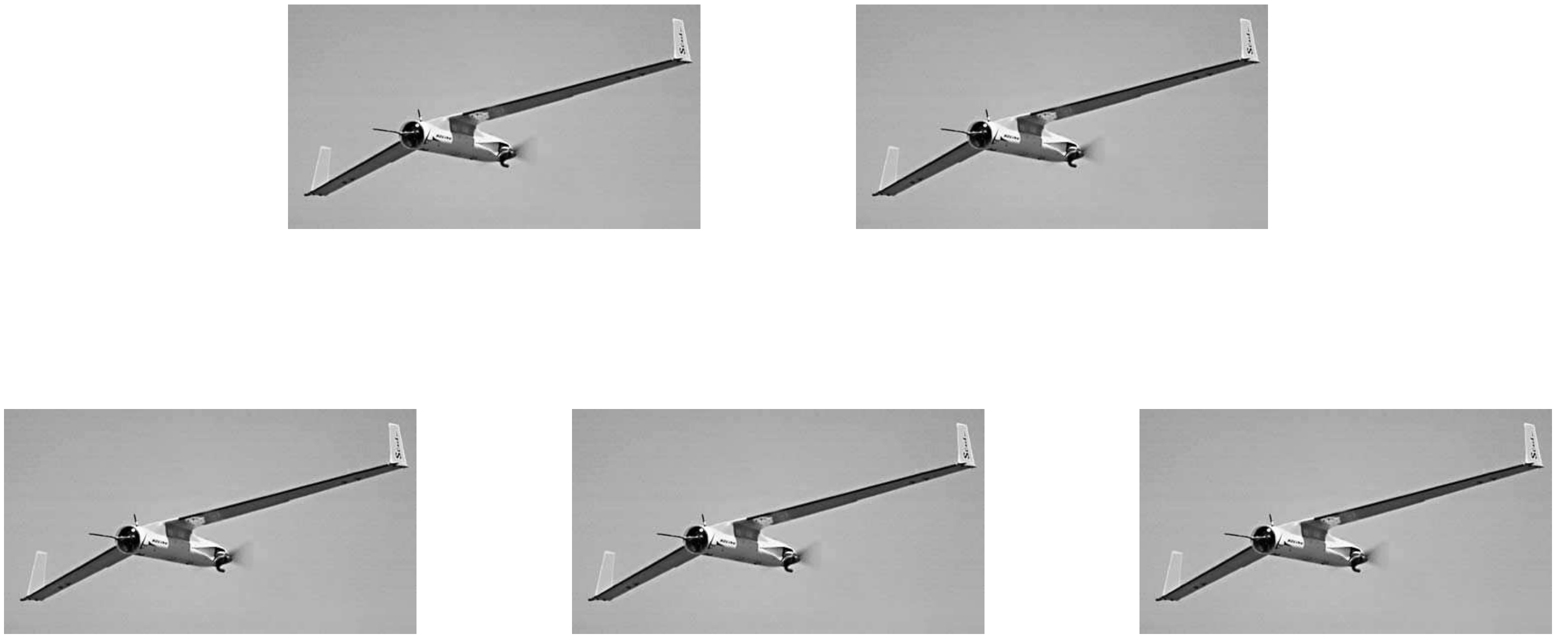


Challenges

The robots are described by nonlinear Lagrangian (passive) dynamics. The communication network introduces additional dynamics: time delays, packet loss.



UAV Formation Control: Heading, velocity and relative distance must be controlled with communication delays, nonlinear dynamics, etc.



DYNAMICS

- Consider N agents with the following dynamics

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i$$
$$y_i = h_i(x_i) \quad i = 1, 2, \dots, N$$

where $x_i \in R^n$, $u_i \in R^m$, $y_i \in R^m$, and f_i, g_i, h_i are smooth of appropriate dimensions with $f(0) = 0, \; h(0) = 0$.

- The above nonlinear system is said to output synchronize if

$$y_1 = y_2 = \dots = y_N \quad as \; t \rightarrow \infty$$

- Note that the N individual systems are independent. Thus, they will be coupled only by our choice of control law u_i, which in turn is dictated by the communication among the agents.

PASSIVITY

Given a (nonlinear) system as above, suppose there exists a C^1 scalar function $V(x) \geq 0$, $V(0) = 0$ and a function $S(x) \geq 0$ such that for all $t \geq 0$:

$$V(x(t)) - V(x(0)) = \int_0^t u^T(s)y(s)ds - \int_0^t S(x(s))ds$$

Such a system is said to be strictly passive for $S(x) > 0$, passive for $S(x) \geq 0$ and lossless for $S(x) = 0$. The function V is called the Storage Function.
In (electro)mechanical systems the product $u^T y$ has units of power and V is thus the energy in the system. Passivity, in effect, says that the change of energy over the time interval $[0, t]$ is due only to the energy supplied by the external input u and the energy dissipated by the term $\int S$. Thus passive systems cannot generate energy. Under some mild additional assumptions, passive systems are also stable.
Note: In much of the literature on synchronization, the systems are represented as first-order (passive) integrators. Also, Lagrangian mechanical systems have a natural passivity property. Hence, the assumption of passivity is not too restrictive.

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NONLINEAR POSITIVE REAL (PR) CONDITION

Assumption: In the analysis that follows we assume that the agents are passive with storage function V and dissipation function S.

We recall the following result (Moylan, IEEE TAC, 1974)

Theorem 1 *Consider a nonlinear system as above. Then TFAE:*

1. *The system is passive*
2. *There exists a C^1 scalar function function $V(x) \geq 0$, $V(0) = 0$, such that*

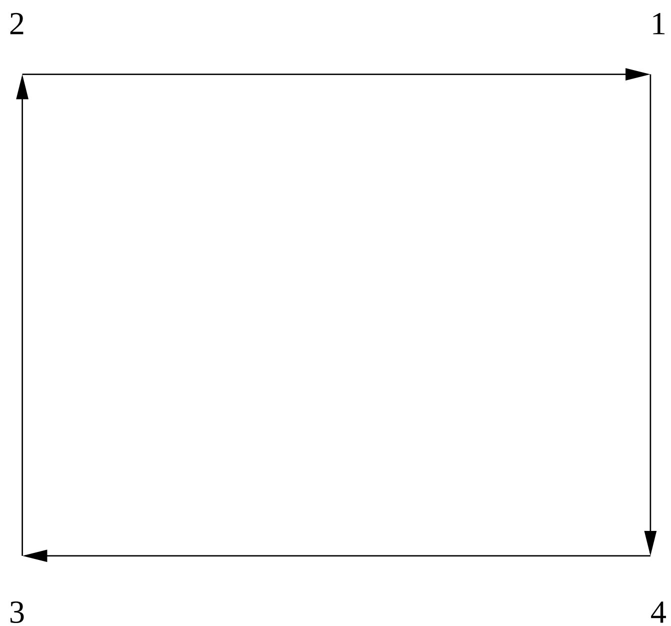
$$\begin{aligned} L_f V(x) &= -S(x) \\ L_g V(x) &= h^T(x) \end{aligned}$$

where $L_f V(x) = \frac{\partial V}{\partial x}^T f(x)$ and $L_g V(x) = \frac{\partial V}{\partial x}^T g(x)$

NETWORK TOPOLOGY

The network topology refers to the way the agents are interconnected, i.e., how the information exchange flows between agents. We make the following assumptions:

- The agents form an *m regular connected graph* with respect to information exchange. This means that each agent has the same number (m) of neighbors.
- Every agent influences *m* agents and is in turn influenced by *m* other agents.
- We note that the agents from which the i^{th} agent receives information may be different from the agents to which the i^{th} agent sends information.
- An example of such a topology with 4 agents and *m=1* is shown below.



MAIN RESULT

Let the agents be coupled together by the following control law

$$u_i = \sum_{j \in \mathcal{N}_i} K(y_j(t-T) - y_i) \ , \ i = 1, \ldots, N$$

where K is a positive constant, $\mathcal{N}_i$ is the set of m agents which are transmitting their outputs to the i^{th} agent, and T is the constant time-delay in the network.

Theorem 2 *Consider the N passive systems coupled together using the above control law. Then for arbitrary initial conditions, all signals in the system are bounded and the systems output synchronize.*

The passivity assumption allows arbitrary time delays in communication. We can also show synchronization with other types of network topologies, including possibly dynamically changing topologies.

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OUTLINE OF THE PROOF

Define a Lyapunov function candidate for the system as

$$V = 2(V_1 + \ldots + V_N) + mK \int_{t-T}^{t} (y_1^T y_1 + \ldots + y_N^T y_N) d\tau$$

The derivative of this Lyapunov function along trajectories of the system is given as

$$\dot{V} = 2\sum_{i=1}^{N} (L_{f_i} V_i + L_{g_i} V_i u_i) + mK \sum_{i=1}^{N} (y_i^T y_i - y_i(t-T)^T y_i(t-T))$$

Using Moylan's result using the above control law for u yields

$$\dot{V} = 2\sum_{i=1}^{N} (y_i^T u_i - S_i(x_i)) + mK \sum_{i=1}^{N} (y_i^T y_i - y_i(t-T)^T y_i(t-T))$$

$$= 2\sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} y_i^T K(y_j(t-T) - y_i) + mK \sum_{i=1}^{N} (y_i^T y_i - y_i(t-T)^T y_i(t-T)) - 2\sum_{i=1}^{N} S_i(x_i)$$

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Using the fact that

$$mK\sum_{i=1}^{N} y_i^T y_i = K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} y_i^T y_i$$

$$-mK\sum_{i=1}^{N} y_i(t-T)^T y_i(t-T) = -K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} y_i(t-T)^T y_i(t-T)$$

The derivative of the Lyapunov function can be written as

$$\dot{V} = -K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}(y_j(t-T)-y_i)^T(y_j(t-T)-y_i) - 2\sum_{i=1}^{N} S_i(x_i)$$

Using an extension of Lasalle's Theorem for time delay systems, we can conclude that output of every i^{th} agent asymptotically converges to that of its neighbors belonging to $\mathcal{N}_i$. Connectivity of the network then implies output synchronization.

EXAMPLE

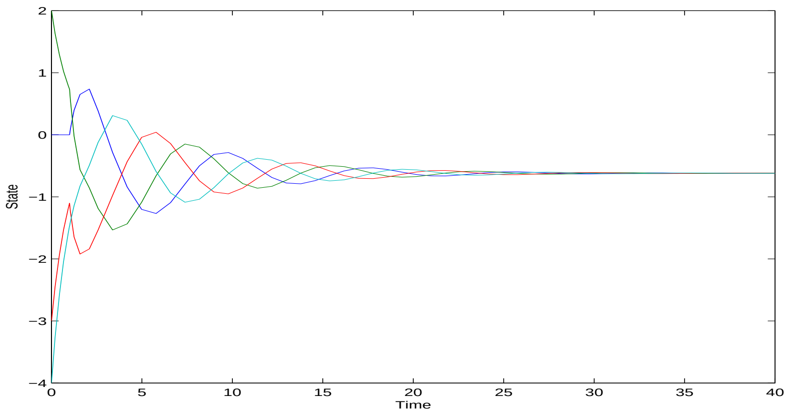
Consider the previous example of four agents coupled via a ring topology. Suppose the dynamics are given as

$$\dot{x}_i = u_i \quad y_i = x_i \;\; i = 1, 2, 3, 4.$$

Let the agents be coupled using the previously defined control which, in this case, leads to

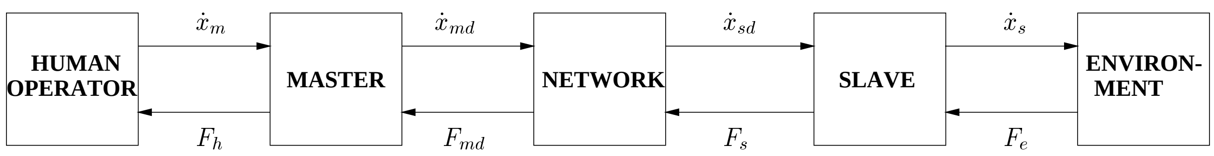
$$\dot{x}_1 = K(x_2(t-T) - x_1) \quad \dot{x}_2 = K(x_3(t-T) - x_2)$$
$$\dot{x}_3 = K(x_4(t-T) - x_3) \quad \dot{x}_4 = K(x_1(t-T) - x_4)$$

It follows that the outputs (states) of the four agents converges asymptotically.



APPLICATION TO BILATERAL TELEOPERATION

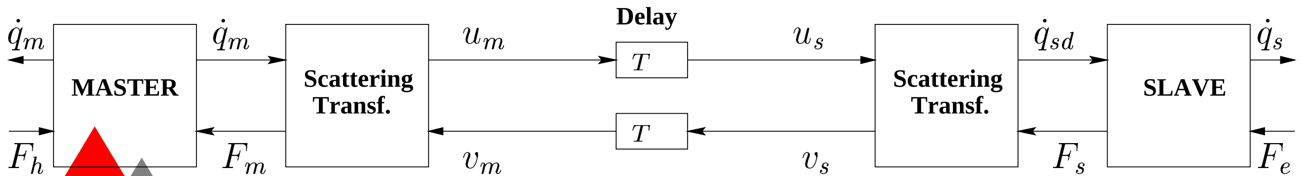
A bilateral teleoperator can be modeled as an interconnection of n-port networks. By designing control laws which impose the passivity property on each of the network blocks, passivity of the interconnection may be guaranteed.



The communication subsystem introduces a time delay, T, and can be made passive by the well-known scattering transformation approach [cf: Anderson and Spong, 1989] where the scattering variables

$$u_m = \frac{1}{\sqrt{2b}}(F_m + b\dot{q}_m)\;; \qquad v_m = \frac{1}{\sqrt{2b}}(F_m - b\dot{q}_m)$$
$$u_s = \frac{1}{\sqrt{2b}}(F_s + b\dot{q}_{sd})\;; \qquad v_s = \frac{1}{\sqrt{2b}}(F_s - b\dot{q}_{sd}) \tag{1}$$

are transmitted across the delay line instead of the original velocities and forces.



DRAWBACKS OF THE TRADITIONAL ARCHITECTURE

Although the traditional scattering-based architecture guarantees stability for all constant delays in the network, there are innate limitations on the transparency of the system

- The objective is to control the position of the remote slave, but the traditional passivity based design of the bilateral teleoperator necessitates that only velocities can be encoded in the scattering variables.
- Therefore potential data losses in the unreliable communication channel can lead to drift in the master and the slave manipulators.
- The transient tracking performance may also degrade with increase in network delay.
- The use of scattering transformation can lead to wave-reflections.

THE PROPOSED ARCHITECTURE

In order to develop an effective coordination strategy within the passivity framework, the following goals need to be accomplished

- A feedback control law for the master and the slave manipulator that renders the manipulator dynamics passive with respect to an output containing both position and velocity information
- A passive coordination control law which uses this output from the master and the slave to *kinematically lock* the motion of the two mechanical systems

Both of these objectives may be achieved within the framework of our results on output synchronization of networked passive systems.

THE CONTROL ALGORITHM

The master and the slave robots are Lagrangian systems and are modeled as

$$\begin{aligned} M_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m + g_m(q_m) &= \tau_m + J_m^T F_h \\ M_s(q_s)\ddot{q}_s + C_s(q_s,\dot{q}_s)\dot{q}_s + g_s(q_s) &= \tau_s - J_s^T F_e \end{aligned}$$

where q_m, q_s are the joint displacements, τ_m, τ_s are the applied torques, F_h, F_e are the human and environment forces, J_m, J_s are the master and slave Jacobians, $M_m(q)$, $M_s(q)$ are the inertia matrices, $C_m(q,\dot{q})$, $C_s(q,\dot{q})$ are Centripetal and Coriolis matrices, $g_m(q)$, $g_s(q)$ the gravitational torques.

In order to achieve the design objectives, the master and slave torques are given, for $i = m, s$ as

$$\tau_i = -M_i(q_i)\lambda\dot{q}_i - C_i(q_i,\dot{q}_i)\lambda q_i + g_i(q_i) + \bar{\tau}_i$$

$\bar{\tau}_m$, $\bar{\tau}_s$ are the additional motor torques required for coordination control

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It is easy to verify that the master and slave dynamics reduce to

$$\begin{aligned} M_m \dot{r}_m + C_m r_m &= \bar{\tau}_m + J_M^T F_h \\ M_s \dot{r}_s + C_s r_s &= \bar{\tau}_s - J_s^T F_e \end{aligned}$$

where r_m and r_s are defined as

$$\begin{aligned} r_m &= \dot{q}_m + \lambda q_m \\ r_s &= \dot{q}_s + \lambda q_s \end{aligned}$$

and will form the passive outputs for the system. Assuming that the human and environment dynamics are themselves passive, the new master and slave dynamics are passive with $(\bar{\tau}_m, r_m)$ and $(\bar{\tau}_s, r_s)$ as the input-output pairs.

PASSIVITY OF THE MASTER/SLAVE DYNAMICS

The feedback interconnection of the master robot along with the human operator together can shown to be passive with respect to the storage function

$$V_m = \frac{1}{2}\left(r_m^T M_m r_m\right) - \int_0^t r_m^T J_m^T F_h ds$$

The interconnection of the slave robot along with environment can shown to be passive with respect to the storage function

$$V_s = \frac{1}{2}\left(r_s^T M_s r_s\right) + \int_0^t r_s^T J_s^T F_e ds$$

Let these two systems be coupled using the control law

$$\bar{\tau}_m = K(r_s(t-T) - r_m)$$
$$\bar{\tau}_s = K(r_m(t-T) - r_s)$$

Thus we have two passive systems which are coupled using their outputs (r_m, r_s).

Using the output synchronization result, the derivative of the Lyapunov function

$$V = 2(V_m + V_s) + K \int_{t-T}^{t} (r_m^T r_m + r_s^T r_s) ds$$

is given by

$$\begin{aligned} \dot{V} &= -K(r_m(t-T) - r_s)^T (r_m(t-T) - r_s) - K(r_s(t-T) - r_m)^T (r_s(t-T) - r_m) \\ &= -\bar{\tau}_m^T K^{-1} \bar{\tau}_m - \bar{\tau}_s^T K^{-1} \bar{\tau}_s \end{aligned}$$

Thus all signals in the system are bounded and $\bar{\tau}_m, \bar{\tau}_s \in \mathcal{L}_2$. Also,

$$K^{-1}\bar{\tau}_m = r_m(t-T) - r_s = \dot{e}_m + \lambda e_m \qquad where \;\; e_m(t) = q_m(t-T) - q_s(t)$$

$$K^{-1}\bar{\tau}_s = r_s(t-T) - r_m = \dot{e}_s + \lambda e_s \qquad where \;\; e_m(t) = q_s(t-T) - q_m(t)$$

SIMULATIONS

Thus we have exponentially stable linear systems with state e_m and e_s driven by $\mathcal{L}_2$ inputs $\bar{\tau}_m$ and $\bar{\tau}_s$ respectively. Thus the coordination errors e_m, e_s exponentially converge to the origin.

Simulations were performed on single-degree of freedom bilateral teleoperator, with the master and slave dynamics given as

$$\begin{aligned} M_m \ddot{q}_m &= F_h + \tau_m \\ M_s \ddot{q}_s &= \tau_s - F_e \end{aligned}$$

The master robot was commanded to follow a sinusoidal trajectory till time t=50s and the human operator command was shut down after this time. To obstruct the motion of the slave, a virtual wall (spring-damper system) was also constructed

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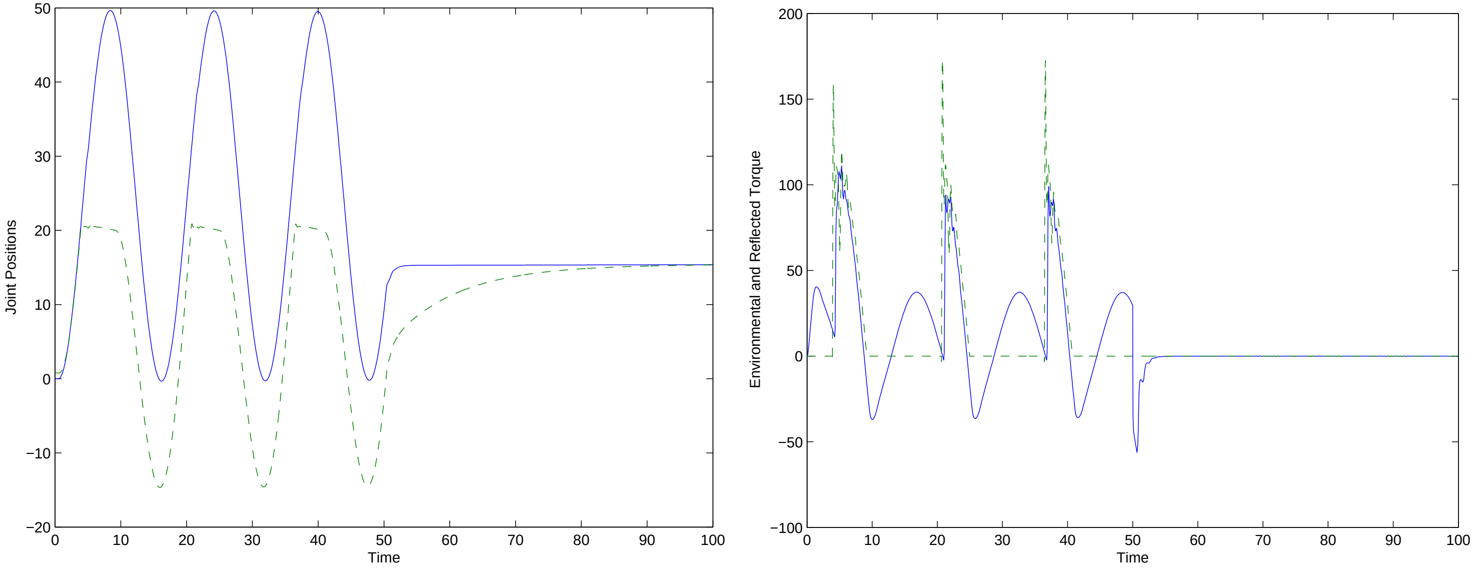

Blue shows the master position (left) and force (right)

Green shows the slave position (left) and force (right)

CONCLUSIONS

- It was shown that agents with passive dynamics, and a regular information graph imposed on them, when coupled together using a proportional control strategy, output synchronize even in the presence of arbitrary delays in the network.
- An important application to the problem of bilateral teleoperation was also demonstrated.
- The result guarantees delay independent exponential stability of the position and force tracking errors without using scattering theory.
- A passivity-based adaptive version of the result is easily derived.
- Future research involves extensions to other network topologies, varying delays, and additional applications.

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Muchas Gracias!