Efficient Similarity Search over Encrypted Data

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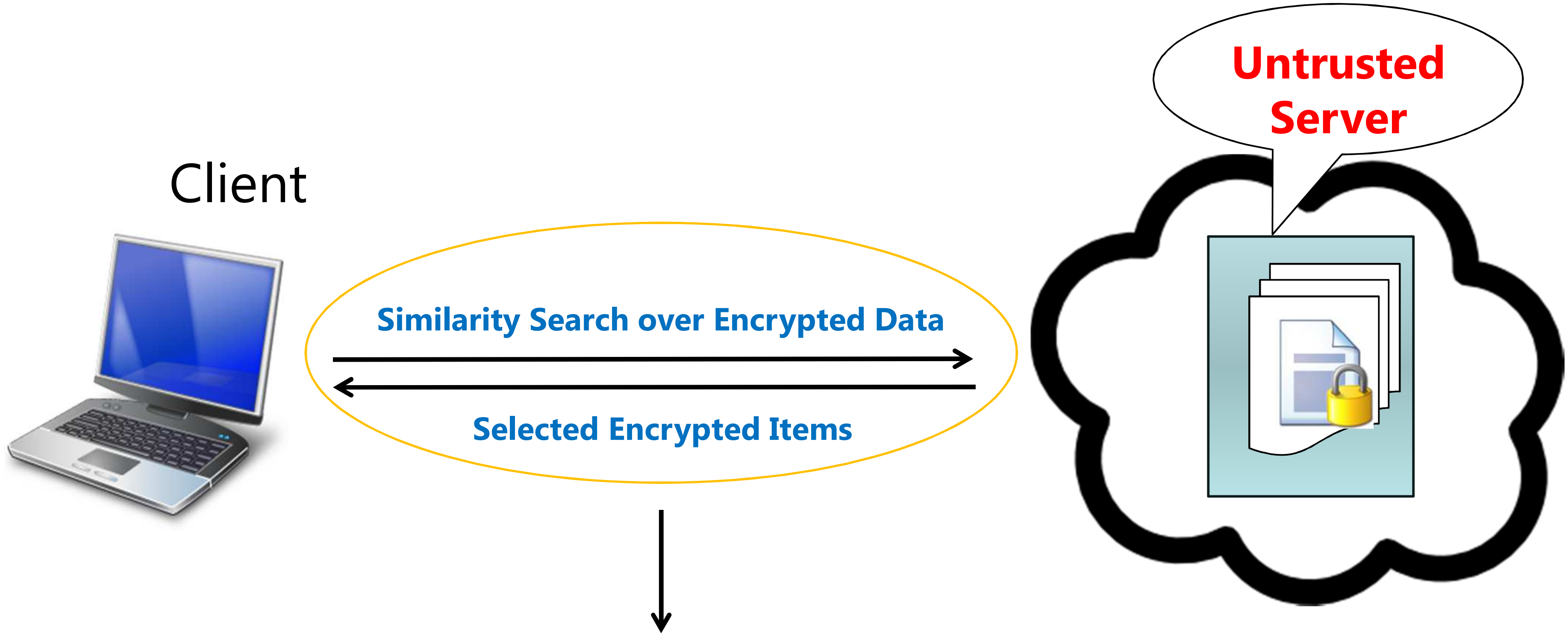
Introduction



Client

Similarity Search over Encrypted Data

Selected Encrypted Items

Untrusted Server

Requires: Efficient and Secure
Similarity Searchable Encryption Protocols

Problem Formulation

- BuildIndex(K, D): Extract feature set for each data item in D and form secure index I with key K.
- Trapdoor (K, f): Generate a trapdoor for a specific feature f with key K and output T.
- Search(I,T): Perform search on I with trapdoor of feature f (T) and output encrypted collection C:

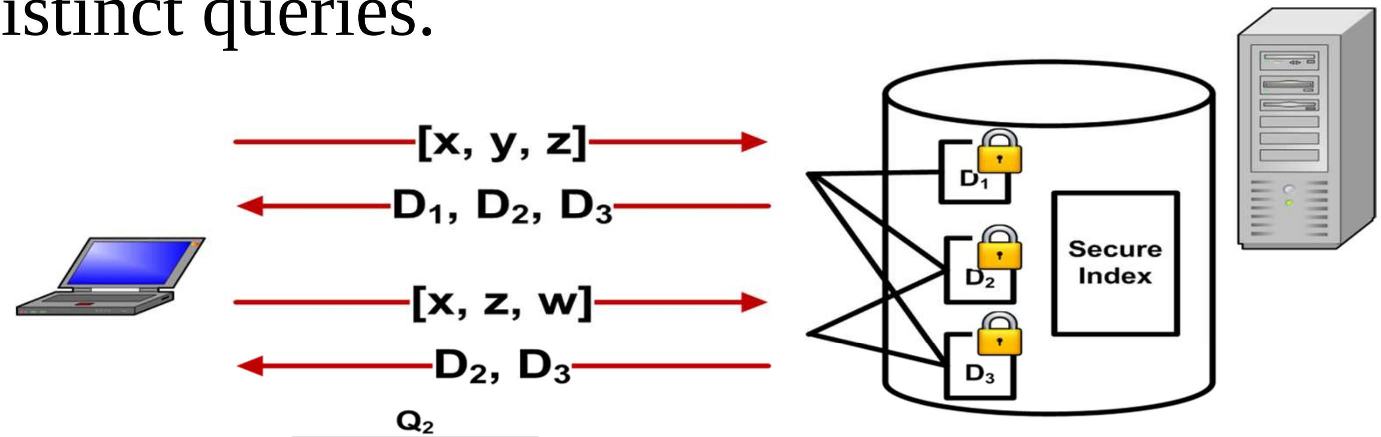
$$C_j \in C \ if \ \exists(f_i \in F_j) \ [dist(f_i, f) \leq \alpha]$$
$$C_j \notin C \ if \ \forall(f_i \in F_j) \ [dist(f_i, f) \geq \beta]$$

Locality Sensitive Hashing

- Family of functions is said to be (r_1, r_2, p_1,p_2)-sensitive if for any x, y ∈ F and for any h ∈ H.
  - if $dist(x, y) \leq r_1$, then $Pr[h(x) = h(y)] \geq p_1$
  - if $dist(x, y) \geq r_2$, then $Pr[h(x) = h(y)] \leq p_2$
- A composite function g: (g_1, …, g_λ) can be formed to push p_1 closer to 1 and p_2 closer to 0 by adjusting the LSH parameters (k, λ).

Security Goals

- Access Pattern (A_p): Identifiers of data items that are in the result set of a specific query.
- Similarity Pattern (S_p): Relative similarity among distinct queries.



$$\begin{array}{c|ccc} & & Q_2 & \\ \hline & 1 & 0 & 0 \\ Q_1 & 0 & 0 & 0 \\ & 0 & 1 & 0 \end{array}$$

$S_p\ (Q_1, Q_2)$

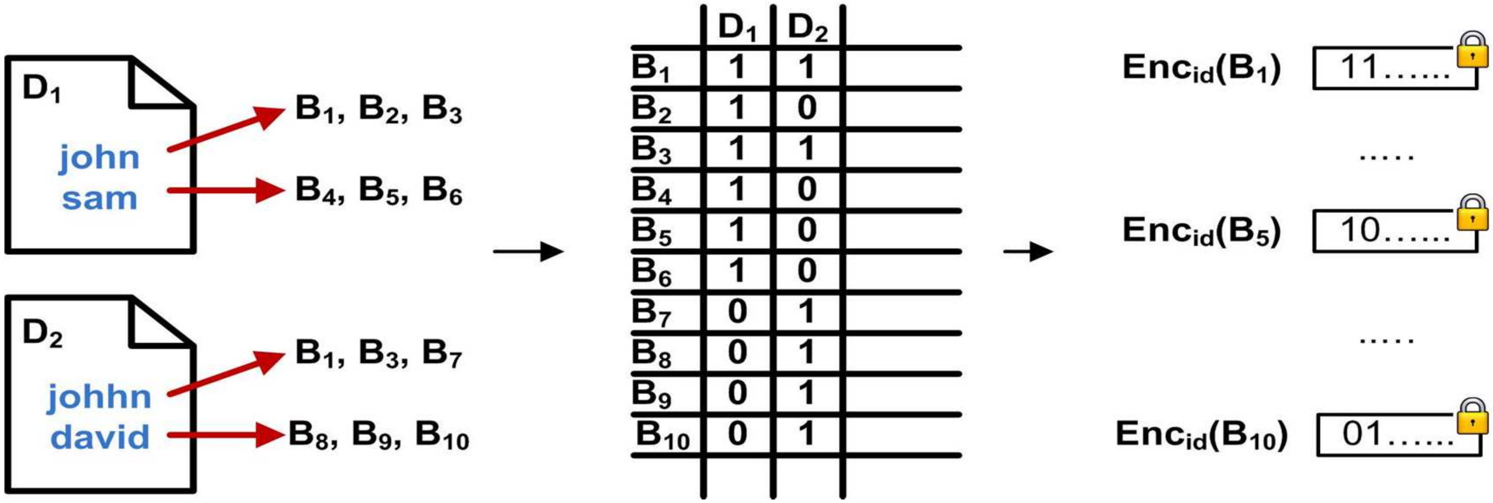
$A_p(Q_1) = \{D_1,D_2,D_3\}$, $A_p(Q_2) = \{D_2, D_3\}$

Secure LSH Index

- Content of any bucket B_k is a bit vector (V_{B_k}):

$$\begin{aligned} V_{B_k}[id(D_z)] &= 1 \quad if\ g_i(f_j) = B_k\ for\ g_i \in g,\ f_j \in D_z \\ V_{B_k}[id(D_z)] &= 0 \qquad\qquad\qquad otherwise \end{aligned}$$

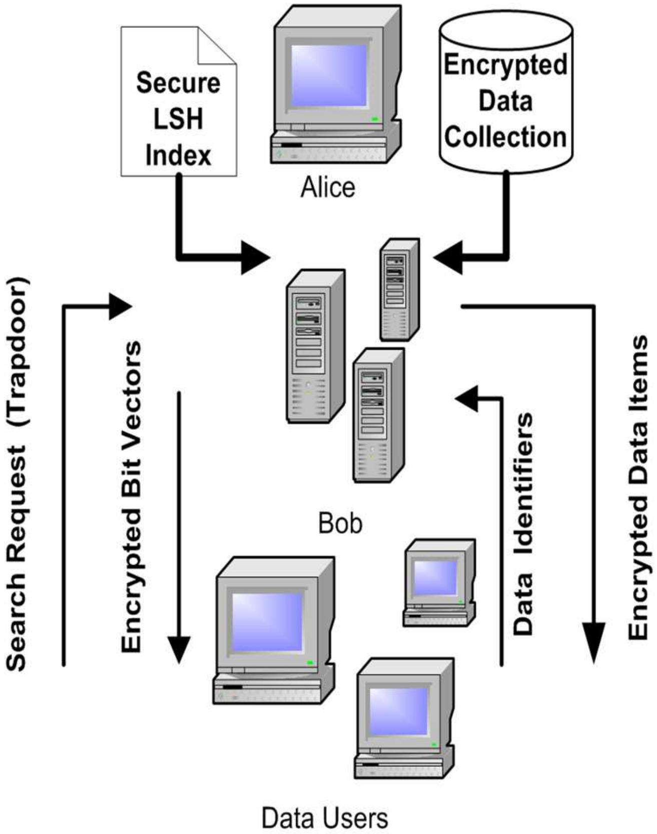
- [$\mathrm{Enc}_{id}(B_k)$, $\mathrm{Enc}_{payload}(V_{B_k})$] ϵ I.



	D_1	D_2
B_1	1	1
B_2	1	0
B_3	1	1
B_4	1	0
B_5	1	0
B_6	1	0
B_7	0	1
B_8	0	1
B_9	0	1
B_{10}	0	1

$\mathrm{Enc}_{id}(B_1)$ 11……
…..
$\mathrm{Enc}_{id}(B_5)$ 10……
…..
$\mathrm{Enc}_{id}(B_{10})$ 01……

Secure Search Scheme



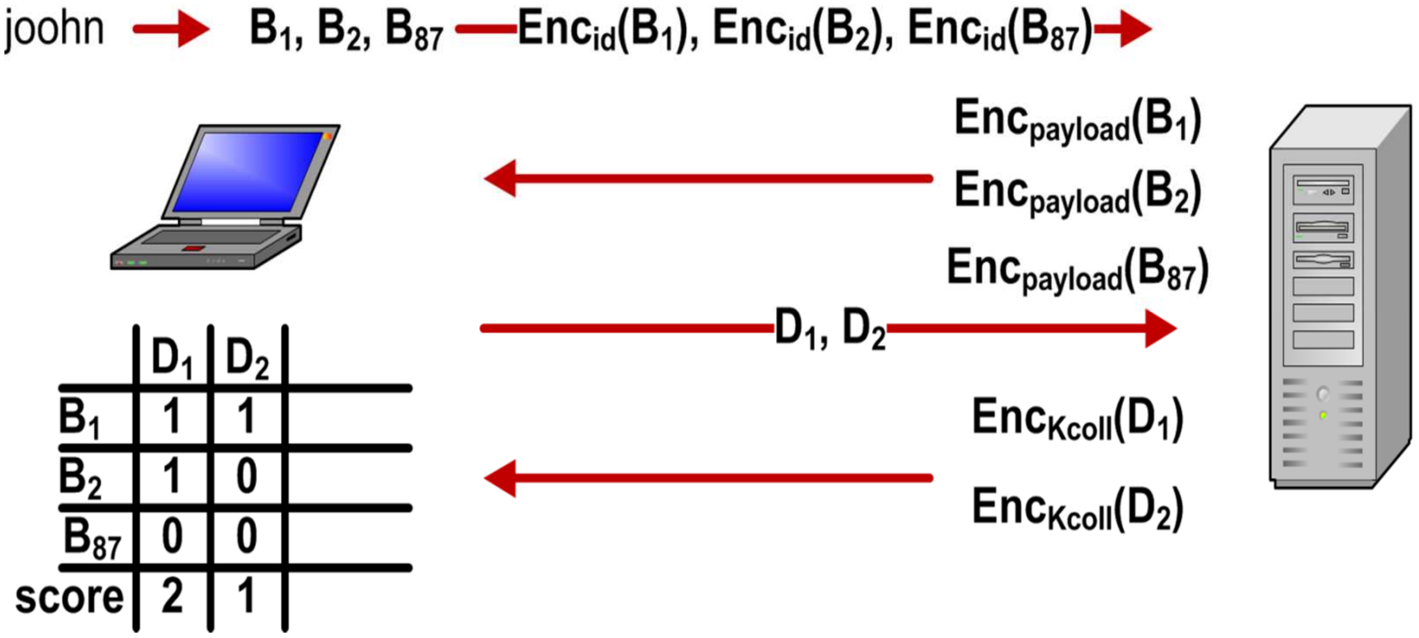
Shared Information

- K_{coll}: Secret key of data collection encryption
- K_{id}, $K_{payload}$: Secret keys of index construction
- ρ: Metric space translation function
- g: Locality sensitive function

Secure Search Scheme

- Trapdoor Construction for feature f_i :

$$T_{f_i} = \{Enc_{id}(g_1(\rho(f_i))), ..., Enc_{id}(g_\lambda(\rho(f_i)))\}$$



joohn → B_1, B_2, B_{87} → $Enc_{id}(B_1)$, $Enc_{id}(B_2)$, $Enc_{id}(B_{87})$ →

$Enc_{payload}(B_1)$

$Enc_{payload}(B_2)$

$Enc_{payload}(B_{87})$

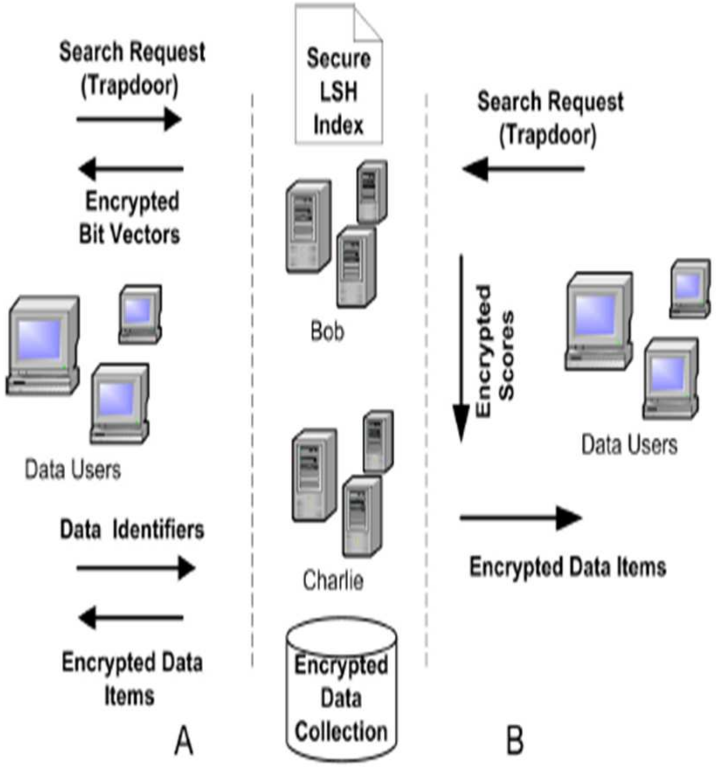
D_1, D_2

$Enc_{Kcoll}(D_1)$

$Enc_{Kcoll}(D_2)$

	D_1	D_2
B_1	1	1
B_2	1	0
B_{87}	0	0
score	2	1

Multi-Server Setting



- Basic search scheme reveals similarity and access patterns.
- It is desirable to separate leaked information to mitigate potential attacks.
- Multi-server setting enables lighter clients.

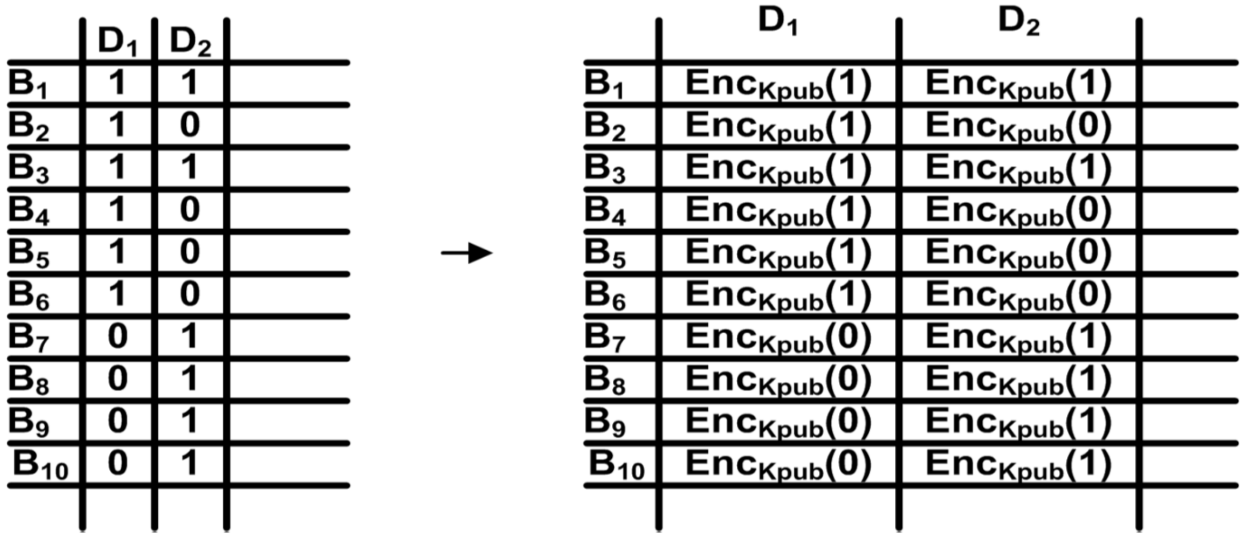
One Round Search Scheme

- This scheme is built on Paillier encryption that is semantically secure and additive homomorphic.

$$if\ \ (\pi_S, \sigma_{V_S}) \in I,\ \ then\ \ (\pi_S, [e_{S_1}, ..., e_{S_\ell}]) \in I'$$

$$e_{S_k} = Enc_{K_{pub}}(1)\ \ if\ \ V_s[id(D_j)] = 1$$

$$e_{S_k} = Enc_{K_{pub}}(0)\ \ otherwise$$



	D_1	D_2
B_1	1	1
B_2	1	0
B_3	1	1
B_4	1	0
B_5	1	0
B_6	1	0
B_7	0	1
B_8	0	1
B_9	0	1
B_{10}	0	1

→

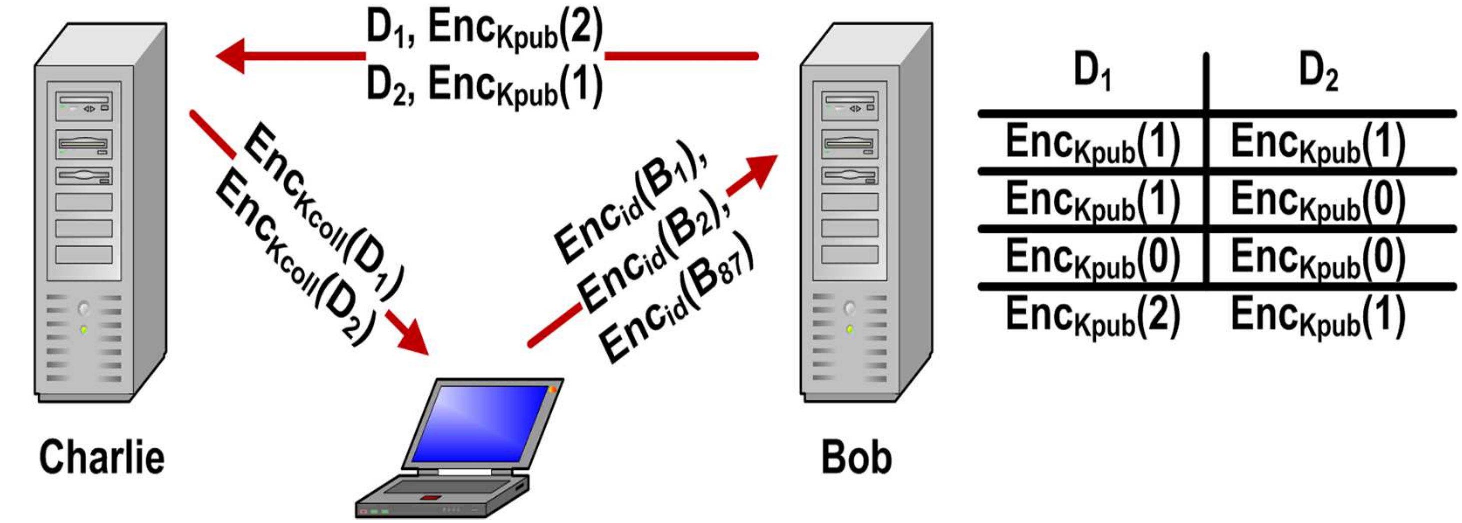
	D_1	D_2
B_1	$Enc_{Kpub}(1)$	$Enc_{Kpub}(1)$
B_2	$Enc_{Kpub}(1)$	$Enc_{Kpub}(0)$
B_3	$Enc_{Kpub}(1)$	$Enc_{Kpub}(1)$
B_4	$Enc_{Kpub}(1)$	$Enc_{Kpub}(0)$
B_5	$Enc_{Kpub}(1)$	$Enc_{Kpub}(0)$
B_6	$Enc_{Kpub}(1)$	$Enc_{Kpub}(0)$
B_7	$Enc_{Kpub}(0)$	$Enc_{Kpub}(1)$
B_8	$Enc_{Kpub}(0)$	$Enc_{Kpub}(1)$
B_9	$Enc_{Kpub}(0)$	$Enc_{Kpub}(1)$
B_{10}	$Enc_{Kpub}(0)$	$Enc_{Kpub}(1)$

One Round Search Scheme

- Bob performs homomorphic addition on the payloads of trapdoor components.

$$\omega_{score(i)} = e_{t_1(i)} \odot .... \odot e_{t_\lambda(i)}$$

$(i, \omega_{score(i)})$ pairs are sent to Charlie



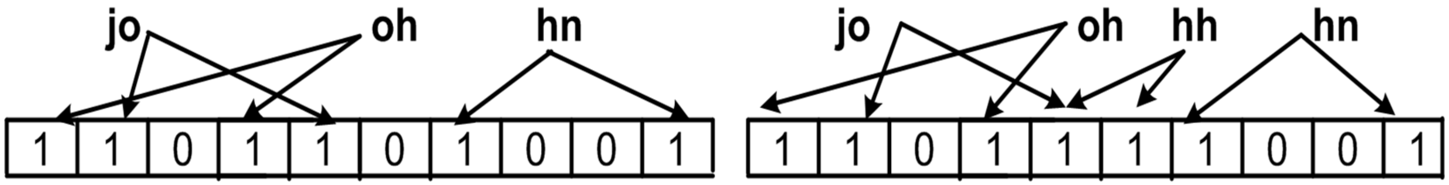
D_1	D_2
$Enc_{Kpub}(1)$	$Enc_{Kpub}(1)$
$Enc_{Kpub}(1)$	$Enc_{Kpub}(0)$
$Enc_{Kpub}(0)$	$Enc_{Kpub}(0)$
$Enc_{Kpub}(2)$	$Enc_{Kpub}(1)$

Error Aware Keyword Search

- Typographical errors are common both in the queries and data sources.
- In this context, data items be the documents, features be the words in the document and query feature be a keyword.
- Bloom filter encoding enables efficient space translation for approximate string matching.

Error Aware Keyword Search

- Elegant locality sensitive family has been designed for Jaccard distance (MinHash) that is [r_1, r_2 ,1-r_1, 1-r_2] sensitive.



A = ρ(john) = {1, 2, 4, 5, 7, 10}

B = ρ(johhn) = {1, 2, 4, 5, 6, 7, 10}

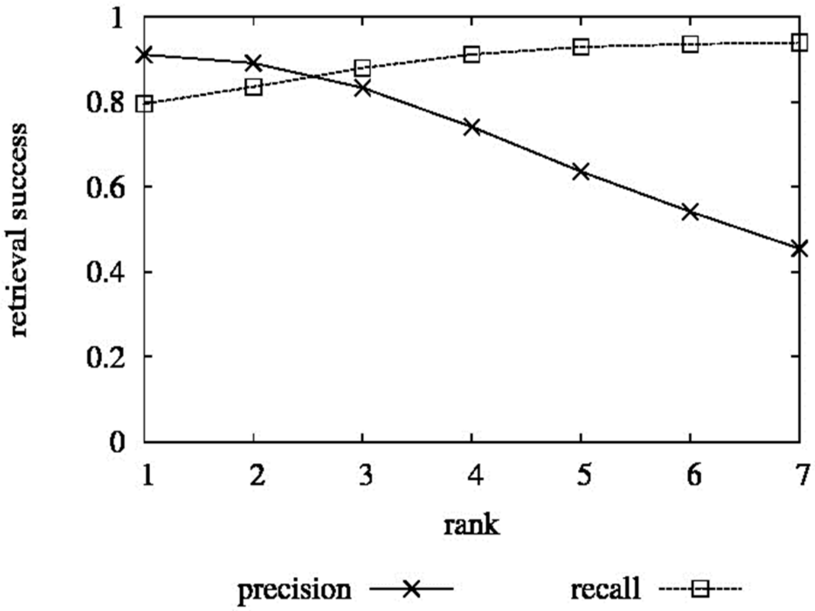
$J_d(A,B) = 1 – |A \cap B|/|A \cup B|$ $J_d(A,B) = 1 – 6/7 = 0.14$

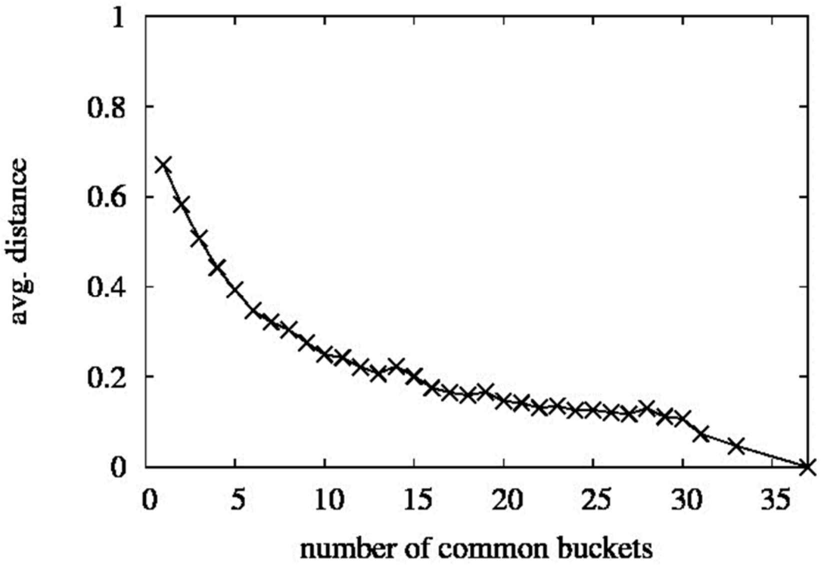
Experimental Setup

- A sample corpus of 5000 emails is constructed from publicly available Enron e-mail dataset.
- Words in e-mails are embedded into 500 bit Bloom filter with 15 hash functions.
- (0.45, 0.8, 0.85, 0.01)-sensitive family is formed from MinHash to tolerate typos. Common typos are introduced into the queries %25 of the time.
- Default Parameters: (Number of documents: 5000, Number of features: 5000, k:5, λ: 37).

Retrieval Evaluation

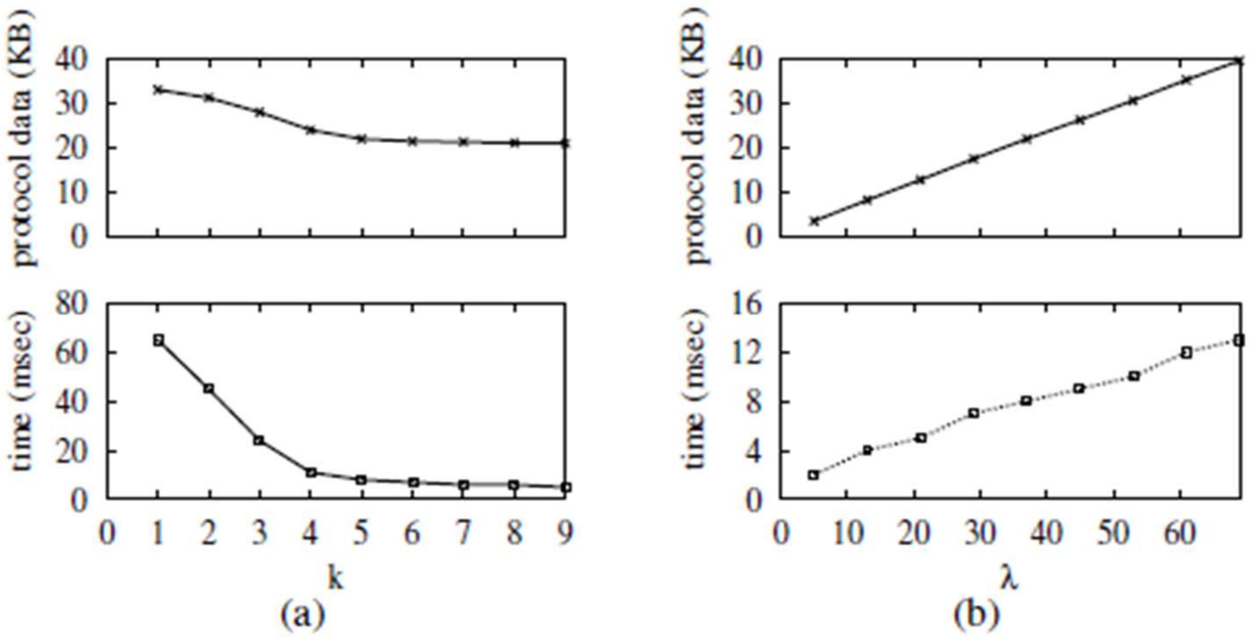
- Ranking limits retrieval of irrelevant items.





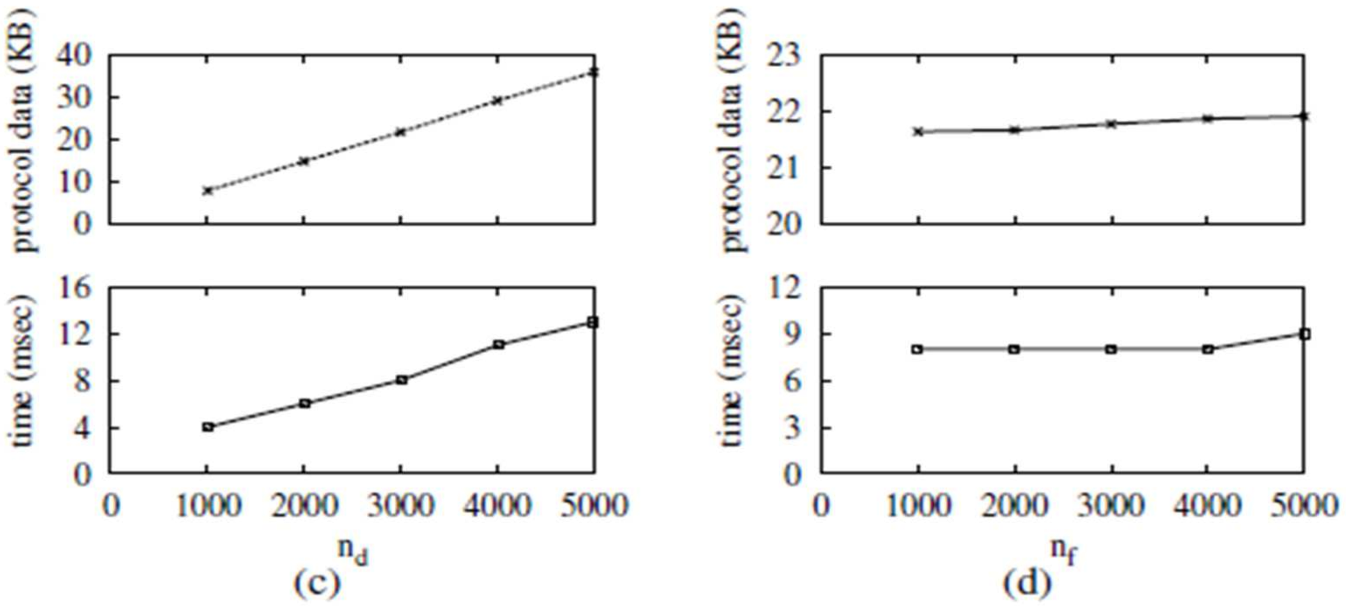
Performance Evaluation (Single Server)

- Increase in k and decrease in λ have similar effects. Decrease in λ leads smaller trapdoors.



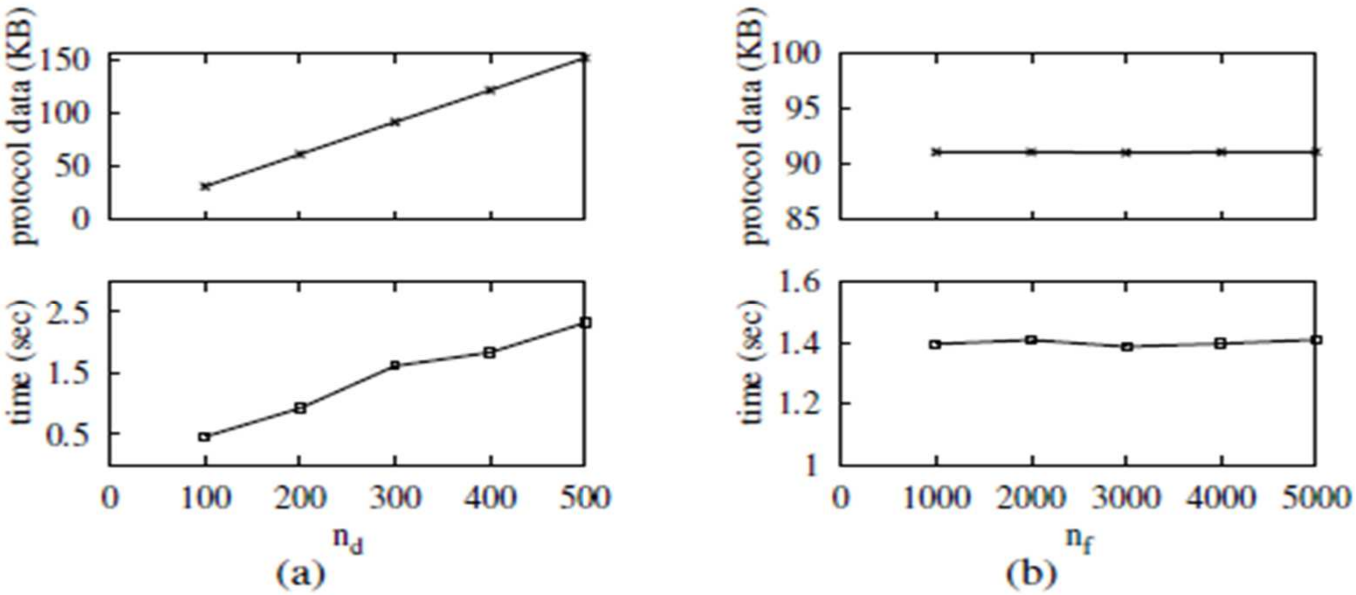
Performance Evaluation (Single Server)

- With increasing n_d, matching documents and the size of transferred bit vectors becomes larger.



Performance Evaluation (Multi-Server)

- Transfer of homomorphic addition results between servers is the main bottleneck.



Conclusion

- We proposed LSH based secure index and search scheme to enable fast similarity search over encrypted data.
- We provided a rigorous security definition and proved the security of the scheme to ensure confidentiality of the sensitive data.
- Efficiency of the proposed scheme is verified with empirical analysis.

Conclusion

THANKS …!

QUESTIONS?