CS 6347

Lecture 22

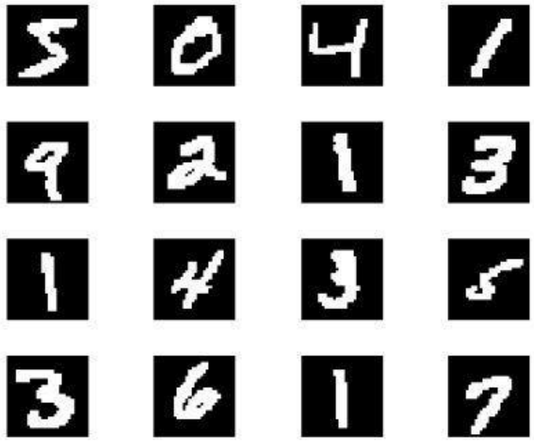
Neural Networks

Classification Problems

- We've been focusing primarily on two different types of learning problems
  - Classification: given a collection of labelled data for training, correctly predict the label of unseen/unlabelled data
  - Structured prediction
- Many natural machine learning tasks can be formulated as classification problems

Handwritten Digit Recognition

- Given a collection of handwritten digits and their corresponding labels, we'd like to be able to correctly classify handwritten digits
  - A simple algorithmic technique can solve this problem with 95% accuracy
    - This seems surprising, in fact, state-of-the-art methods can achieve near 99% accuracy (you've probably seen these in action if you've deposited a check recently)



Digits from the MNIST data set

Neural Networks

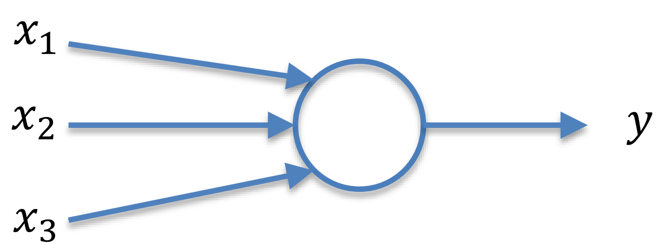
- The basis of neural networks was developed in the 1940s -1960s
  – The idea was to build mathematical models that might “compute” in the same way that neurons in the brain do
  – As a result, neural networks are biologically inspired, though many of the algorithms that are used to work with them are not biologically plausible

Neural Networks

- Neural networks consist of a collection of artificial neurons
- There are different types of neuron models that are commonly studied
  - The perceptron (one of the first studied)
  - The sigmoid neuron (most common)
- A neural network is typically a directed graph consisting of a collection of neurons (the nodes in the graph), directed edges (each with an associated weight), and a collection of fixed binary inputs

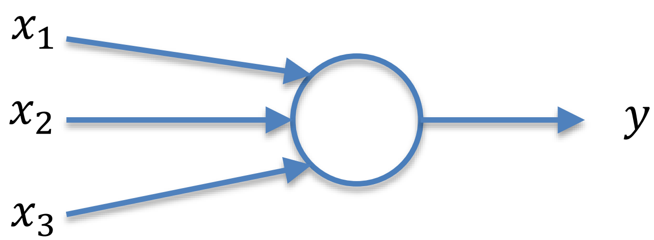
The Perceptron

- A perceptron is an artificial neuron that takes a collection of binary inputs and produces a binary output
  - The output of the perceptron is determined by summing up the weighted inputs and thresholding the result: if the weighted sum is larger than the threshold, the output is one (and zero otherwise)



$$y = \begin{cases} 1 & w_1 x_1 + w_2 x_2 + w_3 x_3 > threshold \\ 0 & otherwise \end{cases}$$

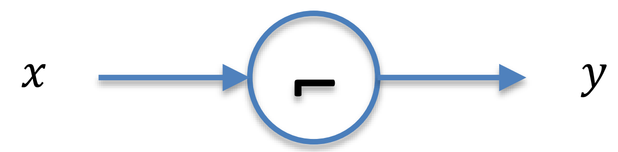
The Perceptron



$$y = \begin{cases} 1 & w_1x_1 + w_2x_2 + w_3x_3 > threshold \\ 0 & otherwise \end{cases}$$

- The weights can be both positive and negative
- Many simple decisions can be modeled using perceptrons
  - Example: AND, OR, NOT

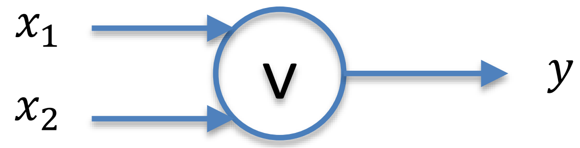
Perceptron for NOT



- Choose $w = -1$, threshold $= -.5$
- $y = \begin{cases} 1 & -x > -.5 \\ 0 & -x \leq -.5 \end{cases}$

Perceptron for OR

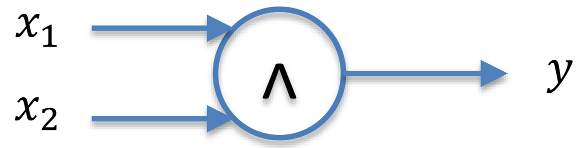
Perceptron for OR



- Choose $w_1 = w_2 = 1$, threshold $= 0$
- $y = \begin{cases} 1 & x_1 + x_2 > 0 \\ 0 & x_1 + x_2 \leq 0 \end{cases}$

Perceptron for AND

Perceptron for AND



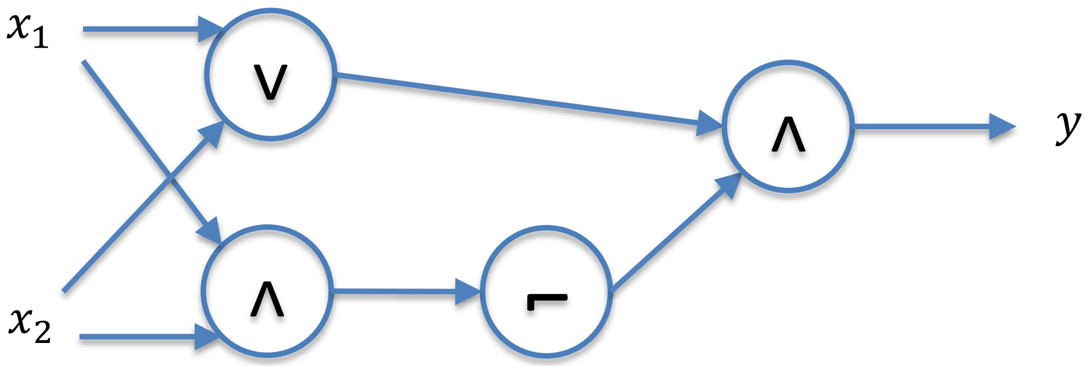
- Choose $w_1 = w_2 = 1$, threshold $= 1.5$

- $y = \begin{cases} 1 & x_1 + x_2 > 1.5 \\ 0 & x_1 + x_2 \leq 1.5 \end{cases}$

Perceptron for XOR

Perceptron for XOR

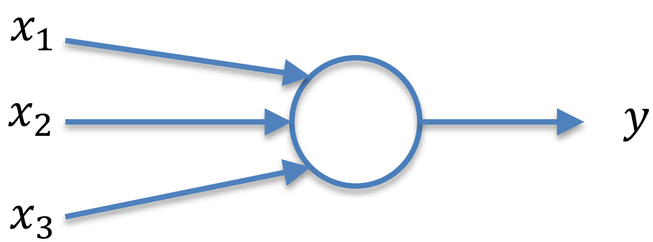
- Need more than one perceptron!



- Weights for incoming edges are chosen as before
  - Networks of perceptrons can encode any circuit!

Perceptrons

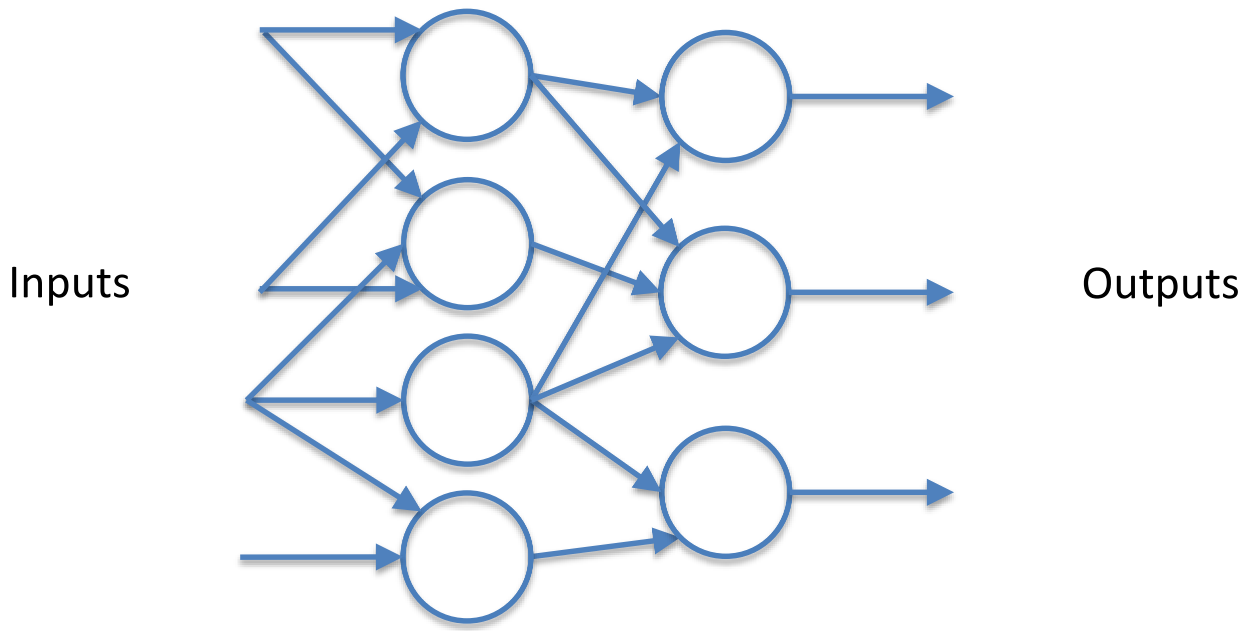
- Perceptrons are usually expressed in terms of a collection of input weights and a bias b (which is the negative threshold)



$$y = \begin{cases} 1 & w_1x_1 + w_2x_2 + w_3x_3 + b > 0 \\ 0 & otherwise \end{cases}$$

Neural Networks

- Gluing a bunch of perceptrons together gives us a neural network
- In general, neural nets have a collection of binary inputs and a collection of binary outputs

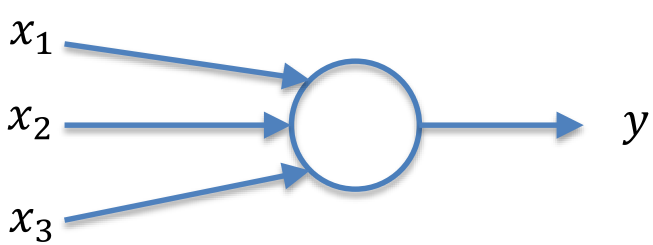


Beyond Perceptrons

- Given a collection of input-output pairs, we'd like to learn the weights of the neural network so that we can correctly predict the ouput of an unseen input
  - We could try learning via gradient descent (e.g., by minimizing the error)
    - This approach doesn't work so well: small changes in the weights can cause dramatic changes in the output
    - This is a consequence of the discontinuity of the sharp thresholding

The Sigmoid Neuron

- A sigmoid neuron is an artificial neuron that takes a collection of inputs in the interval $[0,1]$ and produces an output in the interval $[0,1]$
  - The output is determined by summing up the weighted inputs plus the bias and applying the sigmoid function to the result



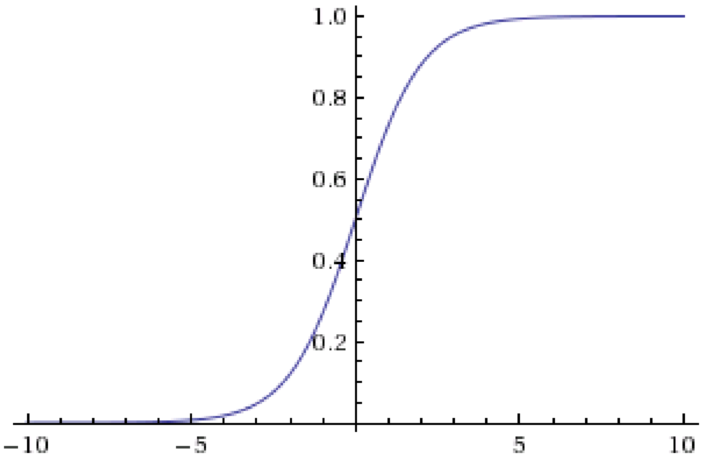
$$y = \sigma(w_1 x_1 + w_2 x_2 + w_3 x_3 + b)$$

where σ is the sigmoid function

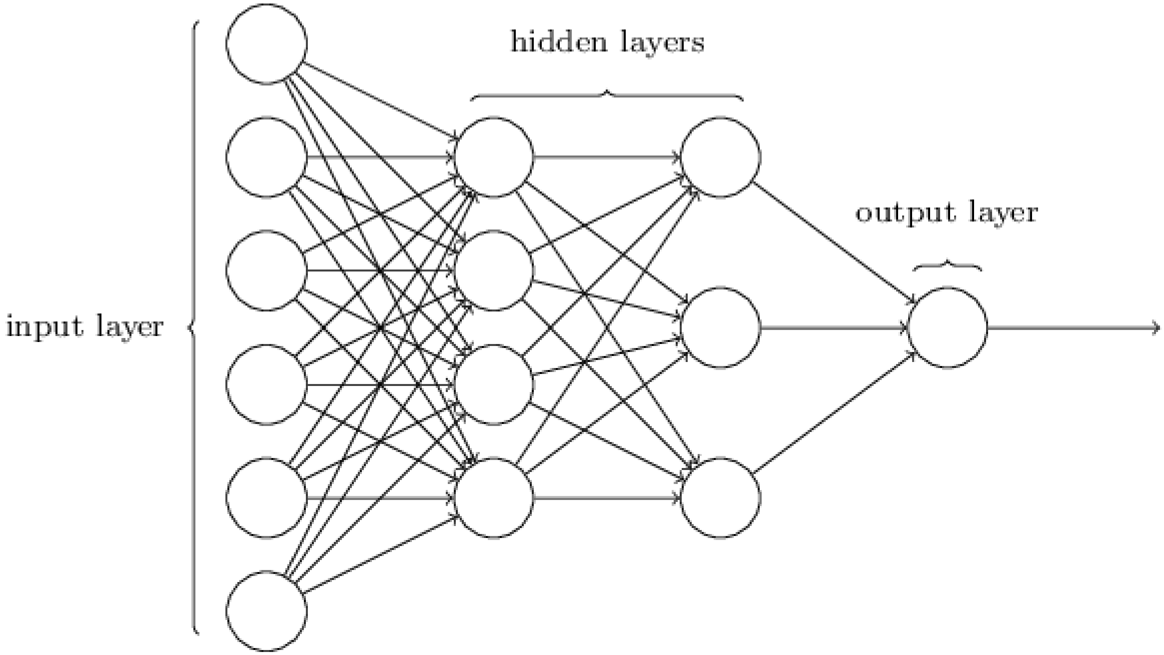
The Sigmoid Function

- The sigmoid function is a continuous function that approximates a step function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



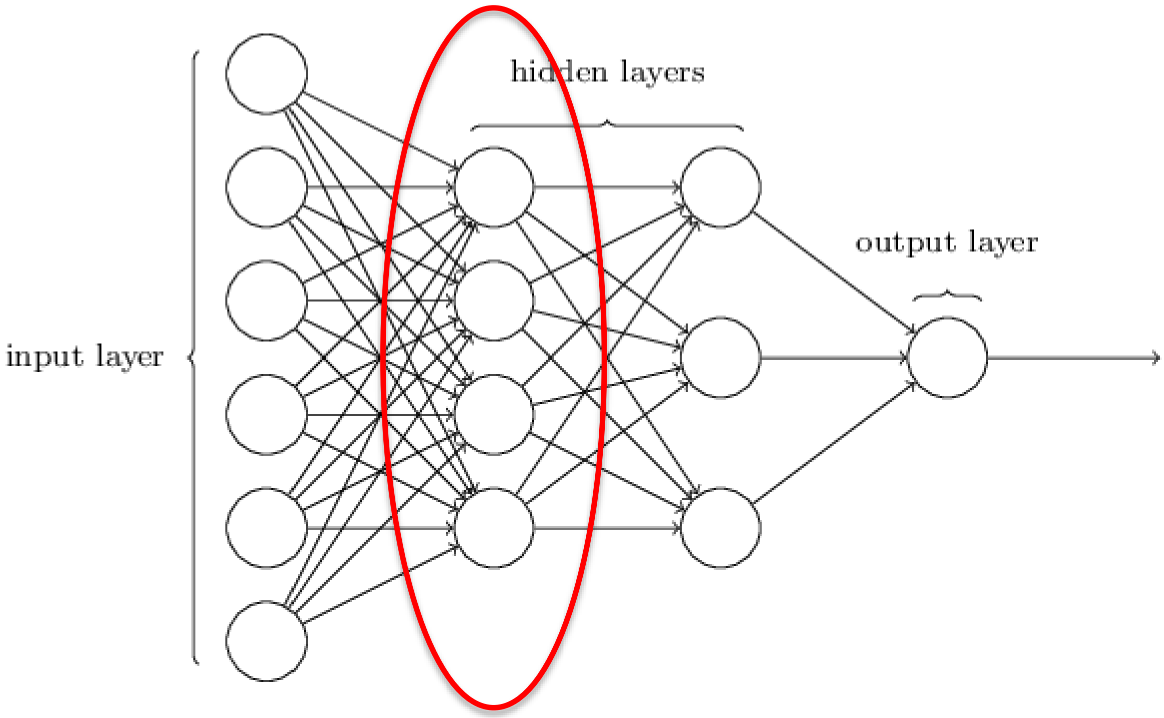
Multilayer Neural Networks



from Neural Networks and Deep Learning by Michael Nielson

Multilayer Neural Networks

NO intralayer connections



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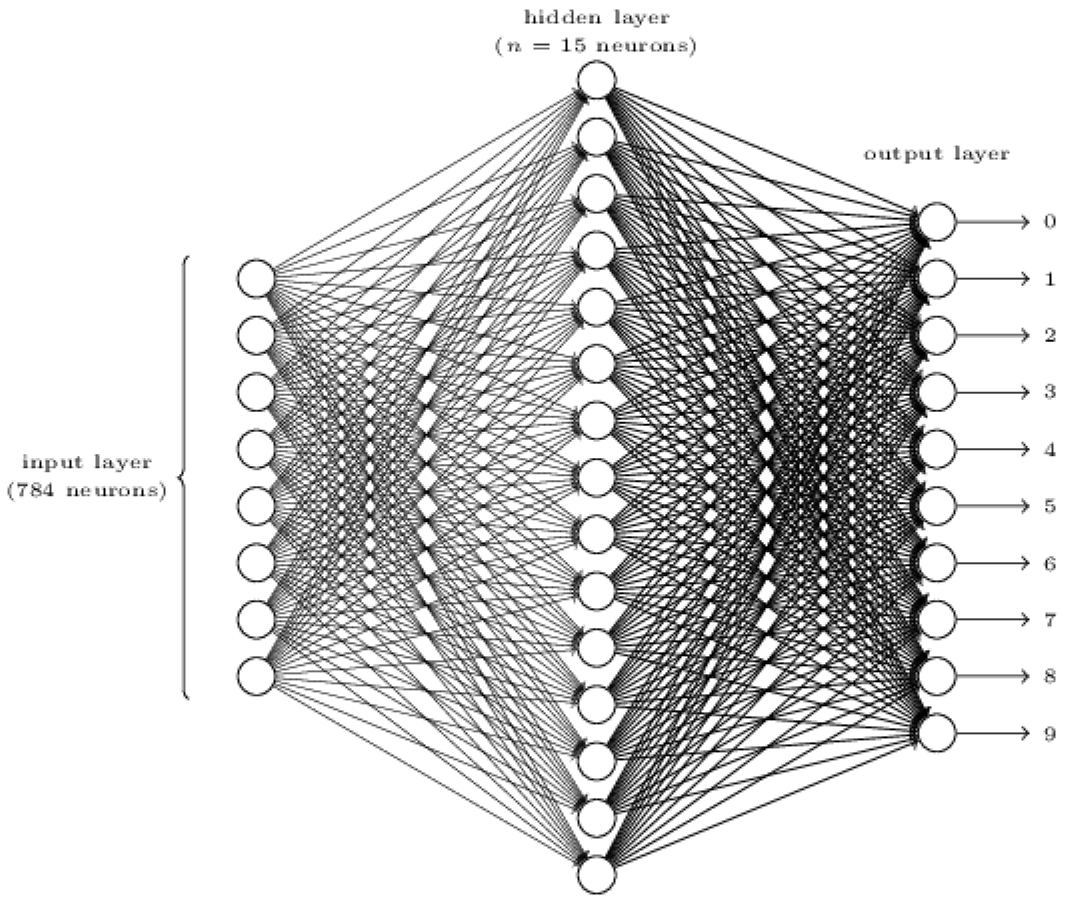
Neural Network for Digit Classification



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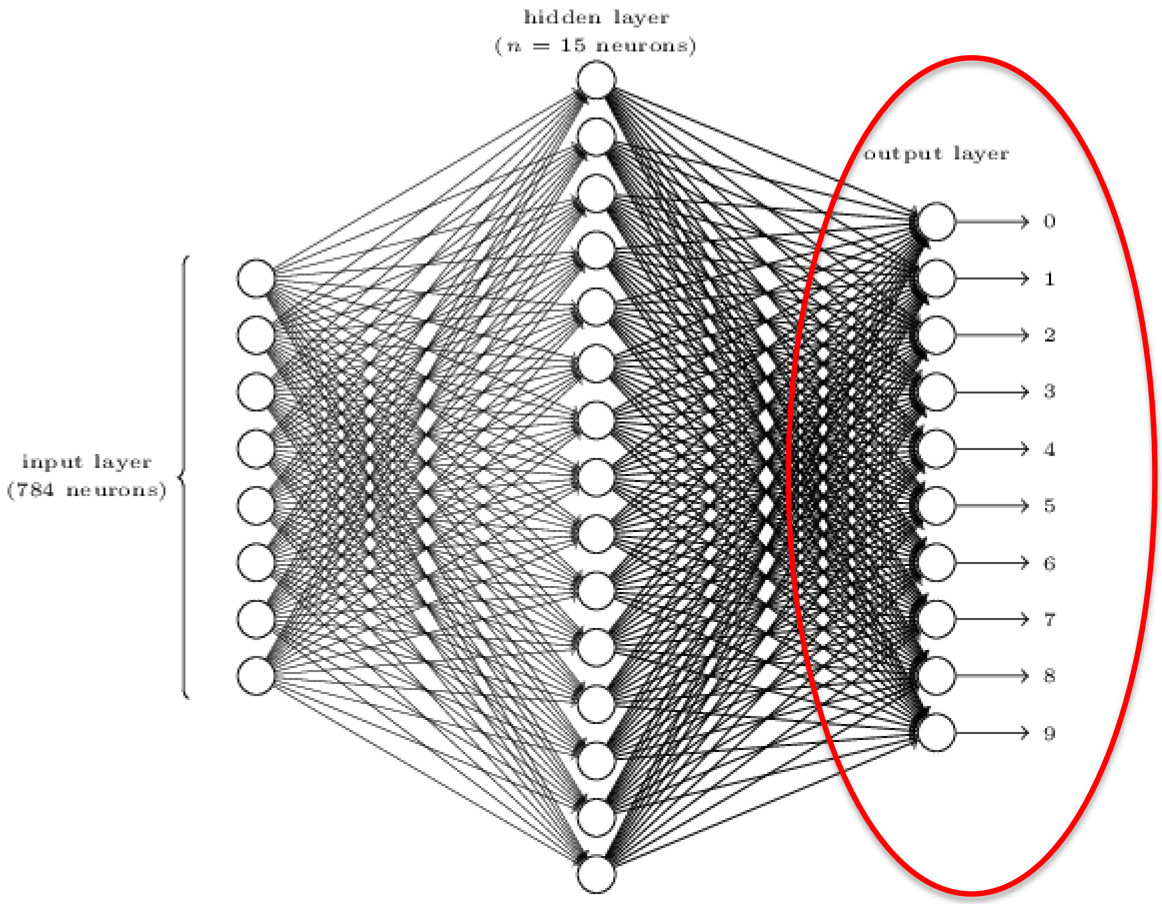
Neural Network for Digit Classification



Why 10
instead of 4?

from Neural Networks and Deep Learning by Michael Nielson

Training Neural Networks

- To do the learning, we first need to define a cost function to minimize

$$C(w,b) = \frac{1}{2M} \sum_m \|y^m - a(x^m, w, b)\|^2$$

- The data consists of input output pairs $(x^1, y^1), \ldots, (x^M, y^M)$
- $a(x, w, b)$ is the output of the neural network for the m^{th} sample
- w and b are the weights an biases

UTD

Gradient of the Cost Function

- The derivative of the cost function is relatively straightforward to calculate

$$\frac{\partial C(w,b)}{\partial w_k} = \frac{1}{M}\sum_m \left[y^m - \frac{\partial a(x^m,w,b)}{\partial w_k}\right]$$

  – To compute the derivative of a, use the chain rule and the derivative of the sigmoid function

$$\frac{d\sigma(z)}{dz} = \sigma(z)\cdot(1-\sigma(z))$$

  – This gets complicated quickly with lots of layers of neurons

Stochastic Gradient Descent

- To make the training more practical, stochastic gradient descent is used instead of standard gradient descent
- The idea of stochastic gradient descent is to approximate the gradient of a sum by sampling a few indices uniformly at random and averaging

$$\nabla_x \sum_{i=1}^{n} f_i(x) \approx \frac{1}{K} \sum_{k=1}^{K} \nabla_x f_{i^k}(x)$$

here, each i^k is sampled uniformly at random from $\{1, \ldots, n\}$