# Reinforcement Learning

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based on the slides of Ronald J. Williams

# Reinforcement Learning

- Autonomous "agent" that interacts with an environment through a series of actions
  - E.g., a robot trying to find its way through a maze
    - Actions include turning and moving through the maze
  - The agent earns rewards from the environment under certain (perhaps unknown) conditions
- The agent's goal is to maximize the reward
  - We say that the agent learns if, over time, it improves its performance

# Reinforcement Learning

- Often formalized (mathematically) as Markov Decision Processes (MDPs) or Partially Observable Markov Decision Processes (POMDPs)

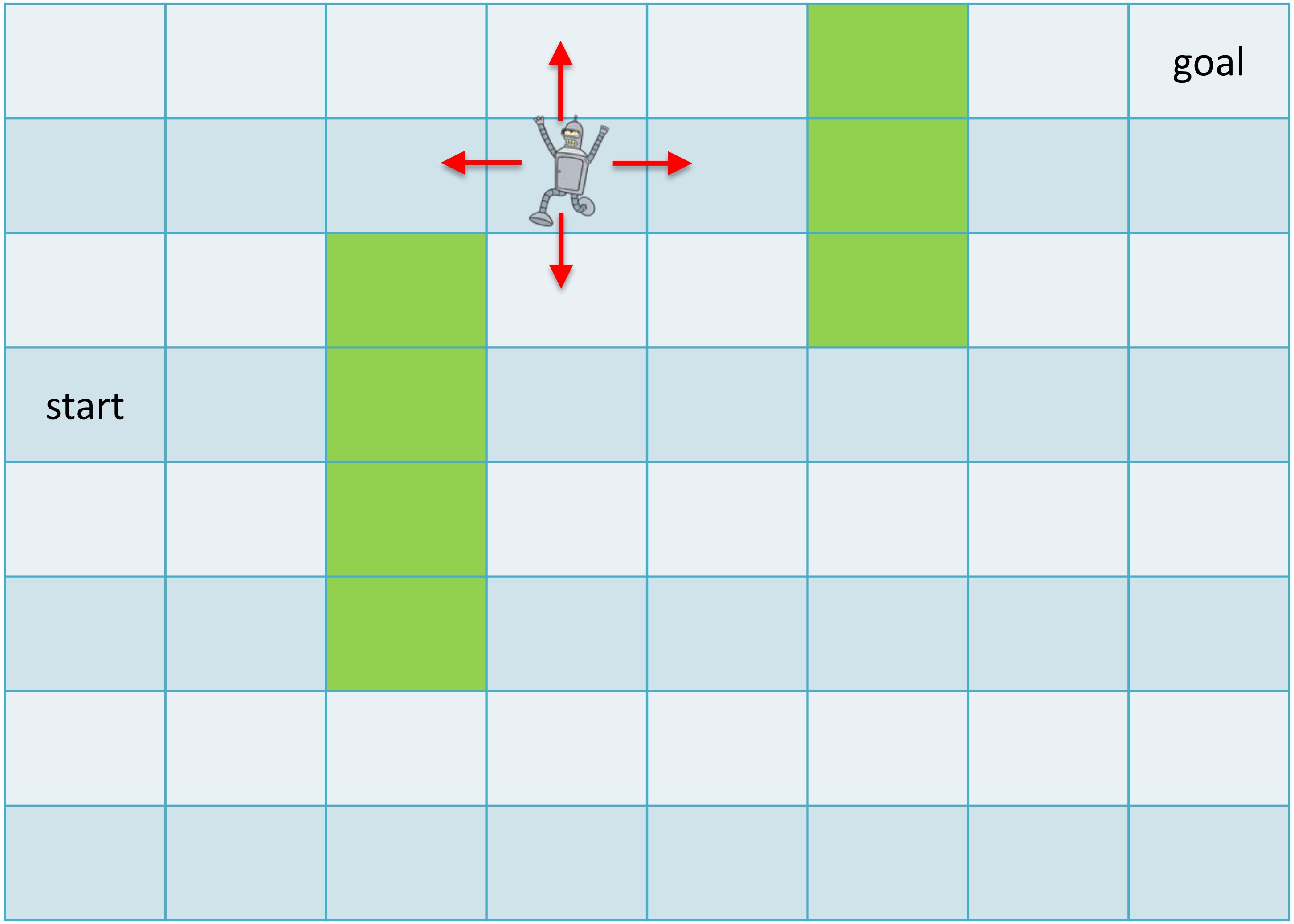
- MDPs are described by series of states (state of the environment) and a collections of actions corresponding to each state (allowable actions that change the state of the environment)

  - The next state depends (perhaps probabilistically) on only the current state and the chosen action

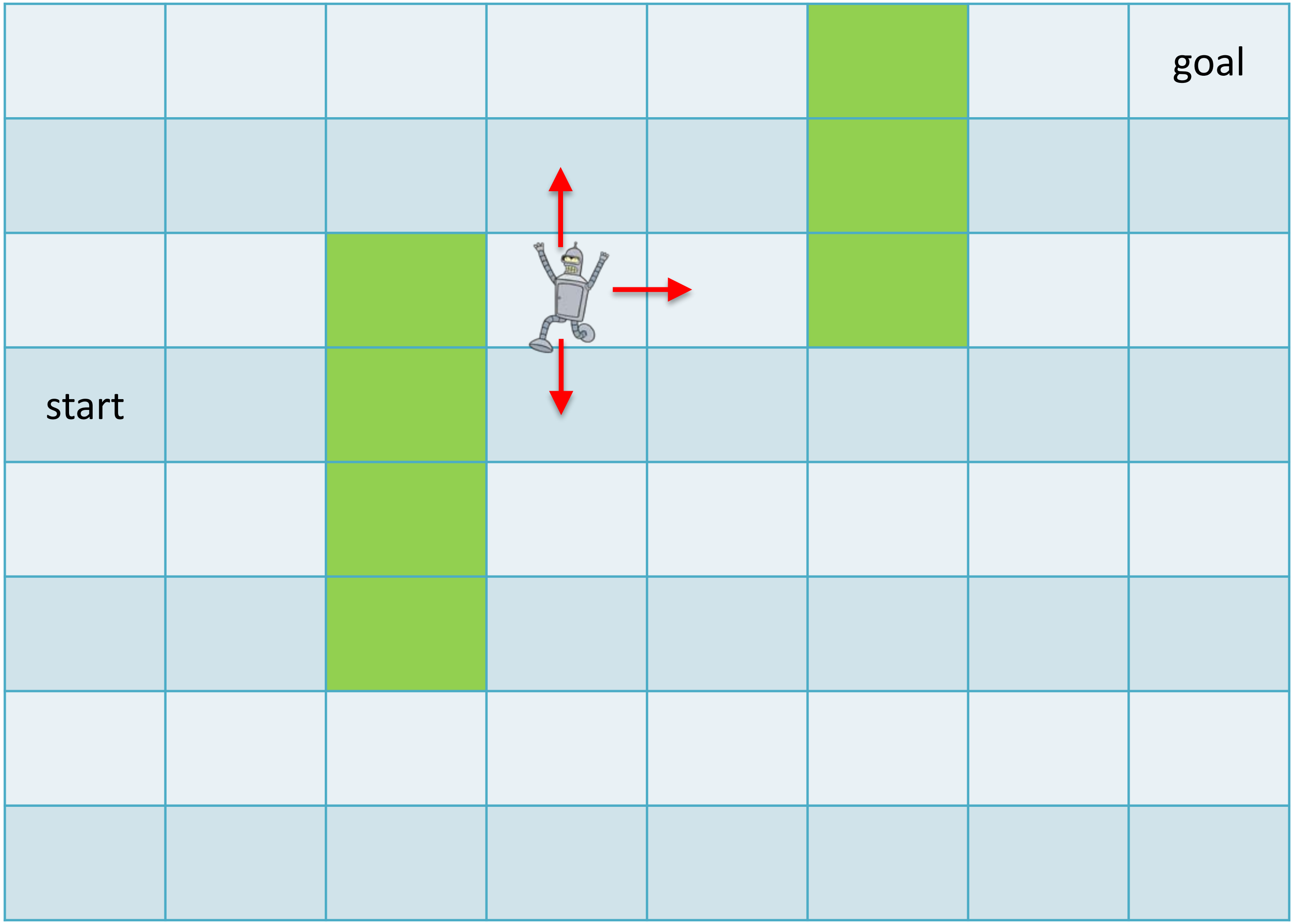
  - Each state/action pair has an associated reward (possibly probabilistic)

- Markov chains are a simple form of MDP with only one action and no rewards

# Example



# Example



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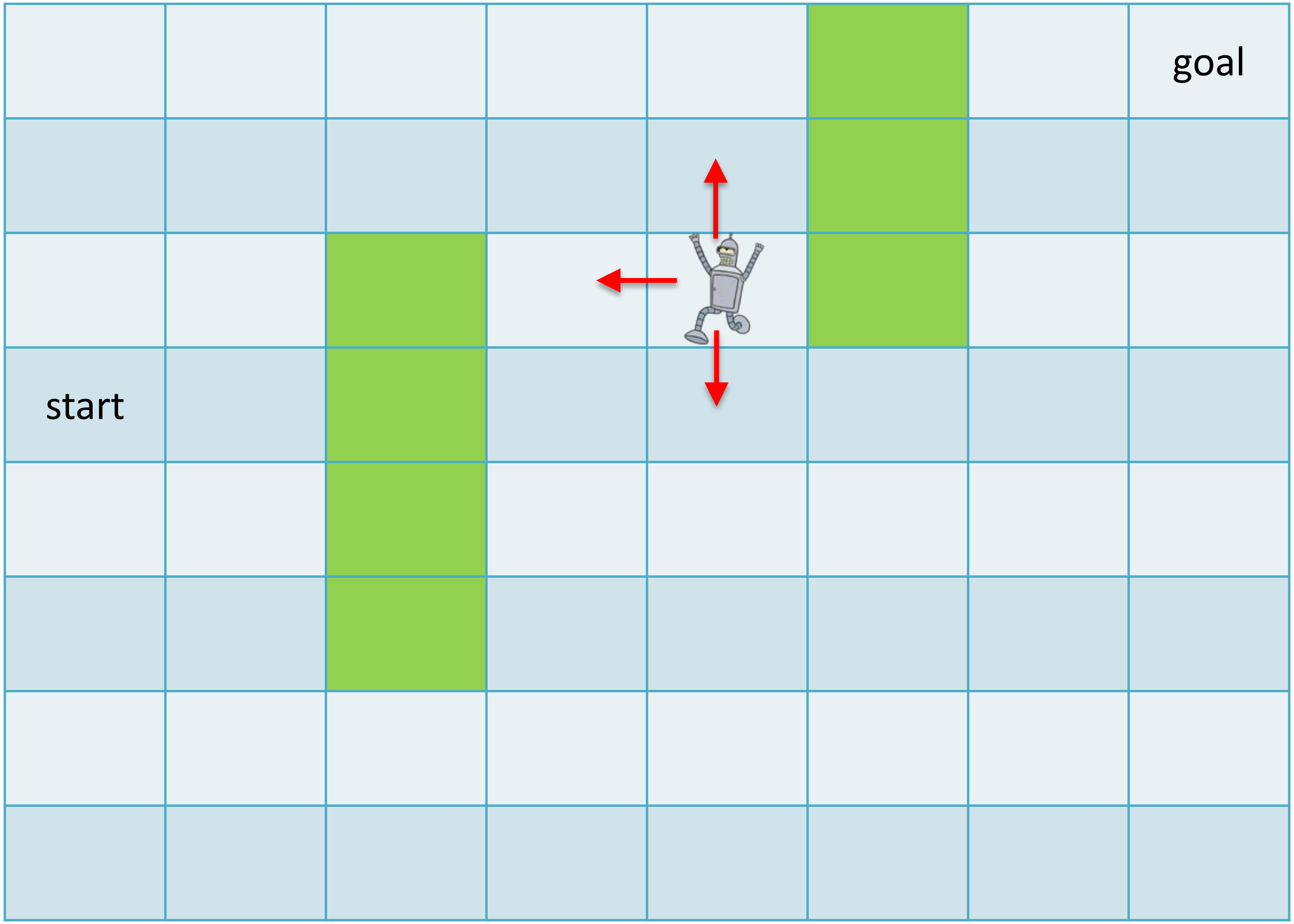


# MDPs

- Rewards can be positive or negative
  - E.g., the robot might receive a small penalty each time it takes a step that does not reach the goal
- Objective of the learning process is to develop a **policy** (a way to choose actions given the current state) to maximize the reward
  - Could be difficult to do as rewards may be delayed
    - E.g., the robot receives a reward for reaching the end of the maze, but only penalties in-between

# MDPs

- Agent at step $t$
  - Observes the state of the system
  - Selects an action to perform
  - Receives some reward
- This process is repeated indefinitely

# Policies

- A policy is the prescription by which the agent selects an action to perform

  - Deterministic: the agent observes the state of the system and chooses an action

  - Stochastic: the agent observes the state of the system and then selects an action, at random, from some probability distribution over possible actions

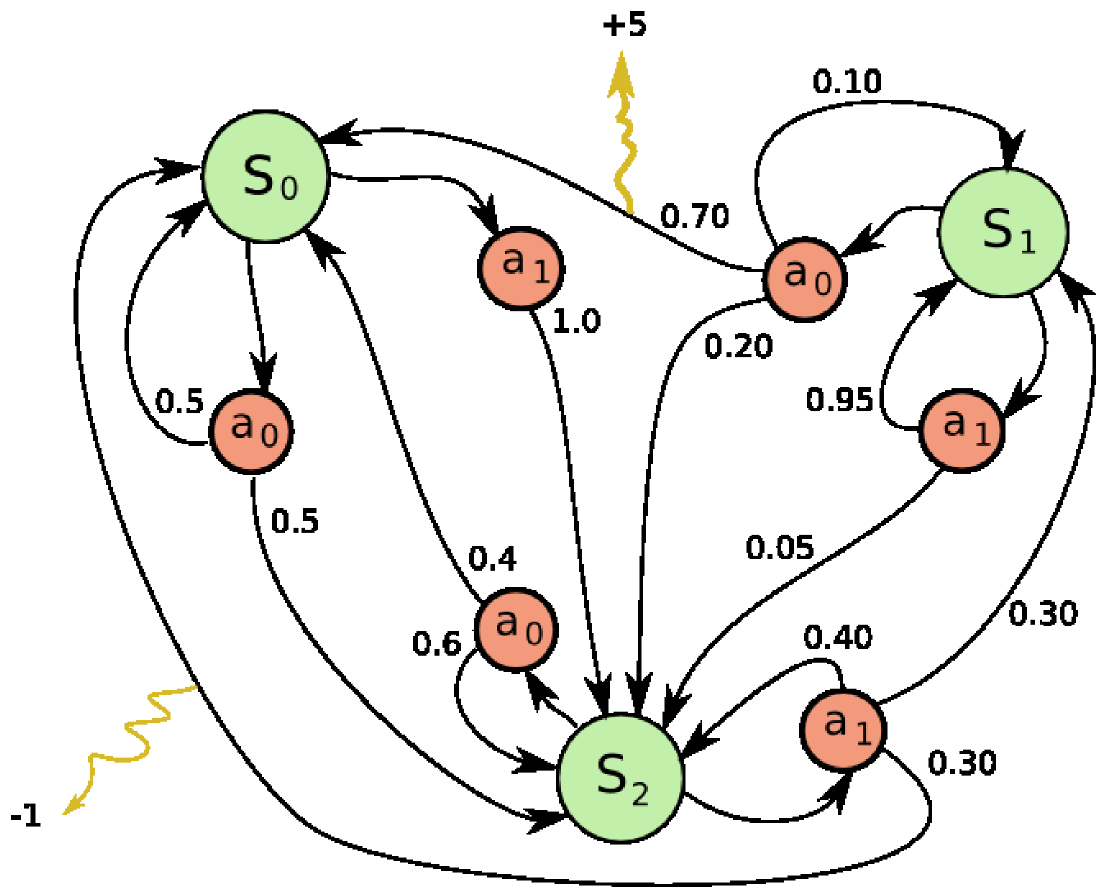
# Applications of MDPs

- Robot pathfinding
- Planning
- Elevator scheduling
- Manufacturing processes
- Network routing
- Game playing

# Formal Definition

- An MDP consists of the following
  - A finite set of states $S$
  - A set of allowable actions $A_s$ for each $s \in S$
  - A transition function $T: S \times A \to S$
  - A reward function $R: S \times A \to \mathbb{R}$
- In the general case, $T$ and $R$ can be stochastic functions (we'll worry about the deterministic case today)

# MDPs




src: Wikipedia

# Cumulative Reward

- A policy is a mapping from states to actions, $\pi: S \to A$
  - Policies can be deterministic or stochastic
- Let $r(t)$ denote the reward at time $t$
- The objective is to find a policy that maximizes the cumulative (discounted) reward

$$r(0) + \gamma r(1) + \gamma^2 r(2) + \cdots$$

where $\gamma \in (0,1)$ is a discount factor necessary to make the sum converge (also applied in economic contexts to prefer future rewards at a discounted rate)

# Value Function

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- How can we evaluate the quality of policy $\pi$?
- A **value** function $V: S \to \mathbb{R}$ assigns a real number to each state
  - A particular value function of interest will be the reward function

$$V^{\pi}(s) = \sum_{t=0}^{\infty} \gamma^t r(t)$$

where the state at time $t$ is generated from the state at time $t-1$ by applying the action dictated by the policy, $\pi(s_{t-1})$

# Technical Notes

- In the case that the rewards, transitions, policy, etc. are stochastic
  - Replace the reward, $r(t)$, with the expected reward under the policy
- An MDP has an absorbing state if there exists a state $s \in S$ such that, with probability one, $T(s, a) = s$ for all $a \in A_s$
  - In this case, if the absorbing state can always be reached, the discount factor is unnecessary

# Objective

- Find a policy $\pi^*: S \to A$ such that

$$V^{\pi^*}(s) \geq V^{\pi}(s)$$

  for all $s \in S$ and all policies $\pi$

- Any policy that satisfies this condition is called an optimal policy (may not be unique)
- There always exists an optimal policy
  - How do we find it?

# Optimal Policies

- Can find an optimal policy via a dynamic programming approach
  - Compute the optimal value, $V^{\pi^*}(s)$, for each state
  - Greedily select the action that maximizes reward
- We can describe the optimal value via a *recurrence relation*

$$V^{\pi^*}(s) = \max_{a \in A_s} \left( R(s,a) + \gamma V^{\pi^*}(T(s,a)) \right)$$

  - This is one of the so-called Bellman equations
  - Justifies the greedy strategy (all optimal strategies are “greedy” in this sense)

# Bellman Equations

$$V^{\pi}(s) = R(s, \pi(s)) + \gamma V^{\pi}(T(s, \pi(s)))$$

$$V^{\pi^*}(s) = \max_{a \in A_s} \left( R(s, a) + \gamma V^{\pi^*}(T(s, a)) \right)$$

- The first equation holds for any policy while the second must hold for any optimal policy
  - Why?

# The Greedy Strategy

- Given a value function $V: S \to \mathbb{R}$, we say that $\pi$ is greedy for $V$ if

$$\pi(s) \in \arg\max_a \big(R(s,a) + \gamma V(T(s,a))\big)$$

- If $\pi$ is not an optimal policy, then $\pi'$ which is greedy for $V^\pi$ must satisfy $V^\pi(s) \leq V^{\pi'}(s)$ for all $s \in S$
  - This suggests that we can, starting from any policy, obtain a better policy (similar to coordinate ascent)
  - Two questions:
    - Does this process converge?
    - If it converges, is the converged policy optimal?

# Value Iteration

- Choose an initial value function $V_0$ (could be anything)
- Repeat until convergence
  - For each $s$

$$V_{t+1}(s) = \max_{a \in A_s}\Big(R(s,a) + \gamma V_t(T(s,a))\Big)$$

- This process always converges to the optimal value, $V_*$, as long as $\gamma \in (0,1)$,

$$||V_{t+1} - V_*||_\infty \leq \gamma ||V_t - V_*||_\infty \leq \gamma^{t+1} ||V_0 - V_*||_\infty$$

# Example (100 reward at goal, -1 for each step)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 100 | 100 | 100 | 100 | 100 | | 100 | 100 |
| 100 | 100 | 100 | 100 | 100 | | 100 | 100 |
| 100 | 100 | | 100 | 100 | | 100 | 100 |
| 100 | 100 | | 100 | 100 | 100 | 100 | 100 |
| 100 | 100 | | 100 | 100 | 100 | 100 | 100 |
| 100 | 100 | | 100 | 100 | 100 | 100 | 100 |
| 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 99 | 99 | 99 | 99 | 99 | | 99 | 100 |
| 99 | 99 | 99 | 99 | 99 | | 99 | 99 |
| 99 | 99 | | 99 | 99 | | 99 | 99 |
| 99 | 99 | | 99 | 99 | 99 | 99 | 99 |
| 99 | 99 | | 99 | 99 | 99 | 99 | 99 |
| 99 | 99 | | 99 | 99 | 99 | 99 | 99 |
| 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 |
| 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 |

# Example (100 reward at goal, -1 for each step)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 98 | 98 | 98 | 98 | 98 | | 99 | 100 |
| 98 | 98 | 98 | 98 | 98 | | 98 | 99 |
| 98 | 98 | | 98 | 98 | | 98 | 98 |
| 98 | 98 | | 98 | 98 | 98 | 98 | 98 |
| 98 | 98 | | 98 | 98 | 98 | 98 | 98 |
| 98 | 98 | | 98 | 98 | 98 | 98 | 98 |
| 98 | 98 | 98 | 98 | 98 | 98 | 98 | 98 |
| 98 | 98 | 98 | 98 | 98 | 98 | 98 | 98 |

# Example (100 reward at goal, -1 for each step)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 87 | 88 | 89 | 90 | 91 | | 99 | 100 |
| 88 | 89 | 90 | 91 | 92 | | 98 | 99 |
| 87 | 88 | | 92 | 93 | | 97 | 98 |
| 86 | 87 | | 93 | 94 | 95 | 96 | 97 |
| 85 | 86 | | 92 | 93 | 94 | 95 | 96 |
| 86 | 87 | | 91 | 92 | 93 | 94 | 95 |
| 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 |
| 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 |

# Policy Iteration

- Choose an initial policy $\pi_0$ (could be anything)
- Repeat until convergence
  - Compute $V^{\pi_t}$
  - Choose $\pi_{t+1}$ to be a greedy policy with respect to $V^{\pi_t}$
- This process always converges to an optimal policy

# Q-Values

- For learning, it will be useful to express value functions in terms of Q-value functions

- For a policy $\pi$, $Q^{\pi}: S \times A \to \mathbb{R}$ is defined to be the value of the policy $\pi$ starting from state $s$ where the first action is taken to be $a$

$$Q^{\pi}(s,a) = R(s,a) + \gamma V^{\pi}(T(s,a))$$

- For any optimal policy $\pi^*$, $V^{\pi^*}(s) = \max_a Q^{\pi^*}(s,a)$

- A policy $\pi$ is said to be greedy with respect to $Q$ if

$$\pi(s) \in \arg\max_a Q(s,a)$$

# Reinforcement Learning

- The above is simply the theory of MDPs
  - We haven’t seen any “learning” yet
    - All transition and reward functions were assumed to be known in advance
- The setting for reinforcement learning:
  - The agent is the learner whose task is to maximize its respective rewards
  - All rewards and transitions are unknown and must be learned through trial and error (key complication in the learning setting)

# Approaches to RL

- Learn the MDP first, then use value/policy iteration
- Learn only the values (don't learn the MDP or explicitly model it)
  - Can be advantageous in practice as MDPs can require a significant amount of storage to specify completely
- Hybrid approaches of learning and planning...

# Q-Learning

- Choose an initial state-value function $Q(s, a)$
- Let $s$ be the initial state of the environment
- Repeat until convergence
  - Choose an action $a$ for the current state $s$ based on $Q$
  - Take action $a$ and observe the reward $r$ and the new state $s'$
  - Set $Q(s, a) = (1 - \alpha)Q(s, a) + \alpha \left( r + \gamma \max_{a'} Q(s', a') \right)$
  - Set $s = s'$

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$\alpha$ is called the learning rate

# Q-Learning

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- How should we pick an action to take based on $Q$?
  - Shouldn't always be greedy (we won't explore much of the state space this way)
  - Shouldn't always be random (will take a long time to generate a good $Q$)
- $\epsilon$-greedy strategy: with some small probability choose a random action, otherwise select the greedy action

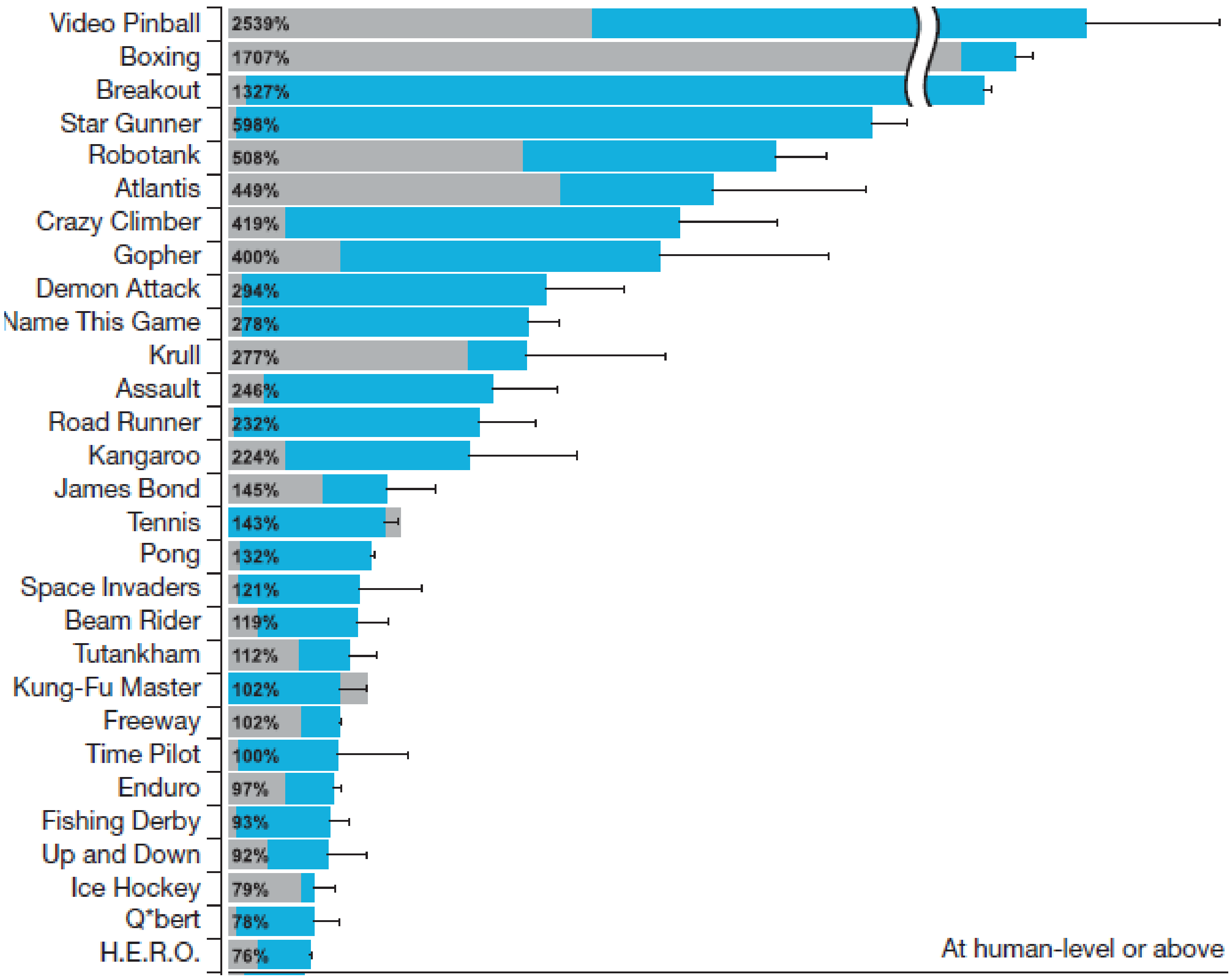
# Reinforcement Learning

- If the state space is large, these techniques are intractable (what if it is continuous?)
  - Need different algorithms for this setting, but we already know a few!
  - If the goal is to learn $Q(s, a)$, we could use techniques from supervised learning
    - Generate a collection of noisy observations using Q-learning
    - Use a supervised learning algorithm (e.g., a neural network, $k - NN$, etc.) to approximate the $Q$ function

# “Deep” Q-Learning

- If the $Q$ function is approximated by a neural network, the correctness guarantees for $Q$-learning no longer apply
  - Learning might converge poorly or not at all
- In practice, experience replay has been shown to result in better learning performance
  - The idea is that every time a state action pair is explored by the $Q$-learner, that pair is added to a replay set with its corresponding reward and transition
  - At each iteration, the replay set is sampled and the samples are used to update the weights of the neural network

# Deep Q-Learning Performance



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