Making the Grade: Product Quality Reporting by Infomediaries

Abstract:
Infomediaries often evaluate products using grades such as thumbs-up or thumbs-down. Their reporting delivers usable information to potential consumers for deciding whether to adopt the product or not. Our interest is to understand how this type of grading should be done from the infomediary’s perspective. Thus, this paper considers the design of rating scales by infomediaries, such as movie critics, used to inform consumers about the product quality of experience goods, such as movies. We use a general modeling setup for the utilities, objectives, and quality distributions. We provide analysis for products that are heterogeneous not just in quality but also in type, and account for reviewers who might be biased. Our method produces recommendations for quality cutoffs, which determine whether the product receives a high or a low grade, in a variety of settings.

Keywords: Information asymmetry; Infomediary; Reporting; Movies; Grading.
1. Introduction

Experience goods are defined as goods where the quality is uncertain until the consumer experiences it (Nelson 1974). Consumer uncertainty about quality is a fundamental problem for sellers (Akerlof 1970). Aspects of marketing such as branding, signaling, reputation, warrantees etc., all have some connection to the need to reduce quality uncertainty. Additionally, there exists a role for intermediaries who deliver information about the product and therefore are called infomediaries. For example, with movies, cars, bonds, restaurants, software etc. an important informational role is played by reviewers or critics who preview the product and grade it for consumers and investors (Chen, Liu and Zhang 2011). Compared to merchant intermediaries like retailers and wholesalers, the decisions of infomediaries are relatively under-examined despite their importance for product adoption. This paper contributes, in particular, to examining the grading function of infomediaries such as movie critics and reviewers, and how they grade experience goods.

In practice, professional movie reviewers provide coarse evaluations for movies they rate, such as awarding stars on a 5-star scale. A discussion of the rating system for movies is given by Bialik (2009). The thumbs up/thumbs down format used on Siskel and Ebert’s show became quite popular. Other reviewers such as the New York Times prefer to not assign stars to movies. The Bialik article notes that for movies, a summary grade is useful because it provides a spoiler-free rating of the film’s worthiness. To quote, “I prefer that critics use some sort of scale, personally, because I don’t want to know much about a movie before seeing it,” said a movie goer. But even without spoiler arguments, having a summary evaluation versus a detailed description can be convenient.
Discrete type of product quality grading occurs in many evaluative settings (e.g., Dubey and Geanakoplos 2010). For example, credit ratings, and restaurant and hotel ratings follow the same pattern. The related fields of reviewing, grading and certification have common practices whereby third parties provide an assurance of quality for buyers. Jamal and Sunder (2011) examine eBay sales data of graded and ungraded baseball cards. The cards are rated on their condition (mint, good, fair, etc.) out of 10 points, with half points possible by intermediaries such as PSA and Beckett, and returned to the owner in a transparent sealed container. Certifications include the Woolmark quality seal for pure wool products, American National Standards Institute (ANSI) certification for many types of industry products; USDA grades for Beef, ISO 9000 for minimum process quality standards etc.

In this paper we provide an analytical study about how reviewers\(^1\) should convert their detailed information into discrete grades for conveying information to consumers. For example, should they assign high ratings only to a few of the top quality items or to most of the items with acceptable quality? We have in mind reviewers whose goal is to maximize their payoff (which can be based on the welfare of consumers interested in purchasing the reviewed product) but whose reviews are constrained to a thumbs up/thumbs down, or a limited number of grades, type of evaluation. To be consistent with institutional details, we use the movie industry as the running example.\(^2\)

In our model, consumers have heterogeneous quality preferences.\(^3\) Consumers observe the reviewer’s rating, update their beliefs, and decide about purchasing the product. For the reviewer, the following tension is important in determining the optimal strategy. Giving a report

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\(^{1}\) Critic, reviewer, and infomediary are used interchangeably.

\(^{2}\) In of itself the movie industry is worth study, e.g., it has economic and cultural significance, and a continuous stream of new products. Eliashberg et al. (2006) and Kumb et al. (2017) review research based on this industry.

\(^{3}\) In addition to heterogeneity about quality, we also consider taste heterogeneity.
of high quality or ‘thumbs up’ only to a few highest-quality products will ensure that most buyers will be happy with their purchase. However, many consumers may have enjoyed some of the products that were reported as low quality due to the high cutoff. On the other hand, giving a high report to many products of medium or high quality will invariably result in more consumers being disappointed by their purchase. Finally, the extremes of giving a high or low rating to every product makes the reporting uninformative.

Our analysis offers several interesting implications. For example, we show that consumer preference for high quality can mean either more lenient or more selective cutoff depending on taste heterogeneity. Having to convey quality and type information (such as mood or acting) increases the communication burden that the grades must carry. In some such cases, the infomediary should forgo using the grading scale to convey quality information and instead use it to convey type information. Cutoffs are consistent with the rating reports of a number of movie critics. The trend and volatility of their ratings is explained by augmenting the model to include investments in product quality. Finally, we examine what happens when the infomediary’s personal taste affects the ability to judge the quality of the product.

Our analysis is distinct from information asymmetry problems caused by misaligned incentives between the buyer and seller. Examples of the latter are the “lemons” car market where market inefficiencies can occur (Akerlof 1970), or “cheap talk” games where incentive misalignment results in, at best, coarse-grained information being conveyed (Crawford and Sobel 1982). Unlike these cases, there is a third party infomediary who has no misaligned incentives on information sharing and consumers who accept that the conveyed information is objective. Nevertheless, the institutional constraint of communicating through a grading scale prevents full information revelation, making the optimization of the communication function worthwhile.
The paper is organized as follows. Section 2 offers a review of the relevant literature; Section 3 sets up the base model, with its analysis presented in Section 4. Sections 5 and 6 extend the analysis to allow for multiple cutoffs and two-dimensional product quality, respectively. Section 5 also discusses the rationale for the existence of coarse grading schemes when reviewers have access to a more refined quality information. Section 7 concludes. Proofs are in the Appendix.

2. Literature

The role of infomediaries is studied in several contexts. There are empirical studies on the role of critics on product sales, and several relate to the movie industry. Eliashberg and Shugan (1997) report a positive correlation between critics reviews and post-release box office revenues but do not have a significant correlation with the release week revenues. From this, they infer that critics act more as leading indicators than as opinion leaders. Basuroy, Chatterjee and Ravid (2003) revisit the issue and argue that critics’ reviews play a dual role, both influencing and predicting movie revenue. In our paper, the critic does influence consumers expected utility through their report about the movie quality. Huang, Ratchford and Strijnev (2015) find that positive critic reviews do increase movie sales especially for minor studios. This is consistent with the role of critics in removing information asymmetry. Elberse and Eliashberg (2003) find that critic ratings have a positive impact on revenues, but a negative impact on screen allocation. Basuroy, Desai and Talukdar (2006) look at the critics’ review consensus, as opposed to the mean level, and find that a positive consensus among reviewers improves revenues and reduces the importance of advertising. Empirical work on recommendation systems wherein consumer ratings (e.g., online book or hotel ratings, Netflix ratings etc.) are aggregated into
making targeted recommendations for potential buyers (e.g., Ying, Feinberg and Wedel 2006). But these papers do not provide a quantitative approach to the design of grading categories and only looks at the use of the information once it is collected.

Beyond the movie category, some papers have looked at a different and more active role of infomediaries in collecting and disseminating information, for example, where the third party measures the information directly and sells this information to buyers (e.g., Sarvary 2002; Sarvary and Parker 1997). Our paper does not consider such report verification since the critic makes information freely available. Thus, our model applies to movie critics, restaurant and hotel guides and consumer reports on many household items.

As to the number of grade categories, Harbaugh and Rasmusen (2018) claim fewer grade categories increase participation and thus can be more informative than more. In contrast, Dubey and Geanakoplos (2010) find that more than two grades motivate individuals and high grades should be much more difficult to achieve. Marketing research looks at scale design, such as assessing the number of response categories of a scale (e.g., Cox 1980) focuses on validity, discriminating power and misresponse. While the notion of quality cutoffs also appears in this paper, we differ from the above papers in that we consider multiple grades and let recipients of the reviewer’s information act strategically on it, as consumers do with critic information.

3. Model

We consider a market for an experience good category, using the movie industry as example. The quality of a given movie is uncertain \textit{a priori}, but the distribution of quality is common knowledge. As is usual, the movie is viewed in advance by the reviewer and the reviewer’s criteria for grading the movie, once settled on, also become known. The reviewer
does not convey full information about quality to consumers and instead issues a report such as high (“thumbs up”) or low (“thumbs down”). Given this rating scheme, the decision for the reviewer is to establish the quality cutoffs for the grading scale. A naïve reviewer might establish the cutoff at the median of the quality distribution, so that half of all movies meet the cutoff and half don’t. We find a median cutoff to be optimal only under some distributional assumptions. In general, a different cutoff will increase the reviewer’s objectives.

Instead of focusing on product quality, we might rephrase the issue in terms of customer preference, which has the benefit of clarifying that consumers do not necessarily agree with the critic or with one another regarding how much they enjoy a given movie. In other words, the critic might recommend a movie that a consumer dislikes, or fail to recommend a movie that the consumer would have liked. A consumer’s taste may even vary between movies, so that a consumer who disliked a movie rated high by the reviewer, may like the next movie so rated.

Heterogeneity in movie quality and in consumer preferences affects the expected utility for the reviewer and consumers. For the reviewer, each movie is a draw from the distribution of movie qualities. The distribution of consumer taste also matters to the reviewer because of the need to estimate how many consumers will be satisfied or dissatisfied with its review and thereby the payoff. The use of an underlying quality variable does mean that everyone agrees on the ordering of product quality. However, once we introduce in a later section the possibility of movies being described on multiple attributes, it will not be the case that consumers and reviewers agree on movie quality ordering either.

A complete description about the model and notation follows next.

**Assumptions:**

(a1). Timeline: The reviewer first selects the cutoff for assigning grades for all movies. For a given movie, the reviewer views and rates the movie. Consumers obtain this rating and
decide whether to see the movie. If they see the movie their utility is based on how they evaluate it. Based on consumer satisfaction, the reviewer receives a payoff.

(a2). About the Product: Movies are characterized by a single attribute which measures quality (or entertainment value) and denote it by $q$. Movies quality is a random variable with a distribution $F(q)$ and density $f(q)$, and support on $[0,1]$. In a later section of the paper, we allow that movies can have more than one attribute.

(a3). About the Consumers: There is a continuum of risk-neutral consumers with unit mass. A consumer with taste $t$ obtains utility $u(q, t)$ from viewing movie of quality $q$. Consumers are heterogeneous in their taste. Let $t$ be a random variable with distribution $G(t)$ and density $g(t)$, and support on $[0,1]$. We assume that their utility is increasing in the quality of the movie and decreasing in the taste parameter. The latter means that low taste individuals can enjoy a movie that a higher taste individual might not enjoy. Examples are a quasi-linear form, $u(q, t) = v(q) - t$, or a step-utility $u(q, t) = U$ if $q \geq t$ or $-D$ otherwise, where $U$ or $D$ are positive. That is, the movie is enjoyed or not. Utility is treated as net of price and called surplus because in the movie industry, ticket prices are uninformative. Finally, if the movie is not viewed, the utility is zero.

(a4). About the Reviewer: The reviewer sets a cutoff, $X \in [0,1]$, and for a movie of quality $q$, makes the report,

$$R(q, X) = \begin{cases} 
    \text{high}, & q \geq X, \\
    \text{low}, & q < X.
\end{cases} \quad (1)$$

Later we allow the possibility of more than one cutoff, which will increase the number of grades that the reviewer can report in an analogous manner.

(a5). About the Objectives: The objective of consumers is to maximize their expected utility with their choice of viewing the movie or not. For the reviewer, the objective is to maximize its expected payoff with the choice of $X$. We assume that the reviewer receives payoff or utility of $E_t[\rho u(q, t)|q, R]$ for a movie of quality $q$ after making a report $R$, where $E_t[\cdot]$ is the expectation operator over taste. This payoff is compatible with incentives such as word of mouth, traffic to the reviewer’s site, advertising revenue or social welfare. Depending on these $\rho$ can be set, so we let $\rho = 1$ as it is a scaling.
Before proceeding, we establish the full information benchmark. In this case, consumers know the movie quality exactly and do not need to compute expected utility. Suppose the quality is $q$. Let $\tau(q)$ be the taste of the consumer who is indifferent between viewing or not viewing the movie, i.e., it solves\(^4\),

$$u(q, \tau) = 0 .$$

Then $G(\tau(q))$ consumers with taste less than $\tau(q)$ will view it, generating a payoff of

$$\int_{0}^{G(\tau(q))} u(q, t) dG(t)$$

to the reviewer. The expected payoff over all movies, therefore, is

$$\int_{0}^{1} \int_{0}^{G(\tau(q))} u(q, t) dG(t) dF(q) .$$

This expression can be evaluated once the functions are specified.

4. Analysis

Let $X$ be the quality cutoff set by the reviewer. Then if the report is $R = high$, which is a probability $\int_{X}^{1} f(q) dq$ event, all consumers of taste less than $X$ view the movie, but consumers of higher taste may or may not. Let $\tau_h \in [X, 1]$ be the location of the indifferent consumer, obtained from,

$$E[u(q, \tau_h) | R = high] = 0 .$$

That is, for the indifferent consumer, the expected utility from viewing the movie equals the utility from not viewing it.

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\(^4\) The indifferent consumer may be at the corner. Thus, implicit is $\tau = 1$ if $u(q, 1) > 0$ and $\tau = 0$ if $u(q, 0) < 0$. 

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Figure 1 shows the location of the indifferent consumer. The indifference condition and other results are collected into the following Lemma:

**Lemma 1A**: If the reviewer reports \( R = \text{high} \), then all consumers with taste \( t \in [0, \tau_h] \) view the movie, where \( \tau_h \in [X, 1] \). Thus, the total number of consumers who view the movie conditional on \( R = \text{high} \) is \( \int_{0}^{\tau_h} g(t)dt \).

(a) Where, \( \tau_h \) is obtained from \( E[u(q, \tau_h) | R = \text{high}] = 0 \).

(b) As \( R = \text{high} \) means movie quality is at or above \( X \), a consumer of taste \( t \) expects utility

\[
\frac{\int_{X}^{t} u(q,t) f(q) dq}{\int_{X}^{1} f(q) dq}.
\]

Thus, the reviewer’s payoff when \( R = \text{high} \) is

\[
\int_{0}^{\tau_h} \frac{\int_{X}^{t} u(q,t) f(q) dq}{\int_{X}^{1} f(q) dq} g(t) dt.
\]

Suppose the reviewer always reports high quality, which is the same as setting the cutoff at \( X = 0 \) and making the report uninformative. Then, from part (a) of the Lemma, the consumer who is indifferent between viewing or not viewing the movies is located at \( \tau_h = u^{-1}(0) \). Part (b) of the Lemma then provides the no-information benchmark: Setting \( X = 0 \) into the expression, the denominator is 1, so the expression becomes

\[
\int_{0}^{\tau_h} \left( \int_{0}^{1} u(q,t) f(q) dq \right) g(t) dt.
\]

This is less than the total expected payoff for the full information benchmark, as some algebraic manipulations will
show. Thus grading scale design should try to improve upon the no-information benchmark and towards the full-information benchmark.

We next consider the case where the report is \( R = \text{low} \), which is a probability \( \int_0^X f(q) dq \) event. In this case, all consumers with taste higher than \( X \) will not view the movie, but some consumers of lower taste may. Let \( \tau_i \in [0, X] \) denote the location of the indifferent consumer. Then,

\[
E[u(q, \tau_i) | R = \text{low}] = 0. \tag{4}
\]

Figure 2 shows the location of the indifferent consumer. The indifference condition and other results are in Lemma 1B.

![Figure 2: Location of Indifferent Consumer when Report = low](image)

**Lemma 1B:** If the reviewer reports \( R = \text{low} \), then all consumers with taste \( t \in [0, \tau_i] \) view the movie, where \( \tau_i \in [0, X] \). Thus, the total number of consumers who view the movie conditional on \( R = \text{low} \) is \( \int_0^{\tau_i} g(t)dt \).

(a) Where, \( \tau_i \) is obtained from \( E[u(q, \tau_i) | R = \text{low}] = 0 \).

(b) As \( R = \text{low} \) means movie quality is below \( X \), a consumer of taste \( t \) expects utility

\[
\frac{\int_0^X u(q, t) f(q) dq}{\int_0^X f(q) dq}.
\]

Thus, the reviewer’s payoff when \( R = \text{low} \) is \( \int_0^{\tau_i} \frac{\int_0^X u(q, t) f(q) dq}{\int_0^X f(q) dq} g(t)dt \).
It may be verified that if the reviewer always reports movies as having low quality, then the indifferent consumer is at the same location as when the reviewer always reports high quality because the report is equally uninformative.

We now consider the reviewer’s decision problem, which is to maximize the ex-ante expected payoff by making a choice on the cutoff. Making use of the results derived above, the problem can be written as,

\[
\max_X \int_X^1 f(q) dq \times \int_0^{\tau_h} \int_X^1 u(q, t) f(q) dq g(t) dt + \int_0^{\tau_i} \int_X^1 u(q, t) f(q) dq g(t) dt,
\]

where the first term is the total expected surplus when the report is high, multiplied by the probability of high, and the second term is the total expected surplus when the report is low, multiplied by the probability it is low. This simplifies to,

\[
\max_X \int_0^{\tau_h} \left( \int_X^1 u(q, t) f(q) dq \right) g(t) dt + \int_0^{\tau_i} \left( \int_0^X u(q, t) f(q) dq \right) g(t) dt.
\]  

(5)

Where \( \tau_h \) and \( \tau_i \) are also functions of \( X \). The solution of this problem is given in Theorem 1.

**Theorem 1:** When deciding the cutoff quality \( X \) that determines whether the report is High or Low, there exists a cutoff that maximizes the reviewer’s payoff and is given by:

\[
\int_{\tau_i}^{\tau_h} u(X, t) g(t) dt = 0,
\]

where \( \tau_h \) and \( \tau_i \) are given by Lemma 1A and Lemma 1B, respectively.

Thus, the grading scale cutoff depends on the quality and taste distributions and on consumer's relative utility of enjoying high quality products and disutility of consuming low quality products. To summarize, the analysis shows that the issue of grading is tied in with
product quality and consumer preferences, or more generally to the issue of conveying discriminating information.\textsuperscript{5}

Theorem 1 gives an implicit relationship solvable for the cutoff and thereby completes the characterization of the reviewer’s decision. While it is generally applicable, comparative statics are not available at this level of generality. Thus, we specify the function for further insights to be obtained.

**Corollary 1.1:** Let the distributions $F(\cdot)$ and $G(\cdot)$ be uniform and assume a linear utility function $u(q,t) = q^\alpha - t$ where $\alpha$ is a positive constant. Then the condition for the optimal cutoff is obtained from $X^{\alpha+1} - (1+2\alpha)X^\alpha + 1 = 0$.

Table 1 shows the optimal cutoffs from Corollary 1.1 for some values of the utility parameter $\alpha$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>0.28</td>
<td>0.35</td>
<td>0.41</td>
<td>0.46</td>
<td>0.5</td>
<td>0.54</td>
<td>0.57</td>
<td>0.59</td>
<td>0.62</td>
<td>0.64</td>
</tr>
</tbody>
</table>

The notable trend from the table is that when $\alpha$ is higher, i.e., when customers have a higher utility from quality of the movie, the cutoff should also be higher. Thus, the critic should be more stringent in giving movies a high grade. The intuition that the cutoff should be at the midpoint is generally belied since this occurs only at a single instance.

\textsuperscript{5} This is information that can be provided on top of description such as of the cast, direction, plot and other movie elements. While conveying information is the goal, a reading of the popular press does not suggest that reviewers go beyond subjective opinion in providing ratings by using a modeling style approach. Nonetheless, subjective judgment by an expert would probably be based on an internal model of how a movie compares to other movies or what type of information may be more useful to consumers. There is an advantage in trying to articulate this explicitly. Indeed, subjectivity was given as the reason by the New York Times for not provide star ratings and instead preferring to provide a longer description for movies (Bialik, 2009).
For the next corollary we use the step utility specification, \( u(q, t) = U \) if \( q \geq t \) or \(-D\) otherwise, where \( U \) or \( D \) are positive, mentioned previously. Then the condition in Theorem 1,

\[
\int_{\tau_i}^{\tau_b} u(X, t) g(t) dt = 0,
\]
yields,

\[
G(X) = \frac{D}{U + D} G(\tau_b) + \frac{U}{U + D} G(\tau_i).
\]

Lemmas 1A and 1B yield \( F(\tau_h) = \frac{U + DF(X)}{U + D} \) and \( F(\tau_i) = \frac{UF(X)}{U + D} \), determining \( \tau_h \) and \( \tau_i \) respectively.

Consider the family of distributions for \( F(\cdot) \) and \( G(\cdot) \) where one distribution is a power function of the other. This encompasses, for example, common choices where distributions are all standard normal or all uniform on the unit interval.

**Corollary 1.2:** Let the distributions \( F(\cdot) \) and \( G(\cdot) \) be related in the following manner: \( G = F^\gamma \) where \( \gamma > 0 \).

(a) Then the condition for the optimal cutoff is

\[
F(X) = \frac{U / D}{\left((1 + U / D)^{\gamma+1} - (U / D)^{\gamma+1}\right)^{\gamma/\gamma-1}}.
\]

(b) If \( F(\cdot) \) and \( G(\cdot) \) are identical, so that \( \gamma = 1 \), this yields \( X = F^{-1}(0.5) \).

If \( \gamma = 1 \), i.e., \( F(\cdot) \) and \( G(\cdot) \) are identical, then \( X = F^{-1}(0.5) \). Then the reviewer should locate the cutoff at the median of movie qualities, rating half the movies as high quality and half as low quality. An interesting feature of this solution is that the cutoff is always at the median, independent of the utility \( U \) or disutility \( D \) values, and irrespective of the distribution.

When the distributions are not identical, we can explore the result by specifying a distribution to see how the parameters affect the cutoff. The utility ratio, \( U / D \), can be treated as
a single parameter while $\gamma$ is a relative skewness parameter. Since Corollary 1.2 provides an explicit solution, the results are readily plotted as shown in Figure 3 using the uniform distribution for $F(\cdot)$ and a rightwards or leftwards skewed distribution for $G(\cdot)$.

\textit{Fig. 3: Cutoff as a Function of the Utility Ratio and Distribution Parameter}

Figure 3 shows that the cutoff increases as $\gamma$ increases. Thus, if the distribution of movies was uniform, an increase in $\gamma$ means that consumer tastes are skewed right, i.e., they are more refined in their tastes. Then the cutoff moves higher. Intuitively, the reviewer would want to give fewer movies a high rating if consumers enjoy higher quality movies in order to avoid disappointing them.

Figure 3 is less intuitive regarding the ratio of utilities, because the cutoff can either increase or decrease with $U / D$. When the utility ratio is higher than 1, however, the optimal cutoff does not change much. To process this analytically, if we write $U / D = \phi$ then a few steps of derivation yields
\[
\lim_{\gamma \to \infty} \frac{\phi}{\left( (\phi + 1)^{\gamma+1} - \phi^{\gamma+1} \right)^{1/\gamma} - 1} = \frac{1}{(1 + \gamma)^{1/\gamma}}
\]
as the asymptote, and from inspecting the figure, it converges quickly. It is known that
\[
\lim_{\gamma \to 0} (1 + \gamma)^{1/\gamma} = e,
\]
the Euler constant, and so the lowest asymptote is \(1/e\), which is about 0.37. So, assuming that the utility ratio is high enough, the reviewer should maintain a lower bound of \(X = F^{-1}(1/e)\), meaning that no less than 37\% of the movies should be rated as low.

Interestingly, we found that well known movie critic Roger Ebert, using a 1 to 4 stars rating scale, consistently assigned about 39\% of movies a rating of 2.5 stars or less. For example, in 1998 and 2002, 38\% of movies reviewed by Ebert received 2.5 stars or less, in 1999 and 2000, the proportion was 41\%, in 2001 it was 36\%. This is near the boundary of where his ratings can be reconciled with the above discussion and it would suggest that if movie quality is drawn from a uniform distribution then consumer tastes are drawn from a highly leftward skewed distribution, i.e., that most consumers enjoy even low quality movies. But one could also interpret a low value of \(\gamma\) as implying that if customers tastes are uniformly distributed, then the quality of movies is highly rightward skewed, i.e., that most movies are high quality. This is perhaps the more natural interpretation that consumers might use when they see that Ebert rates well over half the movies above the midpoint of his scale.\(^6\)

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\(^6\) The distribution of quality should resemble the distribution of movie sales. For the latter, it is known that there are a few blockbuster movies and a large number of average sales movies, and then a number of weak movies. A concave curve or an s-shaped curve would therefore apply to cumulative sales – the distribution \(F(\cdot)\). Tastes on the other hand are difficult to give an objective measure for. In the absence of other information, one might use the uniform distribution since it means that every taste is equally likely. On the other hand, if taste follows some other individual level characteristics, it might have a normal distribution. In the analysis in the Corollary where we used a distribution which is the power function of the other, note that if we take taste to be uniformly distributed, then raising it to a power less than 1 will provide an \(F(\cdot)\) distribution that is concave.
A similar outcome was found on data of some other critics as well. In Figure 4 the entries represent the percentage of reviews less than or equal to 3 out of 5 stars, where are scales are normalized to 5 stars.

**5. Multiple Cutoffs**

We next consider a reviewer capable of issuing finer grades. Whereas these may require a bit more effort compared to a simple high or low assessment, they should convey greater information about the product quality to the consumer. For example, Jin and Leslie (2003) studied a grading system for restaurant hygiene that went from pass/fail to a letter grade system, and found that sales increased for restaurants with A versus C or lower grade, and also that hospital visits for food-related illnesses decreased.

Assume that the report can take on any integer value between 0 and $n$, with a higher value corresponding to higher product quality. Then, the task of the reviewer is to find $n$ cutoff points $X_1, \ldots, X_n$ so that, if the product quality falls in the interval $(X_i, X_{i+1})$, the report is $R = i$, $i = 0, \ldots, n$, where $X_0 = 0$ and $X_{n+1} = 1$. 

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*Fig. 4: Percentage of movies receiving less than 3/5 stars by Year*
The analysis is similar to that for the previous model with a single cutoff. Suppose that the report is $R = i$. Then, all consumers of taste less than $X_i$ will purchase the product, but all consumers of higher taste may not. Let $\tau_i \in (X_i, X_{i+1})$ denote the location of the indifferent consumer. Then, it satisfies $\int_{X_i}^{X_{i+1}} u(q, \tau_i) f(q) dq = 0$. From this, we get Lemma 2.

**Lemma 2**: If the report is $R = i$, then all consumers with taste less than $\tau_i$ will view the movie, where $\tau_i \in (X_i, X_{i+1})$ and where $X_0 = 0$, $X_{n+1} = 1$ and $X_1, \ldots, X_n$ are the $n$ cutoff points.

(a) Where, $\tau_i$ is obtained from $\int_{X_i}^{X_{i+1}} u(q, \tau_i) f(q) dq = 0$ for $i = 1, \ldots, n$.

(b) The payoff conditional on the reviewer’s report of $R = i$ is given by:

$$
\int_0^{\tau_i} \frac{\int_{X_i}^{X_{i+1}} u(q, t) f(q) dq}{\int_{X_i}^{X_{i+1}} f(q) dq} g(t) dt.
$$

We now consider the reviewer’s decision problem, which is to maximize the *ex-ante* expected payoff by making a choice on the cutoffs. The problem can be written as,
The result of the maximization is provided in Theorem 2.

**Theorem 2:** When deciding on multiple cutoffs \( X_i \) for \( i = 1, \ldots, n \), such that report \( R = i \) is issued if observed quality lies in \( (X_i, X_{i+1}) \), there exists a solution that maximizes expected payoff and it is given by the system of equations: 

\[
\int_{\tau_{i-1}}^{\tau_i} u(X_i, t) g(t) dt = 0 \quad \text{for} \quad i = 1, \ldots, n \quad \text{and where} \quad \tau_i \quad \text{for} \quad i = 1, \ldots, n \quad \text{is given by Lemma 2.}
\]

This generalizes the result from a single cutoff in Theorem 1. In an analogous result, if we consider the step utility function, for any distributions where \( F(\cdot) = G(\cdot) \), the theorem yields

\[
X_i = F^{-1} \left( \frac{F(X_{i+1}) + F(X_{i-1})}{2} \right) \quad \text{for} \quad i = 1, \ldots, n, \quad \text{which is independent of the utility parameters} \quad U \quad \text{and} \quad D \quad \text{as before. Full results of this analysis are shown in Corollary 2.}
\]

**Corollary 2:** For the step utility function and if the distributions for taste heterogeneity and movie quality are identical, i.e., \( F(\cdot) = G(\cdot) \), then:

(a) The optimal cutoffs satisfy \( X_i = F^{-1} \left( \frac{i}{n+1} \right) \). Thus, when \( F \) is Uniform on \([0,1]\), the solution is \( X_i = \frac{i}{n+1} \) for \( i = 1, \ldots, n \).

(b) The optimal expected payoff is

\[
\frac{nU}{2(n+1)} + \frac{nU^2}{2(n+1)^2(U + D)}.
\]
According to Corollary 2(a), for the specified conditions, the reviewer should locate the cutoffs at the quantiles of the distribution, so that equal number of movies gets each possible grade.

From the second part of the Corollary, it is clear that the reviewer can achieve the full-information benchmark value by increasing the number of grade categories, i.e., \( n \to \infty \). An obvious question then would be why such fine-grained reporting by reviewers is not observed in practice. Most reviewers use a handful of stars, anywhere from two to five, for grading. If half-stars are allowed, this gives up to ten grades. A finer scale is used by the review aggregation site metacritic.com, which reports on a scale of 1 to 100 using integers only. While metacritic could report fractions, it doesn’t. Indeed one could argue whether consumers would be helped by seeing a rating of 75.39/100 instead of 75/100, or 75/100 instead of 4/5. Clearly there is a convenience factor for consumers in processing coarser information, and likewise for the reviewer in providing coarse grades due to bounded rationality. We can quantify this explanation for fewer grade categories by showing that the benefit from using a finer rating scale has decreasing returns.

In Figure 6, we plot the optimized expected total consumer surplus given in the Corollary against number of grades. For this plot, we take values of \( U = (5, 10, 15) \) and \( D = 5 \). (Other values yield the same pattern.) The resulting function rapidly approaches the full-information benchmark, given by total consumer surplus of \( U / 2 = (2.5, 5, 7.5) \). Even if a small incremental cognitive cost is assumed for the addition of each grade category, a limited number of grades would be close to optimal.
It is commonly observed that the number of grades used by a reviewer does not change from movie to movie. That is, it is decided by the reviewer at the beginning of the review process, perhaps using the cost-benefit analysis just described. Therefore, finite number of grades may be considered an exogenous institutional constraint in each subgame where a movie is rated.

6. Two Product Attributes

The situation that we want to study is when the product has more than one attribute, but the reviewer can only make a summary report using a single item grade scale. We consider the case where, in addition to the vertical differentiation on quality, products can also be horizontally differentiated as belonging to a type. A simple example of type would be subtle characterizations about the mood, acting or direction of the movie.
Specifically, let us suppose that there are two types of the product, denoted $A$ and $B$, and consumers are heterogeneous in their preferences towards them. Consumers cannot observe either quality or type ex ante. The reviewer, on the other hand, perfectly observes both, but issues a summary report such as $R = \text{high}$ or $R = \text{low}$. The problem with reporting about a multi-attribute product with a single item scale is that consumers have to untangle how much information the report conveys about the quality and about the type.

To illustrate, suppose that a movie has attributes (type=$A$, quality=0.9). If the reviewer knows that most people like type $A$, then the reviewer could report $R = \text{high}$. But if more people like type $B$, then the reviewer might report $R = \text{low}$ despite the high quality of the movie. At this point, it is worthwhile to note that since the labels on the grades, $\text{high}$ and $\text{low}$, do not necessarily comport with the actual quality, it is better to give them labels such as $\text{thumbs up}$ and $\text{thumbs down}$. We refer to this issue again at the end.

The analysis in this section is made tractable with the help of the following additional assumptions:

(a6). We assume two types of movies, $A$ and $B$, where type $A$ occurs with probability $\mu$ and type $B$ with probability $1 - \mu$. For each type, the movie quality is a draw from IID distribution $f(\cdot)$. Each consumer cares only about one type of product, with proportion $\lambda$ consumers preferring type $A$ product and $1 - \lambda$ consumers preferring type $B$. We refer to these consumers as type $A$ and type $B$ consumers, respectively. Both customer types have IID taste heterogeneity distribution $g(\cdot)$.

We denote the cutoff quality values as $X_A$ and $X_B$ that will be applied when the reviewer observes type $A$ and type $B$, respectively. This means that if the reviewer observes, for example, a type $A$ movie of quality at or above $X_A$, then it will report $R = \text{high}$. After
observing the report, each consumer decides whether to purchase the product or not. The analysis is similar for both types, so we focus on type A consumers.

Suppose first that the report is \( R = \text{high} \). The customer gets utility \( u_A(A, q, t) \) for movies of type A and quality \( q \). The threshold preference \( \tau_{Ah} \) describes the indifferent type A consumer as shown in Figure 7.

\[
\text{Fig. 7: Location of Indifferent Consumer when Report = high}
\]

Thus, the location \( \tau_{Ah} \) is determined by the condition

\[
\mu \int_{X_A}^{1} u_A(A, q, \tau_{Ah}) f(q) dq + (1 - \mu) \int_{X_B}^{1} u_A(B, q, \tau_{Ah}) f(q) dq = 0 \tag{7}
\]

From the symmetry of this expression, it is clear that \( \tau_{Bh} \) satisfies

\[
\mu \int_{X_A}^{1} u_B(A, q, \tau_{Bh}) f(q) dq + (1 - \mu) \int_{X_B}^{1} u_B(B, q, \tau_{Bh}) f(q) dq = 0 \tag{8}
\]

The reviewer’s expected payoff when \( R = \text{high} \) is thus,

\[
\frac{1}{\int_{X_A}^{1} f(q) dq + \int_{X_B}^{1} f(q) dq} \left[ \lambda \int_{0}^{\tau_{Ah}} \left( \mu \int_{X_A}^{1} u_A(A, q, t) f(q) dq + (1 - \mu) \int_{X_B}^{1} u_A(B, q, t) f(q) dq \right) g(t) dt \right] \times \left[ + (1 - \lambda) \int_{0}^{\tau_{Bh}} \left( \mu \int_{X_A}^{1} u_B(A, q, t) f(q) dq + (1 - \mu) \int_{X_B}^{1} u_B(B, q, t) f(q) dq \right) g(t) dt \right] \tag{9}
\]

When \( R = \text{low} \), the locations \( \tau_{Al} \) and \( \tau_{Bl} \) are determined by the following equations:
The reviewer’s expected payoff when $R_{low}$ is thus,

$$
\frac{1}{\int_0^{X_A} f(q) dq + \int_0^{X_B} f(q) dq} \times \left[ \lambda \int_0^{X_A} u_A(A,q,t) f(q) dq + (1-\lambda) \int_0^{X_B} u_B(B,q,t) f(q) dq \right] g(t) dt
$$

(12)

The reviewer's objective is to choose $X_A$ and $X_B$ to maximize the total expected payoff. The result is in Theorem 3.

**Theorem 3**: When the movie is described by both quality and type attributes then, under the model assumptions ($a_1$ – $a_6$):

(a) The locations of the indifferent consumers $\tau_{Ah}, \tau_{Bh}, \tau_{Al}$ and $\tau_{Bl}$ are given by equations (7) to (11).

(b) The reviewer’s optimal cutoffs $X_A$ and $X_B$ are determined from:

$$
\lambda \int_0^{\tau_{Ah}} u_A(A,X_A,t) g(t) dt + (1-\lambda) \int_0^{\tau_{Al}} u_A(A,X_A,t) g(t) dt = 0,
$$

$$
\lambda \int_0^{\tau_{Bh}} u_B(B,X_B,t) g(t) dt + (1-\lambda) \int_0^{\tau_{Bl}} u_B(B,X_B,t) g(t) dt = 0.
$$

Theorem 3 provides a general result for finding the optimal cutoffs when products differ along two dimensions. To take a closer look at the implications, as before, we make additional assumptions about the taste and quality distribution functions as well as the consumer utility functions. In particular, for the remainder of this section, we assume that

(a7). Both the quality distribution $f(q)$ and the taste distribution $g(t)$ are uniform on $[0,1]$. 

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Corollary 3: Theorem 3 for the step utility function generates the following results:

(a) When \( \lambda = \mu = 0.5 \), i.e., types A and B movies are equally likely and there are equal numbers of consumers of each type, the reviewer’s optimal strategy is \( R = \text{high} \) for type A movies and \( R = \text{low} \) for type B movies, irrespective of movie quality.

(b) When \( \mu = 0.5 \) and \( \lambda > 0.5 \), i.e., both types of movie are equally likely but type A consumers more prevalent, the reviewer’s optimal strategy is to give all type B movies a low rating \( (X_B = 1) \), and give more than half of type A movies a high rating \( (X_A = \frac{2\lambda - 1}{4\lambda - 1}) \).

Similarly, when \( \lambda < 0.5 \), i.e., type B consumers are more prevalent, the reviewer’s optimal strategy is to give all type A movies a low rating \( (X_A = 1) \), and give more than half of type B movies a high rating \( (X_B = \frac{1 - 2\lambda}{3 - 4\lambda}) \).

Observe that the reviewer’s optimal decision is to convey only the type information and leave out the quality information when both types are equally likely. This seems unintuitive, but it makes sense in the context of the constraints put on the reviewer’s ability to communicate. With this rating system, the reviewer provides the most valuable information to the consumers, which is the movie type. This information is more valuable than the information about quality within type because consumers dislike one of the types no matter what the movie quality is. It should be noted that the labels are arbitrary in the optimal solution for Part (a), i.e., we could also use \( R = \text{high} \) for type B movies and \( R = \text{low} \) for type A movies. In either case, the same information is conveyed. The customers would have to infer that the reviewer only likes one type of movie and dislikes the other, but presumably this would quickly become common knowledge.

The reviewer’s choice changes when one of the consumer types becomes more prevalent. Then, the reviewer chooses to convey some quality information as well for the movie type enjoyed by majority of the consumers. Note that, in this case, the review labels are less arbitrary.
as the low quality movies of the type enjoyed by majority of the consumers receive the same rating as all the movies of the other type. This result can be interpreted as suggesting that it is optimal for the reviewer to “pander to the majority” by giving low rating to all “alternative style” movies even if they are exceptionally good quality.

Another message from the result is quite simple, it is that the descriptive passages used for reviews, such as in the New York Times and WSJ, do have an important role to play that cannot completely be satisfied by a summary rating on a single item scale. Possibly a multi-item scale can be developed but a full theory of that seems to be intractable for the moment.

Finally, if type of the movie is known (for example, if the movie trailer is informative enough to convey all the necessary type information), then the analysis reverts back to the basic model or a variant of it because the critic is back to only having to convey quality information in the grade.

6.1. Reviewer’s Type Preference

Suppose now that the reviewer prefers type A and dislikes type B movies. If this preference does not affect the reviewer’s ability to evaluate movie quality of either type, then the reviewer should optimally ignore own preference and produce a report that only reflects consumers’ tastes in order to maximize advertising revenue, etc. This is what we have seen till now. It is possible to assume, however, that a reviewer who dislikes type B movies will also be worse at judging the quality of type B movies. If that is the case then the reviewer’s personal preference will affect the optimal reporting policy and may have implications for market segmentation (consumers who have different tastes from those of the reviewer may choose not to pay attention to his reviews) as illustrated next.
Suppose the reviewer dislikes type B movies and thus cannot observe the true quality of type B movies. But the reviewer observes the quality of type A movies precisely. Under this set of assumptions, the previous analysis must be modified. The indifference conditions giving the locations of the indifferent consumers for the four cases (i.e., two consumer types and two types of observed reports) are provided below.

\[ \mu \int_{X_A}^1 u_A(A, q, \tau_{Ah})dq + (1-X_B)(1-\mu) \int_0^1 u_A(B, q, \tau_{Ah})dq = 0 \]  
(13)

\[ \mu \int_{X_A}^1 u_B(A, q, \tau_{Bh})dq + (1-X_B)(1-\mu) \int_0^1 u_B(B, q, \tau_{Bh})dq = 0 \]  
(14)

\[ \mu \int_0^{X_A} u_A(A, q, \tau_{Al})dq + X_B(1-\mu) \int_0^1 u_A(B, q, \tau_{Al})dq = 0 \]  
(15)

\[ \mu \int_0^{X_A} u_B(A, q, \tau_{Bl})dq + X_B(1-\mu) \int_0^1 u_B(B, q, \tau_{Bl})dq = 0 \]  
(16)

Consider the first indifference condition which determines \( \tau_{Ah} \). It says that when a type A consumer sees a report of high, the consumer infers that if the movie is of type A, its cutoff must be above \( X_A \) but if the movie is of type B, its quality can be anything since the reviewer does not observe the quality. Then first term on the right hand side has \( \mu \), the probability that the movie is type A, multiplied by the expected utility if the movie is type A. In the second term, the multiplier \( (1-X_B)(1-\mu) \) is the probability that the movie is of type B because \( 1-\mu \) fraction of movies are type B to begin with and \( (1-X_B) \) of these receive a report of high. Here we allowed that the reviewer can place a cutoff on type B movies, receiving an uninformative signal or flipping a coin, to determine whether they fall above the cutoff or below. The last term is the utility if the movie turns out to be type B. The remaining three indifference conditions can be explained similarly.

The reviewer’s payoff is given by
The reviewer’s objective is to choose $X_A$ and $X_B$ to maximize total expected payoff. The result is in Theorem 4.

**Theorem 4:** When the reviewer cannot observe the quality of type B movies and can only observe the quality of type A movies, then the cutoffs chosen by the reviewer are determined from the indifference conditions (13-16) and the optimality conditions:

\[
\lambda \int_{t_M}^{t_A} u_A(A, X_A, t) dt + (1 - \lambda) \int_{t_M}^{t_B} u_B(A, X_A, t) dt = 0,
\]

\[
\lambda \int_{t_M}^{t_A} \left( \int_{0}^{1} u_A(B, q, t) dq \right) dt + (1 - \lambda) \int_{t_M}^{t_B} \left( \int_{0}^{1} u_B(B, q, t) dq \right) dt = 0.
\]

It is interesting to compare Theorem 4 to Theorem 3. The difference between the two cases is in the ability of the critic to evaluate the quality of type B movies. As Corollary 4 shows, this may have dramatic effects on the critic’s optimal choices.

**Corollary 4:** When $\mu = 0.5$ and $\lambda > 0.5$, i.e., both types of movie are equally likely but type A consumers are more prevalent, the reviewer’s optimal strategy is to give all type B movies a low rating ($X_B = 1$) and to give more than half of type A movies a high rating ($X_A = \frac{2\lambda - 1}{4\lambda - 1}$). However, when $\lambda \leq 0.5$, the reviewer’s optimal strategy is to give all type A movies a low rating ($X_A = 1$) and to give all type B movies a high rating ($X_B = 0$).
The result in Corollary 4 differs from that in Corollary 3 in that, when the prevalent consumer type is $B$, the reviewer’s report only reflects movie type when the reviewer cannot tell the quality of type $B$ movies, while it also conveys information about the quality of type $B$ movies when it is known to the reviewer. In either case, the report does not convey any information about the quality of type $A$ movies. Given that labeling is arbitrary, as we have noted before, Corollary 4 can be interpreted as suggesting that it is optimal for reviewers to give low rating to all movies of the type that they do not enjoy, even if these are blockbuster movies likely to be enjoyed by the majority of the consumers. This is a surprising result, but it might explain why critics sometimes rate blockbuster movies quite poorly. For example Ebert rated the 2011 movie Transformers: Dark of the Moon as one star ("…a visually ugly film with an incoherent plot, wooden characters and inane dialog. It provided me with one of the more unpleasant experiences I've had at the movies…") even though its worldwide gross proceeded to exceed a billion dollars.

7. Conclusions

The grading function is an important role that third party infomediaries perform for the market. This paper seeks to understand and give normative guidance to infomediaries making grading decisions. We consider the substantive example of a movie reviewer, who in the baseline case needs to determine a quality cutoff or threshold for determining which movies should receive a “thumbs up” or “thumbs down” grade. In doing so, the reviewer conveys information that is useful to consumers for making their own decisions about whether to view the movie or not. Beyond movies, other markets for experience goods have similar issues. Our interest is to see how grade-based reporting can be best employed from the reviewer’s perspective, where
maximization of a consumer utility based payoff function (which may include social welfare) is its objective.

The model allows that consumers can be heterogeneous in their preference for product quality, and that the product (e.g., a movie) has quality drawn from a random distribution. Both the heterogeneity and quality distributions are allowed to be quite flexible. Based on the tension that a high cutoff will lead to more satisfaction among adopters, but will discourage other potential consumers who may otherwise have been satisfied with the product, while a low cutoff will lead to more dissatisfaction among adopters, we are able to link the total expected payoff from the movie as a function of the infomediary’s cutoff decisions. We then derive Theorem 1, for the case of a single cutoff assigning high or low grade, which gives a concise and intuitive condition for optimality of the cutoff.

To generate deeper insights, we considered special cases in Corollary 1.1 and 1.2. The results are quite interesting; for example, if the distribution of customer types and movie quality are identical, then the cutoff will be at the median regardless of the distributions or parameters dealing with the extent to which consumers enjoy good movies. In other words, half the movies should be optimally rated ‘high’ and the other half rated ‘low’. This is a special case of a more general result when the distribution of customer preferences is a constant power of the movie quality distribution. Under this condition, an explicit formula for the cutoff can be obtained and the cutoffs are then not necessarily located at the median. The results in this case are best seen in graphical form in Figure 3 and provide useful qualitative insights for shifting the cutoff of the reviewer under different parametric conditions. Figure 4 shows that the model generates cutoffs consistent with the rating reports of a number of movie critics.
The remaining sections of the paper extend the results and show that more general results can be obtained for the cases of multiple cutoff points and for horizontal type heterogeneity in customer preferences. The results for multiple cutoffs (Theorem 2 and its Corollary) are analogous to those for single cutoff. For example, with identical distributions of preference heterogeneity and product quality, the cutoffs are located at the quantiles of the distribution, so that with two cutoffs for the product to be assigned as ‘high’, ‘medium’ and ‘low’ categories, one-third of the products will fall into each grade. Again, this is for a special case only, and the theorem provides the guidelines for other cases.

In the case of horizontal heterogeneity of movies, we found that the reviewer has to decide whether to communicate type or quality information with the report. In some cases the reviewer conveys through the reported grade only the type of the movie, irrespective of its quality, as shown by Theorem 3 and its corollary. This suggests that a richer means of communication than a summary grade like thumbs up or thumbs down would be quite useful for multi-attribute products. In a further extension we examine the effects of reviewer’s personal preference affecting the quality judgment. There has been some work on the issue of intermediary bias, such as the deliberate slanting of news content to please the target audience (e.g., Mullainathan and Shleifer 2005). The result, given by Theorem 4 and Corollary 4, shows that there is a bias towards giving higher ratings to the preferred type of product although consumers will take this into account in their adoption decisions.

Among limitations, we assumed that the reviewer’s criteria for grading the movie are common knowledge. This assumption means that it does not affect any results if the critic does not review every movie or that consumers do not view every movie, because there is no updating of the grading cutoffs over time. Future research may consider the possibility for relaxing this.
Appendix

Proof of Theorem 1: The objective is

\[
\max_{X} \int_{0}^{\tau_h(X)} \left( \int_{X}^{1} u(q, t) f(q) dq \right) g(t) dt + \int_{0}^{\tau_i(X)} \left( \int_{0}^{X} u(q, t) f(q) dq \right) g(t) dt.
\]

We obtain the first order condition for maximum using Leibniz rule for differentiation under the integral sign. It gives,

\[
\frac{\partial}{\partial X} \left\{ \int_{0}^{\tau_h(X)} \left( \int_{X}^{1} u(q, t) f(q) dq \right) g(t) dt + \int_{0}^{\tau_i(X)} \left( \int_{0}^{X} u(q, t) f(q) dq \right) g(t) dt \right\} = 0
\]

\[
\Rightarrow \frac{\partial}{\partial X} \left\{ \int_{X}^{1} u(q, \tau_h) f(q) dq \right\} - \int_{0}^{\tau_h} u(X, t) f(X) g(t) dt + \frac{\partial}{\partial X} \left( \int_{0}^{X} u(q, \tau_i) f(q) dq \right) + \int_{0}^{\tau_i} u(X, t) f(X) g(t) dt = 0
\]

Recalling from Lemma 1A and 1B that

\[
\int_{X}^{1} u(q, \tau_h) f(q) dq = \int_{0}^{\tau_i} u(q, \tau_i) f(q) dq = 0
\]

The condition simplifies to,

\[
f(X) \int_{0}^{\tau_h} u(X, t) g(t) dt - f(X) \int_{0}^{\tau_i} u(X, t) g(t) dt = 0
\]

\[
\Rightarrow \int_{\tau_h}^{\tau_i} u(X, t) g(t) dt = 0.
\]

Proof of Corollary 1.1: Lemma 1A gives \( \int_{X}^{1} (q^\alpha - \tau_h) dq = 0 \) from which \( \tau_h = \frac{1 - X^\alpha_{\alpha+1}}{(1 - X)(\alpha + 1)} \), and

Lemma 1B gives \( \int_{0}^{X} (q^\alpha - \tau_i) dq = 0 \) from which \( \tau_i = X^\alpha / (\alpha + 1) \). Inserting these into Theorem 1, i.e., \( \int_{\tau_h}^{\tau_i} (X^\alpha - t) dt = 0 \), yields the result.

Proof of Corollary 1.2: The condition in Theorem 1 is \( G(X) = \frac{D}{U + D} G(\tau_h) + \frac{U}{U + D} G(\tau_i) \). Into this, insert \( G(\cdot) = F(\cdot)^\gamma \) and Lemma 1A that \( F(\tau_h) = \frac{U + DF(X)}{U + D} \) and Lemma 1B that \( F(\tau_i) = \frac{UF(X)}{U + D} \):

\[
F(X)^\gamma = \frac{D}{U + D} F(\tau_h)^\gamma + \frac{U}{U + D} F(\tau_i)^\gamma
\]

\[
\Rightarrow F(X) ((U + D)^{\gamma + 1} - U^{\gamma + 1})^{1/\gamma} = D^{1/\gamma} U + D^{(\gamma + 1)/\gamma} F(X)
\]
\[ F(X) = \frac{U / D}{[U / D + 1]^{y+1} - (U / D)^{y+1}} - 1. \]

\[ \Rightarrow \]

Proof of Remark 1: From Lemma 1A, \( \tau_h^{(2)} \) satisfies \( \int_{X^{(2)}} u(q, \tau_h^{(2)}) f^{(1)}(q) dq = 0 \) and \( \tau_h^{(3)} \) satisfies
\[ \int_{X^{(2)}} u(q, \tau_h^{(3)}) f^{(2)}(q) dq + \Pr(X^{(2)}) u(X^{(2)}, \tau_h^{(3)}) = 0. \]

We can rewrite the latter, since \( X^{(2)} = X^{(1)} \) and \( f^{(2)}(q) = f^{(1)}(q) \) for \( q \in (X^{(2)}, 1) \), as
\[ \int_{X^{(1)}} u(q, \tau_h^{(3)}) f^{(1)}(q) dq + \Pr(X^{(1)}) u(X^{(1)}, \tau_h^{(3)}) = 0. \]

The left hand side is decreasing in \( \tau_h^{(3)} \), and is negative at \( \tau_h^{(2)} \) therefore, \( \tau_h^{(3)} < \tau_h^{(2)} \). The analysis is similar for \( \tau_l \). From Theorem 1, the cutoff \( X^{(3)} \) satisfies
\[ \int_{\tau^{(3)}} u(X^{(3)}, t) g(t) dt = 0 \]

The above is increasing in \( X^{(3)} \) and at \( X^{(2)} \) it is positive because \( \tau_h^{(3)} < \tau_h^{(2)} \) and \( \tau_l^{(3)} < \tau_l^{(2)} \). Therefore, \( X^{(3)} < X^{(2)} \). ■

Proof of Lemma 2:
The method closely follows the derivation of Lemma 1 in the text and so it is omitted in the interests of saving space. The proof is available from the authors.

Proof of Theorem 2:
\[ \max_{\{\tau_0, \ldots, \tau_n\}} \sum_{i=0}^{n} \int_{\tau_i}^{\tau_{i+1}} u(q, t) f(q) dq \int_0^\frac{\tau_i}{\tau_{i+1}} g(t) dt \]

Differentiating with respect to \( X_i \),
\[ \tau_i'(X_i) g(\tau_i(X_i)) \int_{X_i}^{X_{i+1}} u(q, \tau_i(X_i)) f(q) dq - \int_0^{\tau_i} u(X_i, t) f(X_i) g(t) dt \]
\[ + \tau_{i-1}'(X_i) g(\tau_{i-1}(X_i)) \int_{X_{i-1}}^{X_i} u(q, \tau_{i-1}(X_i)) f(q) dq + \int_0^{\tau_{i-1}} u(X_i, t) f(X_i) g(t) dt = 0 \]

Simplifying this using Lemma 2 that \( \int_{X_i}^{X_{i+1}} u(q, \tau_i) f(q) dq = 0 \) for \( i = 1, \ldots, n \), we get
\[ \int_{\tau_{i-1}}^{\tau_i} u(X_i, t) f(X_i) g(t) dt = 0 \]
from which \( f(X_i) \) may be cancelled out to yield the desired result:

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\[
\int_{\tau_i}^{t_i} u(X_i, t) g(t) dt = 0.
\]

Proof of Corollary 2: For the step-utility specification, Lemma 2 provides that \( \tau_i \) is obtained from

\[
F(\tau_i) = \frac{U}{U+D} F(X_{i+1}) + \frac{D}{U+D} F(X_i)
\]

for \( i = 1, \ldots, n \). The condition in Theorem 2 becomes

\[
G(X_i) = \frac{U}{U+D} G(\tau_{i-1}) + \frac{D}{U+D} G(\tau_i)
\]

for \( i = 1, \ldots, n \). We proceed from this.

Part (a): When \( F(\cdot) = G(\cdot) \),

\[
F(X_i) = \frac{DF(\tau_i)}{U+D} + \frac{UF(\tau_{i-1})}{U+D}
\]

\[
\Rightarrow -F(X_i) + \left( \frac{D}{U+D} \right) UF(X_{i+1}) + DF(X_i) + \left( \frac{U}{U+D} \right) UF(X_i) + DF(X_{i-1}) = 0
\]

\[
\Rightarrow F(X_i) = 1/2 [F(X_{i+1}) + F(X_{i-1})]
\]

Because \( F(X_{n+1}) = 1 \) and \( F(X_0) = 0 \), the unit length is divided into \( n \)-quartiles. We can conclude that \( F(X_j) = i/(n+1) \), which, in case of the uniform distribution, reduces to \( X_j = i/(n+1) \).

Part (b): The following results when \( F(\cdot) = G(\cdot) \) will be useful. From part (a) of this Corollary, \( F(X_i) = i/(n+1) \), and so \( F(X_{i+1}) - F(X_i) = 1/(n+1) \). From Lemma 2 that \( F(\tau_i) = uF(X_{i+1}) + \delta F(X_i) \), where \( u = U/(U+D) \) and \( \delta = D/(U+D) \), part (a) of the corollary implies \( F(\tau_i) = (i+u)/(n+1) \), where we used that \( u + \delta = 1 \).

We write the objective function using \( u \) and \( \delta \), and that \( F(\cdot) = G(\cdot) \), as:

\[
(U+D) \sum_{i=0}^{n} \left[ F(X_i) u \int_{X_{i+1}}^{X_i} f(q) dq + \int_{X_{i-1}}^{X_i} \left( u \int_{X_{i+1}}^{X_i} f(q) dq - \delta \int_{X_{i-1}}^{X_i} f(q) dq \right) f(t) dt \right]
\]

And now we insert the results collected above, to simplify the optimized value of this objective function, as follows:

\[
(U+D) \sum_{i=0}^{n} \left[ \frac{F(X_i) u}{n+1} + \int_{X_{i-1}}^{X_i} (F(\tau_i) - F(t)) f(t) dt \right]
\]

\[
= (U+D) \sum_{i=0}^{n} \left[ \frac{iu}{(n+1)^2} + \frac{(F(\tau_i) - F(X_i))^2}{2} \right]
\]
\[ (U + D) \sum_{i=0}^{n} \left[ \frac{iu}{(n+1)^2} + \frac{u^2}{2(n+1)^2} \right] = \frac{nU}{2(n+1)} + \frac{nU^2}{2(n+1)^2 (U + D)}. \]

**Proof of Theorem 3:** The objective of the reviewer can be written as

\[
\left( \int_{X_A}^{1} f(q) dq + \int_{X_B}^{1} f(q) dq \right) \pi(R = \text{high}) + \left( \int_{0}^{X_A} f(q) dq + \int_{0}^{X_B} f(q) dq \right) \pi(R = \text{low}),
\]

Where \( \pi(R) \) represents the payoff of the reviewer when the report is \( R \). When \( R = \text{high} \), this payoff is given by (9), and when \( R = \text{low} \), it is given by (12). Inserting these expressions, the objective can be stated as maximizing:

\[
\begin{align*}
\lambda \int_{0}^{\tau_{Ah}} \left( \mu \int_{X_A}^{1} u_A(A, q, t) f(q) dq + (1 - \mu) \int_{X_B}^{1} u_A(B, q, t) f(q) dq \right) g(t) dt \\
+ (1 - \lambda) \int_{0}^{\tau_{Bh}} \left( \mu \int_{X_A}^{1} u_B(A, q, t) f(q) dq + (1 - \mu) \int_{X_B}^{1} u_B(B, q, t) f(q) dq \right) g(t) dt \\
+ \lambda \int_{0}^{\tau_{Al}} \left( \mu \int_{0}^{X_A} u_A(A, q, t) f(q) dq + (1 - \mu) \int_{0}^{X_B} u_A(B, q, t) f(q) dq \right) g(t) dt \\
+ (1 - \lambda) \int_{0}^{\tau_{Bl}} \left( \mu \int_{0}^{X_A} u_B(A, q, t) f(q) dq + (1 - \mu) \int_{0}^{X_B} u_B(B, q, t) f(q) dq \right) g(t) dt.
\end{align*}
\]

The first order conditions in the statement of the theorem are then obtained using equations (7), (8), (10), and (11).

**Proof of Corollary 3:** When \( f(\cdot) \) and \( g(\cdot) \) are uniform and \( \mu = \frac{1}{2} \), the location of the indifferent consumers are: \( \tau_{Ah} = \tau_{Bl} = 0; \tau_{Bh} = 0.5(X_A + X_B); \) and \( \tau_{Al} = 0.5(X_A - X_B) \) for \( X_A > X_B \) and \( \tau_{Bh} = \tau_{Al} = 0; \tau_{Ah} = 0.5(X_A + X_B); \) and \( \tau_{Bl} = 0.5(X_B - X_A) \) for \( X_A \leq X_B \). Because the functions differ depending on the relative size of \( X_A \) and \( X_B \), we first find the local maximum of the objective when \( X_A > X_B \), then find the local maximum when \( X_A \leq X_B \), and then find the global maximum by comparing the two local ones.

For \( X_A > X_B \), the derivative of the reviewer’s objective with respect to \( X_A \) (the left hand side of the first equation in Theorem 3(b) can be written as

\[ X_A + X_B - 2\lambda X_B. \]

The above is positive for any admissible \( X_A > X_B \), implying that the optimal choice for the reviewer is \( X_A = 1 \). At \( X_A = 1 \), the first order condition with respect to \( X_B \) can be written as
which implies that

$$X_B = \frac{1 - 2\lambda}{3 - 4\lambda}$$

Similarly, for $X_A \leq X_B$, the derivative of the reviewer’s objective with respect to $X_B$ (the left hand side of the first equation in Theorem 3(b)) can be written as

$$X_A + X_B - 2\lambda X_A.$$

The above is positive for any admissible $X_B \geq X_A$, implying that the optimal choice for the reviewer is $X_B = 1$. At $X_B = 1$, the first order condition with respect to $X_A$ produces

$$X_A = \frac{2\lambda - 1}{4\lambda - 1}.$$

Comparing the values of the reviewer’s objective at the above two pairs of choices for $(X_A, X_B)$ shows that $(1, \frac{1-2\lambda}{3-4\lambda})$ is preferred for $\lambda < \frac{1}{2}$, and $(\frac{2\lambda-1}{4\lambda-1}, 1)$ is preferred for $\lambda \geq \frac{1}{2}$.

**Proof of Theorem 4:** The proof is similar to those for Theorems 1 and 2. The objective function is in the text. The first order conditions in the statement of the theorem are then obtained by differentiating (17) with respect to $X_A$ and $X_B$ and using equations (13) through (16). Details are available from the authors.

**Proof of Corollary 4:** When $f(\cdot)$ and $g(\cdot)$ are uniform, $\mu = \frac{1}{2}$, and $\lambda \geq \frac{1}{2}$, the reviewer optimally chooses $X_B = 1$ and $(X_A = \frac{2\lambda-1}{4\lambda-1})$ even if the reviewer observes the quality of type $B$ movies. Since the less informed reviewer, who does not observe the quality of type $B$ movies, can perfectly replicate this reporting strategy, the less informed reviewer also finds this strategy optimal.

Consider next the case when $\mu = \frac{1}{2}$, and $\lambda < \frac{1}{2}$. In this case, when $X_A > X_B$, the reviewer’s objective can be restated as

$$0.125(1 - 2\lambda)(1 - 2X_B) - X_B.\]

The above is increasing in $X_A$, and therefore the optimal choice is $X_A = 1$. Given this choice, the objective becomes
0.125((1 - \lambda)(1 - X_B) - 0.5(1 - \lambda) + 0.5\lambda(1 - X_B)^2),

which is decreasing in \(X_B\), implying that the optimal choice is \(X_B = 0\). Similar calculations show that when \(X_A \leq X_B\), the optimal choice is \(X_A = 0\) and \(X_B = 1\). Given that the labeling is not important, the above analysis implies that the global maximum is achieved at \(X_A = 0, X_B = 1\).
REFERENCES


