

CS 6347

Lecture 3

More Bayesian Networks

- Last time:
 - Complexity challenges
 - Representing distributions
 - Computing probabilities/doing inference
 - Introduction to Bayesian networks
- Today:
 - D-separation, I-maps, limits of Bayesian networks

- A **Bayesian network** is a directed graphical model that represents a subset of the independence relationships of a given probability distribution
- Directed acyclic graph (DAG), $G = (V, E)$
 - Edges are still pairs of vertices, but the edges (1,2) and (2,1) are now distinct in this model
- One node for each random variable
- One conditional probability distribution per node
- Directed edge represents a direct statistical dependence

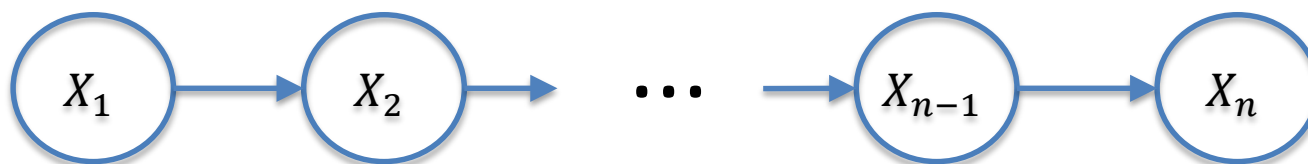
- A **Bayesian network** is a directed graphical model that represents a subset of the independence relationships of a given probability distribution
- Encodes **local Markov** independence assumptions that each node is independent of its non-descendants given its parents
- Corresponds to a **factorization** of the joint distribution

$$p(x_1, \dots, x_n) = \prod_i p(x_i | x_{parents(i)})$$

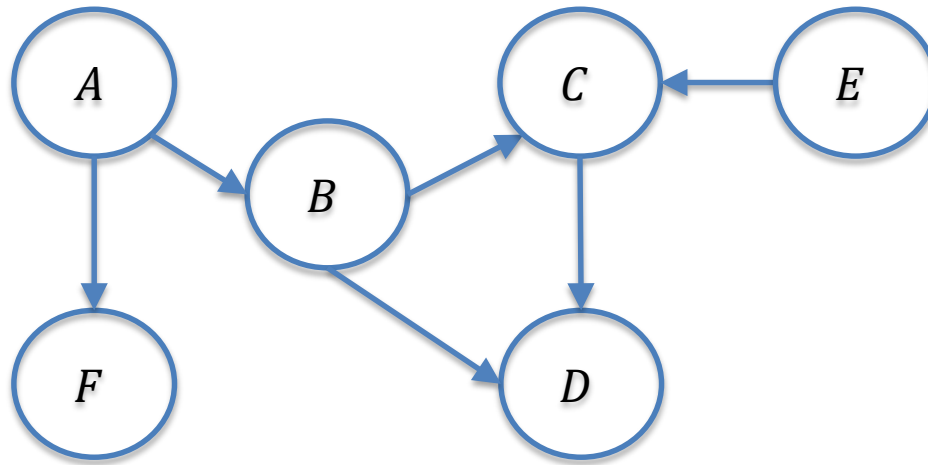
Directed Chain



$$p(x_1, \dots, x_n) = p(x_1)p(x_2|x_1)p(x_3|x_2) \dots p(x_n|x_{n-1})$$



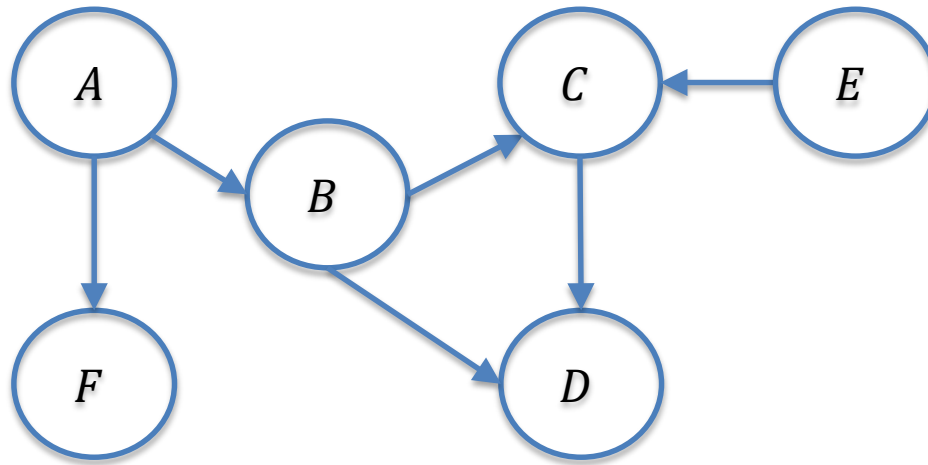
Example:



Suppose that a joint distribution factorizes over this graph...

- Local Markov independence relations?
- Joint distribution?

Example:



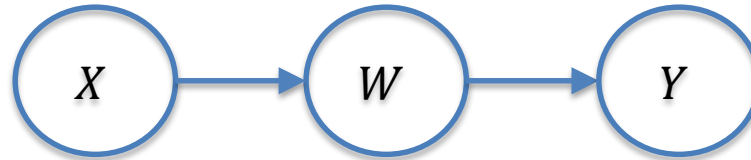
The local Markov independence relations are not exhaustive:

- How can we figure out which independence relationships the model represents?

- Independence relationships can be figured out by looking at the graph structure!
 - Easier than looking at the tables and plugging into the definition
- We look at all of the paths from X to Y in the graph and determine whether or not they are **blocked**
 - $X \subset V$ is **d-separated** from $Y \subset V$ given $Z \subset V$ **iff** every path from X to Y in the graph is blocked by Z

- Three types of situations can occur along any given path

(1) Sequential

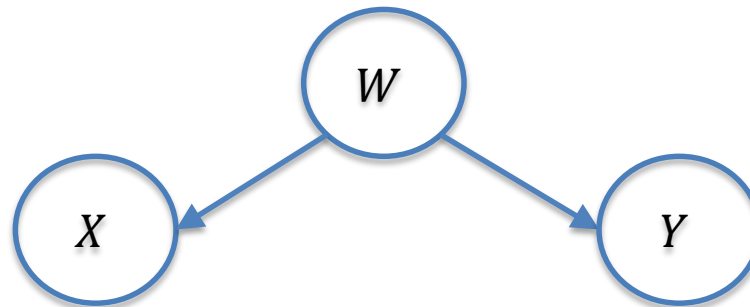


The path from X to Y is blocked if we condition on W

Intuitively, if we condition on W , then information about X does not affect Y and vice versa

- Three types of situations can occur along any given path

(2) Divergent

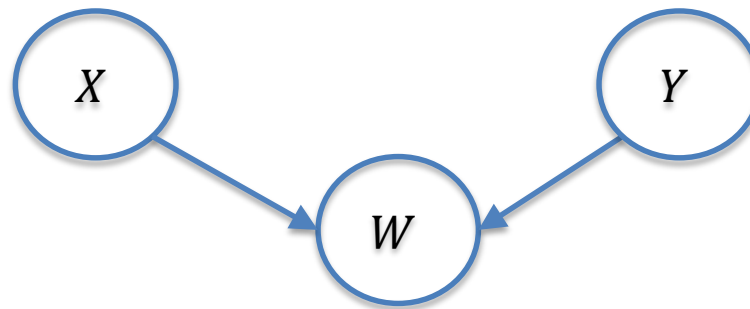


The path from X to Y is blocked if we condition on W

If we don't condition on W , then information about W could affect the probability of observing either X or Y

- Three types of situations can occur along any given path

(3) Convergent

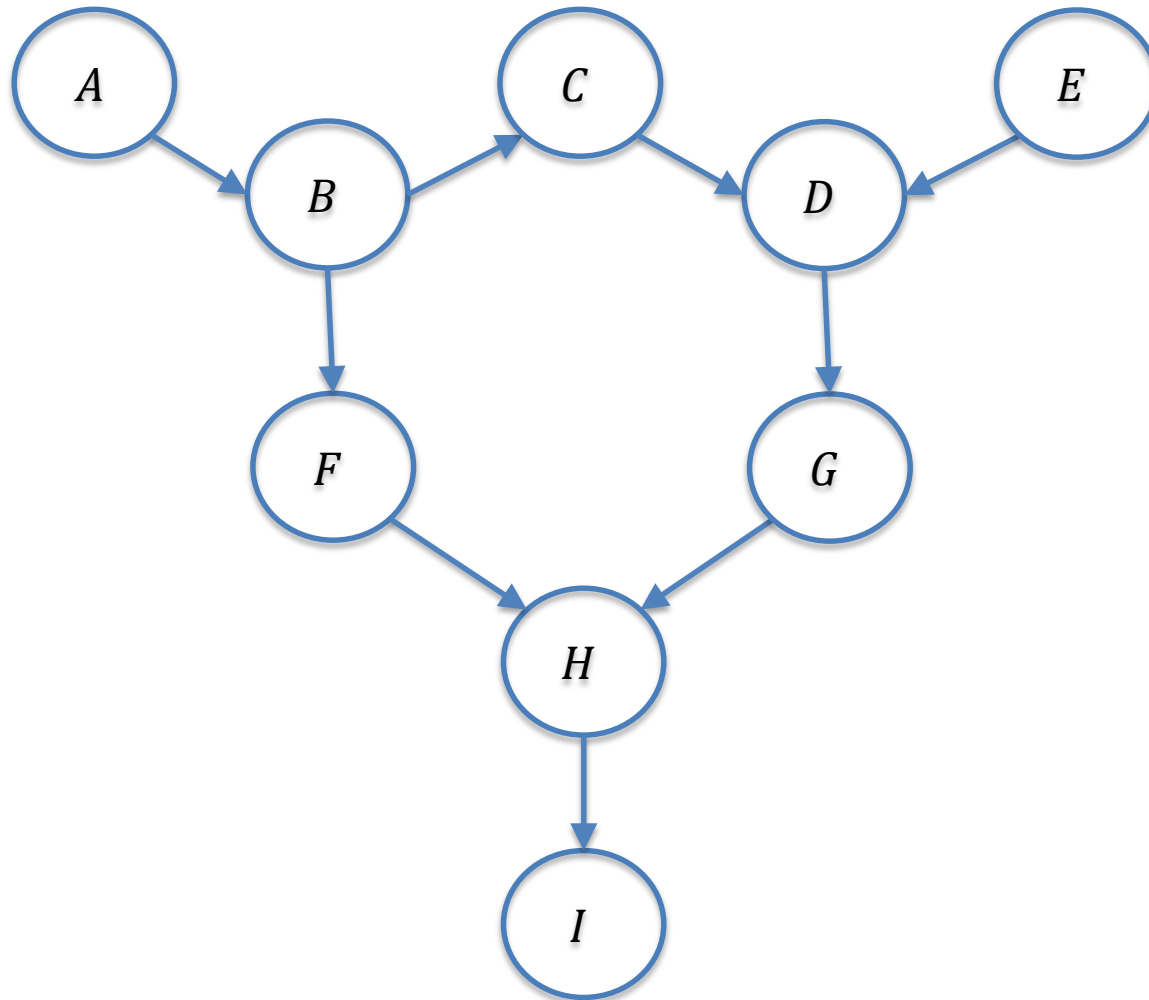


The path from X to Y is blocked if we **do not** condition on W or any of its descendants

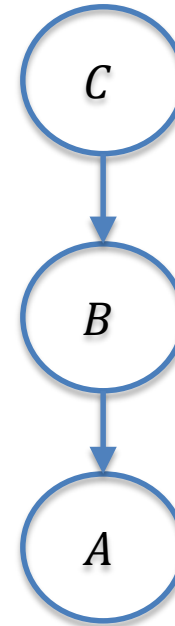
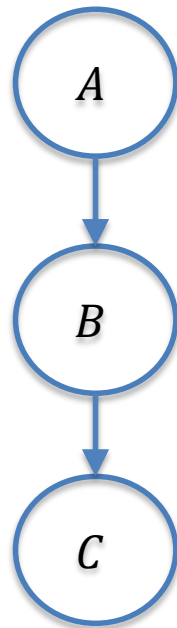
Conditioning on W couples the variables X and Y : knowing whether or not X occurs impacts the probability that Y occurs

- If the joint probability distribution factorizes with respect to the DAG $G = (V, E)$, then X is d-separated from Y given Z implies $X \perp Y \mid Z$
 - We can use this to quickly check independence assertions by using the graph
 - In general, these are **only a subset** of all independence relationships that are actually present in the joint distribution
 - If X and Y are not d-separated in G given Z , then there is some distribution that factorizes over G in which X and Y dependent

D-separation Example

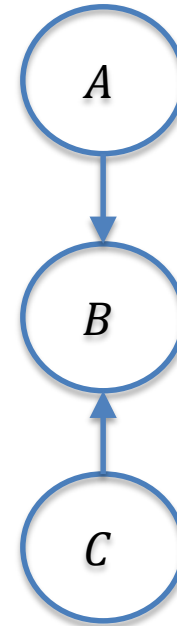
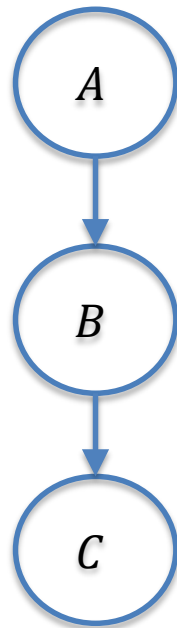


Equivalent Models?



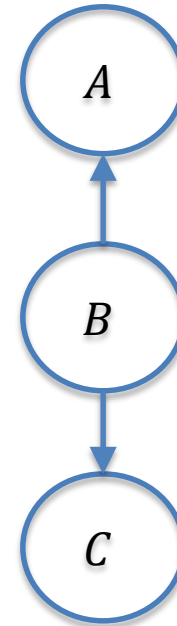
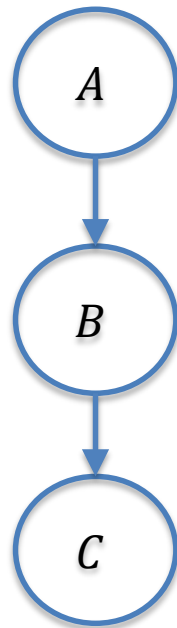
Do these models represent the same independence relations?

Equivalent Models?



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Equivalent Models?



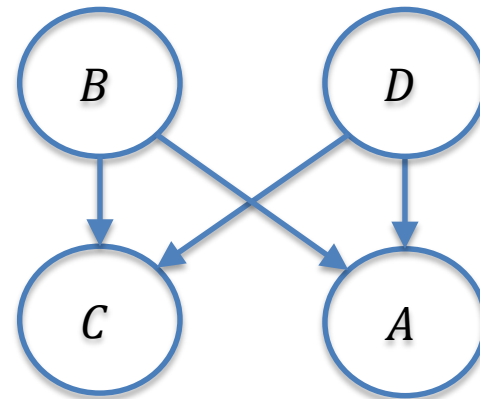
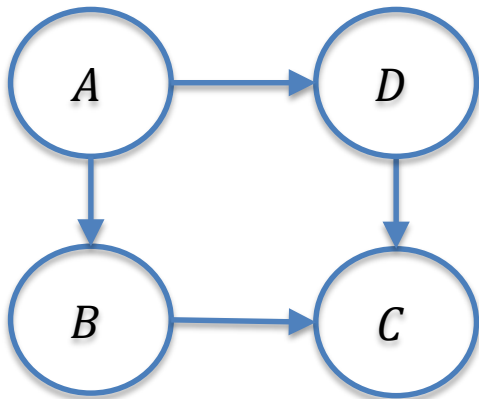
Do these models represent the same independence relations?

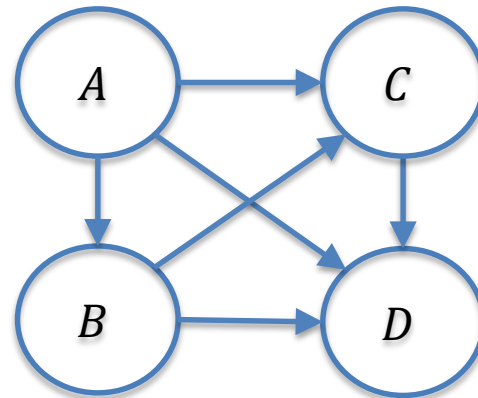
- Let $I(p)$ be the set of all independence relationships in the joint distribution p and $I(G)$ be the set of all independence relationships implied by the graph G
- We say that G is an **I-map** for $I(p)$ if $I(G) \subseteq I(p)$
- Theorem: the joint probability distribution, p , **factorizes** with respect to the DAG $G = (V, E)$ iff G is an I-map for $I(p)$
- An I-map is **perfect** if $I(G) = I(p)$
 - Not always possible to perfectly represent all of the independence relations with a graph

Limits of Bayesian Networks

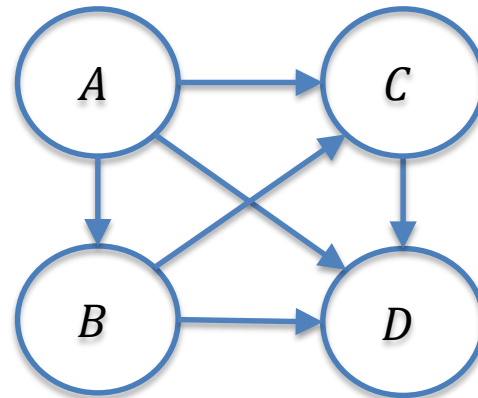


- Not all sets of independence relations can be captured by a Bayesian network
 - $A \perp C \mid B, D$
 - $B \perp D \mid A, C$
- Possible DAGs that represent these independence relationships?





What independence relations does this model imply?



$I(G) = \emptyset$, this is an I-map for any joint distribution on four variables!