

Proposal All Reviews: 1955092

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Agency Name: National Science Foundation

Agency Tracking Number: 1955092

Organization:

NSF Program: Combinatorics

PI/PD: Williams, Nathan

Application Title: Combinatorial Tools in Representation Theory

Review 1

Rating:

Very Good

Review:

Summary

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to intellectual merit.

The PI has a collection of projects in algebraic combinatorics, centered around some outstanding conjectures for representation theoretic and/or combinatorial explanations of positivity.

1. Potential to advance knowledge (within the field and across fields)

The PI describes some novel approaches to some conjectures that can advance understanding of combinatorics surrounding Macdonald polynomials.

2. To what extent do the proposed activities suggest and explore creative, original, or potentially transformative concepts?

While the ideas do not rise to the level of "transformative" there are some good ideas for attacking some established conjectures with novel, largely combinatorial approaches with the sweep and zeta maps as primary tools.

3. Is the plan for carrying out the proposed activities well-reasoned, well-organized, and based on a sound rationale? Does the plan incorporate a mechanism to assess success?

The proposal is well written and the PI seems familiar with the current state of work on the problems described.

4. How well qualified is the individual, team, or institution to conduct the proposed activities?

The PI and Thomas have several recent works that are directly applicable to some of the described projects. The PI has a strong track record in both algebraic and dynamical combinatorics.

5. Are there adequate resources available to the PI (either at the home institution or through collaborations) to carry out the proposed activities?

Yes.

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to broader impacts.

The proposal's broader impacts include dissemination of work, service to the community and teaching and mentoring of students (both undergraduate and graduate). The PI has experience with organizing conferences and workshops. As an early career mathematician the broader impacts are fair.

Please evaluate the strengths and weaknesses of the proposal with respect to any additional solicitation-specific review criteria, if applicable

Summary Statement

This well-written proposal is squarely in the field of mainstream algebraic combinatorics, with problems inspired by classical invariant theory as well as more modern questions like combinatorial expansions of the (q,t) -Kostka numbers. The PI has an early-career mathematician with a strong track record in the field. The broader impacts are fair.

Review 2

Rating:

Multiple Rating: (Very Good/Good)

Review:

Summary

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to intellectual merit.

The ultimate goal of this proposal is to find a combinatorial interpretation for the modified Macdonald polynomials of Garsia and Haiman when expanded in the Schur basis. This involves finding a combinatorial interpretation for the (q,t) -Kostka polynomials. The proposal proposes to use zeta and sweep maps. The proposal contains a number of problems related to this main problem along with several problems that will give partial results. The proposal is divided into three projects the first one relates to series arising from coinvariant rings of complex reflection groups, the second relates to diagonal coinvariants, the two variable version and the third project connects to Macdonald polynomials.

A classical result in algebraic combinatorics is the combinatorial interpretations for the Hilbert series of the coinvariant ring of the symmetric group. This result shows that this Hilbert series is obtained using either the major index statistic or the inversion statistic. The first project proposed is generalizing this result to all complex reflection groups. This involves finding a statistic such that when we sum powers of a variable to this statistic over all elements in the group we would get the Hilbert series. The PI mentions that a possible statistic is to take number of inversions. I found this confusing because earlier the PI said that there was no notion of inversion statistic for complex reflection groups. So, would the problem involve finding an inversion statistic for these groups? This made me wonder if the PI had tried examples that back gives a hint about the statistic. I found Problem 1 a bit vague and although some indication is given for how to get started, it would be useful to have some partial computations and a little more details on the methods that the PI plans to use. Problems 2 and 3 are ask questions related to polynomials arising from complex reflection groups. These problems are an ongoing project with the PI and a collaborator.

The main part of the proposal relates to the problems relating to series related to diagonal coinvariants and Macdonald polynomials. The problems that many researchers have been trying to solve relates to finding combinatorial statistics on permutations to express Hilbert (or Frobenius) series using these statistics. The PI has defined a sweep map that is a bijection on certain lattice paths on a rectangle, this map has already been used by other authors. And Problem 4 and Problem 5 are related to this map; unfortunately there was little motivation for these problems and I think that they should have been elaborated on the plan on how the PI would solve them. This section also contains problems on rational parking functions and hyperplane arrangements. The last section on Macdonald polynomials proposes to define a zeta map that would allow for a combinatorial interpretation for the modified Kostka two variable polynomials. This is a very nice idea if it were to work.

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to broader impacts.

The PI already has two Ph.D. students. He has also mentored undergraduates. He is very involved in the organization of conferences, workshops and AMS special sessions.

Please evaluate the strengths and weaknesses of the proposal with respect to any additional solicitation-specific review criteria, if applicable

Summary Statement

This is a good proposal with a good range of problems. The proposal would have read better if the PI would have started with the diagonal coinvariants and Macdonald polynomials and left the coinvariant ring related to complex reflection groups as a minor project since this was the least interesting part. Also, the proposal needs more concrete conjectures based perhaps on computations or partial results. The PI is a very active researcher and this proposal contains beautiful mathematics and interesting problems.

No previous NSF support

Review 3

Rating:

Very Good

Review:

Summary

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to intellectual merit.

The first project, studying the coinvariant rings of complex reflection groups, is a natural generalization of the known theory for real reflection groups. Due to the complicated nature of complex reflection groups, it would be interesting if there were anything approaching a uniform, combinatorial proof for Problem 1. However, not much discussion is given to whether this is actually feasible at the moment. Problems 2 and 3 seem somewhat more well-reasoned in this regard.

As for the other projects, studying the dynamics of the zeta/sweep maps is important, but it is not so clear whether this study will help make significant progress on some of the problems proposed.

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to broader impacts.

Broader impacts mostly take the form of mentoring undergraduate and graduate students, as well as a notable amount of conference organization. The PI notes that many of his problems have led to successful REU projects in the past.

Please evaluate the strengths and weaknesses of the proposal with respect to any additional solicitation-specific review criteria, if applicable

Summary Statement

Overall this proposal suggests a number of interesting lines of study, although it suffers somewhat from a lack of justification for why the desired goals are within reach.