

## Proposal All Reviews: 2054560

[Back to Proposal](#)

Agency Name: National Science Foundation

Agency Tracking Number: 2054560

Organization:

NSF Program: Combinatorics

PI/PD: Williams, Nathan

Application Title: Independence Posets

### Review 1

**Rating:**

**Very Good**

**Review:**

#### Summary

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to intellectual merit.

The PI is an extraordinarily prolific researcher with many results both in pure algebraic combinatorics, and in the relationship between algebraic combinatorics and representation theory. The current proposal describes a construction to assign a poset to an acyclic directed graph, motivated by the notion of "simple minded collections" in representation theory. The elements of this poset are the independent sets (antichains) in the graph; the order on the poset depends on the orientation of the graph in a complicated manner.

I found the details of this definition a bit hard to follow and think about. There are a lot of steps -- the notion of tight orthogonal pairs is not completely clear, then the connection to independent sets, and the notion of flips. I was also puzzled by the boldfaced sentence on the first page: "What if a distributive lattice weren't a lattice?" All the examples the PI describes are in fact lattices (though not distributive ones). So I was unsatisfied with the exposition here.

However, if I trust that the definitions work, the proposed applications are interesting and seem plausible. Propp and Wilson's "coupling from the past method" allows random sampling from large distributive lattices, and has been very valuable in the generation of conjectures about random rhombus tilings, alternating sign matrices and other related objects. The PI proposes that his construction could allow similar random sampling from independent sets in many graphs, which is important for studying the hard-core model in statistical mechanics.

The PI also proposes building a piecewise linear version of his flip construction, analogous to other recent popular work on "dynamical combinatorics" such as "row motion". There has been a lot of recent work in this area, much of it by the PI, and this project seems doable.

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to broader impacts.

The PI is a frequent mentor of undergraduate research projects and proposes more based on the current proposal. He is also an active organizer of conferences.

Please evaluate the strengths and weaknesses of the proposal with respect to any additional solicitation-specific review criteria, if applicable

#### Summary Statement

The proposal is fairly technical, and it is a bit hard to appreciate all its connections. But the PI describes connections to very interesting areas of math, and has a past record of success with problems like this.

### Review 2

**Rating:**

**Good**

**Review:**

#### Summary

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to intellectual merit.

A key ingredient to this proposal was the notion (introduced by the PI in joint work with Hugh Thomas) of independence posets. The PI does define these and gives an example, but still I somehow found this discussion very difficult to follow. For problem 2 asking for an extension from the finite case to the infinite case, it would be better to get some ideas towards approaching this question to give evidence the PI has a good chance of success, though he does have a track record in this area.

The PI gives a number of projects regarding independence posets, which sound like good projects, provided that independence posets are indeed well motivated -- I have not yet seen enough explanation of their motivations. Sections 1.2 and 1.3 purportedly describe motivations to representation theory and to lattice theory, respectively, but the former seems to digress immediately to other things such as definitions and open questions and not actually explain the motivations, while the latter does not make clear exactly what fundamental lattice theoretic structure is being captured by independence posets.

That said, problem 4 asks whether mutation equivalence of digraphs implies that two digraphs have the same independence poset, a fact which if true would seem to actually give some added motivation for the notion of independence posets. Problem 7 also seems to give another possible motivation -- that independence posets could help in the study of independent sets in graphs via Propp and Wilson's CFTP (coupling from the past) by giving a way to encode independent sets as poset order ideals (therefore amenable to a trick of Shor and Winkler).

Problem 5 (compiling a database) seems more like a broader impact than a research project. Problem 6 (seeking a generalization of mutations on digraphs) seems kind of vague, or at a minimum seems to require a more careful explanation to understand what exactly the PI is proposing.

The PI has not had prior NSF support.

My biggest hesitation is that most of the PI's work related to this proposal is joint work with Hugh Thomas, and it is not clear what is the PI's role in the collaboration. It seems like nice work, but the lack of clarity in the explanations in the proposal are part of what led to my concern as to what exactly is the PI's role in this work.

Section 3 (regarding coupling from the past), Section 4 (regarding integer points in polytopes and getting into counting lattice points in polytopes), and Section 5 (on rowmotion and dynamical algebraic combinatorics) seem somewhat more independent, but I worry whether there is enough evidence provided for these projects of likelihood of success. Certainly the task of enumerating lattice points in polytopes is very challenging in general. The description here of coupling from the past and the associated project is a little vague.

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to broader impacts.

Problem 5 involves creating a digital database for various researchers to use. The PI plans to organize an annual online conference. He has mentored several undergraduate research projects, including mentoring several women in math. He is currently advising two Ph.D. students. He has organized a lot of workshops. He serves on the editorial board for Annals of Combinatorics and does a fair bit of refereeing. This seems like an appropriate quantity and style of broader impact for the PI's career stage, though not as strong as some other proposals I evaluated this year in terms of broader impact.

Please evaluate the strengths and weaknesses of the proposal with respect to any additional solicitation-specific review criteria, if applicable

#### Summary Statement

The PI is doing nice work and certainly has some interesting ideas and research plans here, but nonetheless this proposal is in the bottom third of proposals I reviewed this year due to the fierce competition and hesitations outlined above. I would certainly encourage the PI to reapply if he is not successful this year, trying to address the above concerns.

### Review 3

**Rating:**

**Multiple Rating: (Excellent/Very Good)**

**Review:**

#### Summary

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to intellectual merit.

The PI is part of a small group of very active researchers who have identified and categorized an existing phenomenon in algebraic combinatorics of dynamic algebraic combinatorics. His current proposal is a program to substantially broaden the cases in which these studies are applied via his newly defined independence posets and to study these objects in general. He gives a convincing list of objects on which this work can be applied, including a number of interest in various areas of algebraic combinatorics and representation theory. The combination of both his broad goals for the project and his specific applications, when combined with his impressive record of publications, makes this a very attractive proposal.

I'll offer a minor expository suggestion for the PI on future proposals in this area: I think the specifics of his independence posets would be more effectively communicated if he combined his definition with an immediately occurring concrete example on a broadly familiar combinatorial object.

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to broader impacts.

The PI gives a strong argument for his broader impacts, and in his three years at his current position has already shown a good concrete record for encouraging undergraduate research and pursuit of graduate school, as well as allowed him to begin developing relationships with his first graduate students. He has an impressive record of organizing successful courses and workshops, refereeing for journals, and appears to have a research program that is well designed to encouraging meaningful research and engagement by undergraduate and graduate researchers. One minor observation in a otherwise very well written proposal: the PI writes that he has "substantial past experience in involving students and underrepresented students in research," but in contrast to both the remainder of this section and other comparable proposals is not very concrete on this point.

Please evaluate the strengths and weaknesses of the proposal with respect to any additional solicitation-specific review criteria, if applicable

#### Summary Statement

This is a strong proposal, both in terms of broader impacts and research content. It does a good job of balancing a strong vision of an abstract mathematical goal with a number of concrete examples of what such a goal will likely apply to.