An On-line Addendum to: “Strategic Commitments for an Optimal Capacity Decision Under Asymmetric Forecast Information”

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1. Remaining Proofs:

Proof of Theorem 4. For \( w_a \in [c + c_k, \frac{r c_k}{w - c}] \), we have the boundary equilibrium. From its definition, \( y^b(\xi) \) is decreasing in \( w_a \) (that is, \( \frac{dy^b(\xi)}{dw_a} < 0 \)) and \( y^b(\xi) = \mu + \xi + G^{-1}(\frac{r - c - c_k}{r}) \) when \( w_a = c + c_k \). The total supply chain profit at the boundary equilibrium is \( \Pi^{\text{tot}}(y^b(\xi), y^b(\xi), \xi) = rE \min(\mu + \xi + \epsilon, y^b(\xi)) - (c + c_k)y^b(\xi) \) from Equations (16) and (17). This function is concave and increasing in \( y^b(\xi) \) for all \( y^b(\xi) \) \( \leq \mu + \xi + G^{-1}(\frac{r - c - c_k}{r}) \) (that is, \( \frac{d\Pi^{\text{tot}}(y^b(\xi), y^b(\xi), \xi)}{dy^b(\xi)} > 0 \)). Hence \( \frac{d\Pi^{\text{tot}}(y^b(\xi), y^b(\xi), \xi)}{dw_a} < 0 \).

For \( w_a \in \left[ \frac{r c_k}{w - c}, w + \frac{(r - w) c_k}{w - c} \right] \), we have the interior equilibrium. From its definition, \( y^i(\xi) \) is also decreasing in \( w_a \) \( \left( \frac{dy^i(\xi)}{dw_a} < 0 \right) \). From Lemma 2 the optimal capacity \( K^{\text{ap}}(\xi) \) is independent of \( w_a \). The total supply chain profit in the interior equilibrium is \( \Pi^{\text{tot}}(y^i(\xi), K^{\text{ap}}(\xi), \xi) = E[r \min(\mu + \xi + \epsilon, K^{\text{ap}}(\xi)) - cK^{\text{ap}}(\xi) \] and it is decreasing in \( y^i(\xi) \) \( \left( \frac{d\Pi^{\text{tot}}(y^i(\xi), K^{\text{ap}}(\xi), \xi)}{dy^i(\xi)} < 0 \right) \). Hence \( \frac{d\Pi^{\text{tot}}(y^i(\xi), K^{\text{ap}}(\xi), \xi)}{dw_a} > 0 \).

For Part 2, note that the manufacturer’s profit function in Equation (16) decreases uniformly in \( w_a \), hence it decreases at the equilibrium as well.

Part 3 follows from Parts 1 and 2 immediately since supplier’s profit is the difference between the total profit and the manufacturer’s profit.

For Part 4, we differentiate the equilibrium profit functions with respect to \( \xi \). At the interior equilibrium, we have \( \frac{\partial \Pi^{\text{tot}}(y^i(\xi), K^{\text{ap}}(\xi), \xi)}{\partial \xi} = w_a - c - c_k \geq 0 \) and \( \frac{\partial \Pi^{\text{m}}(y^i(\xi), K^{\text{ap}}(\xi), \xi)}{\partial \xi} = r - w_a \geq 0 \). Similarly, at the boundary equilibrium, we have \( \frac{\partial \Pi^{\text{tot}}(y^b(\xi), y^b(\xi), \xi)}{\partial \xi} = w_a - c - c_k \geq 0 \), \( \frac{\partial \Pi^{\text{m}}(y^b(\xi), y^b(\xi), \xi)}{\partial \xi} = r - w_a \geq 0 \).

For Part 5, from Theorem 3 we know that an advance purchase contract with \( w_a \in [w, w + \frac{r - w - c_k}{w - c}] \) leads to the interior equilibrium and credible information sharing. The equilibrium profit under such a contract is \( \Pi^{\text{ap}}_a(\xi) = (w - c)E(\min(\mu + \xi + \epsilon, K^{\text{ap}}(\xi)) - y^i(\xi))^{+} + (w_a - c)y^i(\xi) - cK^{\text{ap}}(\xi) \geq \)

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with probability one; if the advance purchase is \( \xi \in \) is a separating equilibrium with the following belief: if the manufacturer orders \( y \) when \( K < y \), then the supplier infers \( \mu = \xi \), which maximizes \( (w - c)E\min(\mu + \xi + \epsilon, K^{ws}(\xi)) - y^{i}(\xi) \) and the last inequality is due to the optimality of \( K^{ws}(\xi) \), which maximizes \( (w - c)E\min(\mu + \xi + \epsilon, K^{ws}(\xi)) - c_{k}K^{wa} = \Pi_{a}(\xi) \). The first equality is due to \( w_{a} \geq w \) and the last inequality is due to the optimality of \( K^{ws}(\xi) \), which maximizes \( (w - c)E\min(\mu + \xi + \epsilon, K) - c_{k}K \).

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**Proof of Theorem 5.** To prove the first part, notice that when \( w_{a} \geq w \), \( \frac{\partial \Pi_{a}(y,K,\xi)}{\partial y} \) < 0 both when \( K < y \) and \( K > y \). Hence the manufacturer’s profit function is always decreasing in \( y \) when \( w_{a} \geq w \). So to induce an advance purchase the supplier should set \( w_{a} < w \). The proof of the second part is exactly the same as that of Lemma 2.

Knowing the supplier’s optimal response, the manufacturer solves: \( \max_{y} \Pi_{m}(y, \max(K^{ap}, y), \xi) = \max\{\max_{y \leq K^{ap}} \Pi_{m}(y, K^{ap}, \xi), \max_{y > K^{ap}} \Pi_{m}(y, y, \xi)\} \). It is easy to verify that \( \Pi_{m}(y, K^{ap}, \xi) \) is concave and maximized at \( y^{is} \equiv \mu_{m} + G^{-1}(\frac{w - w_{a}}{w}) \). Similarly, \( \Pi_{m}(y, y, \xi) \) is concave and maximized at \( y^{b} \equiv \mu_{m} + \xi + G^{-1}(\frac{w - w_{a}}{w}) \). Hence we have the following 3 cases.

Case 1: If \( w_{a} \leq \frac{w_{c}}{w - c} \), then \( K^{ap} \leq y^{is} < y^{b} \). Hence \( \Pi_{m}(y, \max(K^{ap}, y), \xi) \) is maximized at \( y^{b} \).

Case 2: If \( w_{a} \geq \frac{w_{c}}{w - c} \), then \( y^{is} < y^{b} \leq K^{ap} \). Hence \( \Pi_{m}(y, \max(K^{ap}, y), \xi) \) is maximized at \( y^{is} \).

Case 3: If \( w_{a} \in \left( \frac{w_{c}}{w - c}, \frac{w_{c}}{w - c} \right) \), then \( y^{is} < K^{ap} < y^{b} \). Hence, \( \max_{y} \Pi_{m}(y, \max(K^{ap}, y), \xi) = \max\{\Pi_{m}(y^{is}, K^{ap}, \xi), \Pi_{m}(y^{b}, y^{b}, \xi)\} \); the maximizer is either \( y^{is} \) or \( y^{b} \).

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**Proof of Theorem 6.** It follows directly from the discussion prior to its statement in § 7.

**Proof of Lemma 3.** The manufacturer’s and the supplier’s expected profits are similar to Equations (16) and (17) but in this case the manufacturer also pays \( \tau E(K - \max(D, y))^{+} \) to the supplier. Given these profit functions, the proof is similar to that of Lemma 2.

**Proof of Theorem 7.** From Lemma 3, a necessary condition is \( K^{\tau}(\xi) = K^{cs}(\xi) \). This is satisfied by setting \( \tau = \frac{(r - w)\xi}{\tau - c - e_{k}} \). At an interior separating equilibrium, manufacturer’s profit function is \( \Pi_{m}(y^{\tau}(\xi), K^{cs}(\xi), \xi) = E[y\min(D, K^{cs}(\xi)) - w(\min(D, K^{cs}(\xi)) - y^{\tau}(\xi))^{+} - w_{a}y^{\tau}(\xi) - \tau(K^{cs}(\xi) - \max(D, y^{\tau}(\xi))^{+}] \). For this to be an equilibrium, the manufacturer’s profit must be maximized at \( \xi = \xi \). Thus the first order condition must hold at \( \xi = \xi \).

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Since \( y^{\tau}(\xi) \) cannot be a constant in a separating equilibrium, it must be the case that \( w(1 - G(y^{\tau}(\xi) - \mu - \xi)) - w_{a}y^{\tau}(\xi) - \tau G(y^{\tau}(\xi) - \mu - \xi) = 0 \). The solution to this equation is \( y^{\tau}(\xi) = \mu + \xi + G^{-1}(\frac{w - w_{a}}{w}) \). With a similar argument as in the proof of Theorem 3, we can verify that for \( w_{a} \geq \frac{w_{c}}{r - c - e_{k}} \), \( y^{\tau}(\xi, K^{cs}(\xi)) \) is a separating equilibrium with the following belief: if the manufacturer orders \( y = y^{\tau}(\xi) \) for any \( \xi \in [\xi, \bar{\xi}] \), then the supplier infers that the manufacturer’s forecast information is \( \xi \) with probability 1. Similarly, the off-the-equilibrium belief is such that if the advance purchase is \( y < y^{\tau}(\xi) \) then the supplier infers \( \xi \) with probability one; if the advance purchase is \( y > y^{\tau}(\xi) \) then the supplier infers \( \bar{\xi} \) with probability one.

Note that when \( w_{a} = w \) and \( \tau = \frac{(r - w)\xi}{\tau - c - e_{k}} \), the advance purchase quantity is \( y^{\tau}(\xi) = \mu + \xi \) units. Since this quantity is equal to the minimum demand when the forecast information is \( \xi \), there is no
risk of overproduction. The signal is costless, and we achieve channel coordination.

Note that in contrast to Theorem 3, a minimum advance purchase is not necessary in this case. To see the reason, consider the advance purchase with payback under symmetric forecast information. In the interior case, the manufacturer solves \( \max_y \Pi^m(y, K^{cs}(\xi), \xi) \). The first order condition is \( w(1 - G(y^{cs}(\xi) - \mu - \xi)) - w_a + \tau G(y^{rs} - \mu - \xi) = 0 \). Hence the optimal advance purchase quantity under symmetric information is \( y^{rs}(\xi) = \mu + \xi + G^{-1}(\frac{w - w_a}{w - r}) \). Note that the manufacturer with the lowest possible private forecast information \( \xi \) doesn’t have any incentive to signal her forecast information (since the supplier knows that the forecast is at least \( \xi \)). Therefore when the information is asymmetric, the manufacturer can still chose to place the optimal advance order \( y^{rs} \), but this is the same as the \( y^{r}(\xi) \). Therefore, the supplier does not need to impose any restriction on the manufacturer’s advance purchase. \( \square \)

**Proof of Theorem 8.** To identify an optimal menu of contracts, the supplier solves a problem similar to the one in Equation (11), but now both the supplier’s and manufacturer's profits are functions of \((K(\xi), P(\xi), w(\xi), \xi)\). Using a similar argument as in Lemma 1, we can replace the PC with PC’ that is \( \Pi^m(K(\xi), P(\xi), w(\xi), \xi) = \pi^m_{\min} \). Note that manufacturer’s optimal profit is \( \pi^m(\xi) = \Pi^m(K(\xi), P(\xi), w(\xi), \xi) \). From this definition and the IC we have \( \pi^m(\xi) = \max_{\xi} \Pi^m(K(\xi), P(\xi), w(\xi), \xi) \). The envelope theorem implies \( \frac{d\pi^m(\xi)}{d\xi} = \frac{\partial \Pi^m(K(\xi), P(\xi), w(\xi), \xi)}{\partial \xi} \). Note that \( \partial \Pi^m(K(\xi), P(\xi), w(\xi), \xi) \) is a function of \((w, K, \xi)\) and \( \Pi^m(\xi) \) is increasing.

To prove Part 1, we define

\[
\pi_i^*(w) \equiv \max_{w_a \in [c + e_k, \frac{r_k}{w - c}]} E[\Pi^i(y^b(\xi), y^b(\xi), \xi)], \quad \text{and} \quad \pi_i^*(w) \equiv \max_{w_a \in [\frac{c_k}{w - c}, \frac{r_k}{w - c}]} E[\Pi^i(y^c(\xi), K^{ap}(\xi), \xi)]
\]

To maximize the above objective function, the supplier would first set \( w = r \) and eliminate the manufacturer’s information rent, which is the second term. The remaining problem then is to choose a capacity level to maximize the total supply chain profit, that is to maximize \( \Pi^{tot}(K(\xi), \xi) \). The solution for this problem is to set \( K^{cr} = K^{cs} = \mu + \xi + G^{-1}(\frac{c - c_k}{r - c}) \). The supplier’s profit is exactly equal to the centralized system’s profit. The manufacturer’s optimal profit \( \pi^m(\xi) = \pi^m_{\min} \).

Hence, using Equation 8 and solving for the transfer payment, we obtain \( P^{cr} = -\pi^m_{\min} \). \( \square \)

**Proof of Theorem 9** To prove Part 1, we define

\[
\pi_i^*(w) \equiv \max_{w_a \in [c + e_k, \frac{r_k}{w - c}]} E[\Pi^i(y^b(\xi), y^b(\xi), \xi)], \quad \text{and} \quad \pi_i^*(w) \equiv \max_{w_a \in [\frac{c_k}{w - c}, \frac{r_k}{w - c}]} E[\Pi^i(y^c(\xi), K^{ap}(\xi), \xi)]
\]
as the supplier’s profit as functions of \( w \) in the boundary equilibrium and interior equilibrium, respectively. In the boundary equilibrium, we have \( E[\Pi^s(y^b(\xi), y^b(\xi), \xi)] = (w_a - c - c_k)(\mu + G^{-1}(\frac{r-w_a}{r})) \) If \( G(\cdot) \) is IFR, from Lemma 4 (provided later in this appendix), the profit function \( E[\Pi^s(y^b(\xi), y^b(\xi), \xi)] \) is unimodal in \( w_a \). Therefore there exists a \( w_a^* \) such that \( \pi^s(w) \) is increasing in \( w_a \) for \( w_a \leq w_a^* \) and decreasing in \( w_a \) for \( w_a \geq w_a^* \). Hence the optimal \( w_a = \min(\frac{rc_k}{w-c}, w_a^*) \). The result for the interior equilibrium follows from Theorem 4 Part 3, that is, the supplier’s profit is increasing in the advance purchase price \( w_a \) for each \( \xi \). Therefore, \( E[\Pi^s(y^i(\xi), K^{ap}(\xi), \xi)] \) is maximized at \( w_a = w + \frac{(r-w)c_k}{w-c} \).

To prove Part 2, note that in the boundary equilibrium, the manufacturer orders only prior to capacity decision. Note also that if \( w \leq \frac{rc_k}{w_a} + c \), then the maximizer of the supplier’s profit in the boundary equilibrium is attainable, that is \( \min(\frac{rc_k}{w-c}, w_a^*) = w_a^* \). At this equilibrium, \( \Pi^s(y^b(\xi), y^b(\xi), \xi) \geq 0 \). Hence \( w_a^* \) is feasible.

To prove the convexity of supplier’s profit at the interior equilibrium, we substitute the optimal \( w_a = w + \frac{(r-w)c_k}{w-c} \) into \( \pi^s(w) \) and obtain

\[
\pi^s_i(w) = (w-c-c_k)\mu + (w-c)E[\min(\epsilon, G^{-1}(\frac{w-c-c_k}{w-c})] - c_kG^{-1}(\frac{w-c-c_k}{w-c}) + \frac{(r-w)c_k}{w-c} (\mu + \epsilon).
\]

Note that in the above function, \( V(w) = \max_y(w-c)E[\min(\epsilon, y)] - c_ky \). By the envelope theorem, we have \( \frac{dV}{dw} = E[\min(\epsilon, G^{-1}(w))] \). Therefore, \( \frac{d\pi^s_i(w)}{dw} = \mu + E[\min(\epsilon, G^{-1}(\frac{w-c-c_k}{w-c})] - \frac{(r-c)c_k}{w-c} (\mu + \epsilon). \)

Since \( G^{-1}(\cdot) \) is increasing in \( w \), \( \pi^s_i(w) \) is convex in \( w \). Hence, \( \pi^s_i(w) \) is maximized at either \( w = c + c_k \) or \( w = r \). When \( w = c + c_k \), we have \( \pi^s_i(c + c_k) = (r - c - c_k)(\mu + \epsilon) \). When \( w = r \), recall that \( E[K^{cs}] = \mu + G^{-1}(\frac{r-c-c_k}{w-c}) \), hence we have \( \pi^s_i(r) = E[\Pi^s(K^{cs}(\xi), \xi)] \), the optimal centralized supply chain profit, where \( \Pi^s(K, \xi) \) is defined in Equation (1). The manufacturer’s expected profit at the interior equilibrium is

\[
\Pi^m(y^i(\xi), K^{ap}(\xi), \xi) = (r-w) \left\{ \mu + \epsilon + E[\min(\epsilon, G^{-1}(\frac{w-c-c_k}{w-c})] - \frac{(r-w)c_k}{w-c} (\mu + \epsilon) \right\},
\]

which is equal to 0, when \( w = r \). Therefore with \( w_a = w = r \), the supplier maximizes his profit to be equal to the centralized supply chain profit.

Part 3 follows immediately from Parts 1 and 2.

2. Omitted Results:

**Lemma 4** The function \( E[\Pi^s(y^b(\xi), y^b(\xi), \xi)] = (w_a - c - c_k)(\mu + G^{-1}(\frac{r-w_a}{r})) \) is unimodal in \( w_a \)

**Proof of Lemma 4** The proof of this Lemma uses exactly the same arguments used for a different problem setting in Özer, Uncu and Wei (2003). For completeness, we include the proof here. We define \( v \equiv G^{-1}(\frac{r-w_a}{r}) \). Hence, \( w_a = r(1 - G(v)) \) and \( E[\Pi^s(y^b(\xi), y^b(\xi), \xi)] \) can equivalently be written as \( \Pi^m(v) = [r(1 - G(v)) - c - c_k] (\mu + v) \). Hence, \( \frac{d\Pi^m(v)}{dv} = r(1 - G(v)) \left( 1 - \frac{g(v)(\mu + v)}{1 - G(v)} \right) - c - c_k \). Next we will show that \( \Pi^m(v) \) is unimodal in \( v \). Once we show this result, since \( v \) is monotone in \( w_a \), the profit function is also unimodal in \( w_a \).
Next we show that $\Pi^0(v)$ is unimodal in $v$. Let $\tilde{v}_o$ be the supremum of the set of points such that $\frac{g(v)\epsilon}{1-G(v)} \leq 1$. First, we prove that $\tilde{v}_o$ is finite. Let $t \in (0, b)$ and define the truncated random variable $\epsilon_t$ on $[t, b)$, whose cdf is $G_t(v) = \frac{G(v)-G(t)}{1-G(t)}$ for all $v \in [t, b)$. Note that $\epsilon_t$ has a finite mean since $\epsilon$ has a finite mean. Note also that the failure rate of $\epsilon_t$ is equal to the failure rate of $\epsilon$ for all $v \in [t, b)$. Denote the failure rate of $\epsilon_t$ with $h_t(\cdot)$. Assume for a contradiction argument that $\tilde{v}_o = \infty$. Then, $h_t(v) \leq \frac{1}{\mu}$ for all $v \in [t, b)$. This implies that $\epsilon_t$ is stochastically larger than a random variable $\eta$ with a failure rate $h_\eta(v) = \frac{1}{\mu}$ for $v \geq t$ (Ross 1983). We have $1 - G_\eta(v) = \frac{v}{\mu}$ for all $v \geq t$ since $1 - G_\eta(v) = Exp[- \int_t^v h_\eta(v)dv]$ (Ross 1983). Then, $\eta$ has an infinite mean implying that $\epsilon_t$ has an infinite mean, too. This contradicts the fact that $\epsilon_t$ has finite mean. Therefore, $\tilde{v}_o$ must be finite. This result implies that $\tilde{v}$, the supremum of the set of points such that $\frac{g(v)(\mu+\epsilon)}{1-G(v)} \leq 1$, is also finite because $\mu \geq 0$. The second derivative of $\Pi^0_b(v)$ is
\[
\frac{d^2 \Pi^0_b(v)}{dv^2} = -rg(v) \left( 1 - g(v)(\mu+\epsilon) \right) - r(1-G(v)) \frac{d}{dv} \left( g(v)(\mu+\epsilon) \right).
\]
For $v \in (-\infty, a)$, we have $\frac{d \Pi^0_b(v)}{dv} = r-c-c_k > 0$ and hence $\Pi^0_b(v)$ is increasing. Since $G(\cdot)$ has IFR, we have $\frac{d}{dv} \left( \frac{g(v)(\mu+\epsilon)}{1-G(v)} \right) > 0$ for all $v \in [a, b)$. Then for $v \in [a, b]$, we have $\frac{d^2 \Pi^0_b(v)}{dv^2} < 0$ and hence $\Pi^0_b(v)$ is strictly concave. For $v \in (\tilde{v}, \infty)$, we have $\frac{d \Pi^0_b(v)}{dv} < 0$ and hence $\Pi^0_b(v)$ is decreasing. Therefore, $\Pi^0_b(v)$ is unimodal and its maximizer lies on $[a, \tilde{v}]$.

**Pooling Equilibrium:** We return to the asymmetric information case and provide a pooling equilibrium example. In a pooling equilibrium, the manufacturer orders the same advance purchase quantity $y$ regardless of her private forecast information $\xi$. Hence the supplier cannot update his belief and he secures component capacity according to his prior belief about $\xi$. Note that several pooling equilibria may exist in this game as in many other signaling games (Fudenberg and Tirole [17]).

**Theorem 10** For $w_o \geq w$, there exists a continuum of pooling equilibrium with advance purchase quantity $y \in [0, y^\ast]$ where $y^\ast$ is defined in the proof. If the advance purchase is $y \not\in [0, y^\ast]$, the supplier updates his prior belief to $\xi$ and secures $K \equiv K^{ws}(\xi)$ units of component capacity. Otherwise, he cannot update his prior belief.

**Proof of Theorem 10.** In a pooling equilibrium, the supplier cannot update his belief hence, he determines the component capacity based on his prior belief about $\xi$. Given advance purchase $z$ and capacity $K$, the supplier’s expected profit is $E_\xi \Pi^*(z, K, \xi)$ where $\Pi^*(z, K, \xi)$ is defined in Equation (17). A similar proof as in Lemma 2 shows that to maximize his profit, the supplier secures $\max(K^{wa}, z)$ units of component capacity, where $K^{wa}$ is the same component capacity that the supplier would have built under the wholesale price contract. The best off-the-equilibrium belief to support such a pooling equilibrium is that the supplier believes the manufacturer’s forecast information is $\xi$ and builds capacity $K$ if he observes any other advance purchase quantity $y \neq z$.

If the manufacturer with forecast information $\xi$ were to deviate from the above pooling equilibrium, the best deviation can be determined as follows. Suppose the manufacturer choose advance
purchase quantity $y \neq z$. Her profit from deviation is maximized by $\max_y r E m\in(D(\xi), K) - w E (\min(D(\xi), K) - y)^+ - w_a y$. Note that when $w_a \geq w$, the optimal deviation is $y = 0$. The optimal profit from deviation is $\tilde{\Pi}^m(\xi) = (r - w) E m\in(D(\xi), K)$. The pooling equilibrium profit for the manufacturer with $\xi$ is $\Pi^m(\xi, y) = r E m\in(D(\xi), K^{wa}) - w E (\min(D(\xi), K^{wa}) - y)^+ - w_a y$. Note that $\tilde{\Pi}^m(\xi)$ is independent of $y$; $\Pi^m(\xi, y)$ is decreasing in $y$; and $\Pi^m(\xi, 0) > \tilde{\Pi}^m(\xi)$. Hence, there exists a unique $y(\xi) > 0$ such that $\tilde{\Pi}^m(\xi) = \Pi^m(\xi, y(\xi))$. For $y \leq y(\xi)$, the manufacturer of type $\xi$ would not deviate. For $y \in [0, y^*]$, $y^* \equiv \min_{\xi \leq \xi \leq \xi} y(\xi)$, no manufacturer of any forecast information $\xi$ would deviate.

Next we show that $y^* = y(\xi)$ in two steps. First, note that $\Pi^m(\xi, 0) - \tilde{\Pi}^m(\xi) = (r - w)(E m\in(\mu + \xi + \epsilon, K^{wa}) - E m\in(\mu + \xi + \epsilon, K))$ is increasing in $\xi$ since $\frac{\partial^2}{\partial^2 K} E m\in(\mu + \xi + \epsilon, K) = g(K - \mu - \xi) > 0$ and $K^{wa} > K$. Second, note that $\frac{\partial^2 \Pi^m(\xi, y)}{\partial^2 y} = wg(y - \mu - \xi) > 0$ while $\frac{\partial \Pi^m(\xi, y)}{\partial y} = w(1 - G(y - \mu - \xi)) - w_a < 0$. Therefore $\Pi^m(\xi, y)$ decreases in $y$ and the rate of decrease also decreasing in $y$. Hence $y(\xi)$ is increasing in $\xi$ and $y^* = y(\xi)$.

References