Suppose that measurement $y(n)$, is an M-Point sequence as:

$$y(n) = x(n) + v(n), \qquad n = 0, 1, 2, \dots, (M-1) \tag{1}$$

Where $x(n)$ is periodic with fundamental period of N samples and $v(n)$ is the zero mean, uncorrelated noise.

M-point periodic extension of $y(n)$ is:

$$y_p(n) = \sum_{l=-\infty}^{\infty} y(n - lM) \tag{2}$$

We generate a periodic impulse train as:

$$\delta_M(n) = \sum_{q=0}^{L-1} \delta(n - qN) \tag{3}$$

Where $L = \left\lfloor \frac{M}{N} \right\rfloor$ is the number of complete cycles of $x(n)$ in the measurement record.

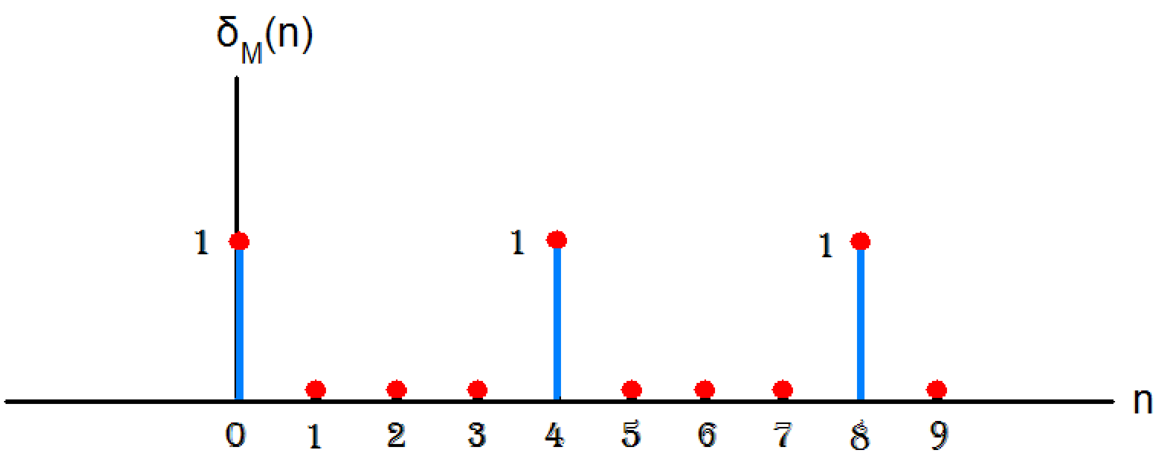

Figure (1). Illustration of $\delta_M(n)$ with $M = 10, N = 4$.

The M-Point periodic extension of $\delta_M(n)$ is given by:

$$\delta_p(n) = \sum_{l=-\infty}^{\infty} \delta_M(n - lM) \tag{4}$$

Circular/Periodic cross correlation of $y_p(n)$ and $\delta_p(n)$ is:

$$r_{y\delta}(l) = \frac{1}{M}\sum_{n=0}^{M-1} y(n)\cdot\delta_M(n-l) \tag{5}$$

Where $y(n)$ corresponds to one period of $y_p(n)$ and $\delta_M(n)$ corresponds to one period of $\delta_p(n)$. For both sequences the fundamental period is M samples.

$$\begin{aligned}
r_{y\delta}(l) &= \frac{1}{M}\sum_{n=0}^{M-1} y(n)\cdot\delta_M(n-l) \\
&= \frac{1}{M}\sum_{n=0}^{M-1}\left[y(n)\cdot\sum_{q=0}^{L-1}\delta(n-(qN+l))\right] \\
&= \frac{1}{M}\sum_{q=0}^{L-1}\sum_{n=0}^{M-1} y(n)\cdot\delta(n-(qN+l)) \\
&= \frac{1}{M}\sum_{q=0}^{L-1} y(qN+l) \\
&= \frac{1}{M}\sum_{q=0}^{L-1}[x(qN+l)+\ v(qN+l)] \\
&= \frac{L}{M}x(l)+\frac{1}{M}\sum_{q=0}^{L-1} v(qN+l) \\
&= \frac{L}{M}x(l)+\varepsilon(l)
\end{aligned} \tag{6}$$

Where $x(l)$ is the periodic signal and $\varepsilon(l)$ is the averaged noise.

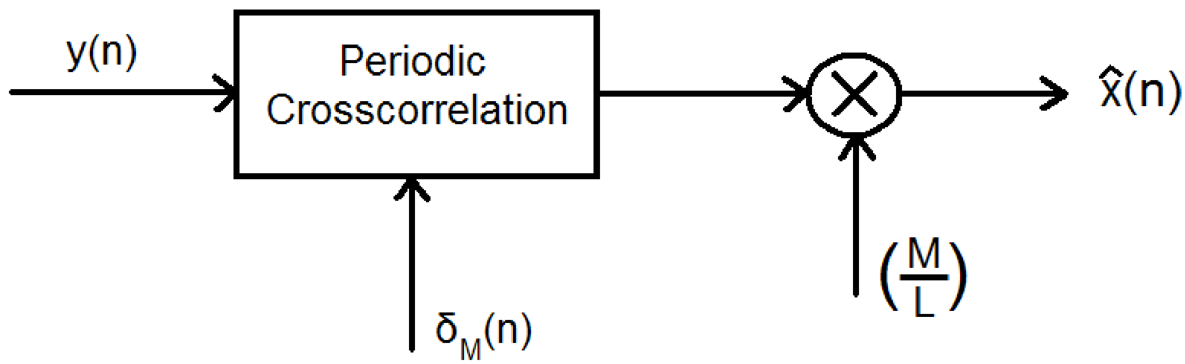

Figure (2). Generating $\hat{x}(n)$ from measurement $y(n)$.

$\hat{x}(n)$ is the estimate of $x(n)$ and

$$\hat{x}(n) = x(n) + \frac{1}{L}\sum_{q=0}^{L-1} v(qN+n) \tag{7}$$

Note that our estimate $\hat{x}(n)$ will be "more accurate" as L increases, i.e. the measurement record contains increasing number of complete cycles of the periodic signal.