4/4/06

DSP Spring 06

Chapter 4 Summary

(notes as taken by Cale Dingman)

Thanks, Cale!

3/30 Announcements

* Linear Phase Sequence - Notes online [ch 8.3.1]
* Exam 2 - 4/11 ⇒ Portions thru next Tuesday

Ch4

- Discrete Fourier Series
  - Computable [Finite Sums]
- ~~DTFS~~ DTFT

$\}$ ⇒ Fourier Analysis

LTI - Fourier Analysis

(Relaxed)

$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$

- sinusoid,
- periodic, etc
- suddenly applied sinusoid $\longrightarrow$ $y(n) = y_{ss}(n) + y_{ts}(n)$ [Assuming stable]

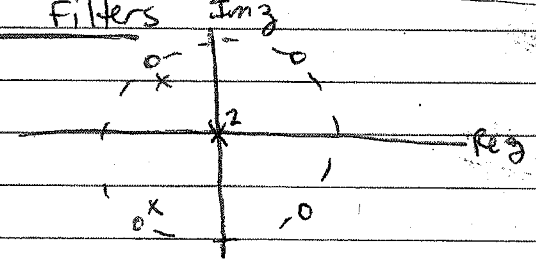
?

$\}$ All involve $H(\omega)$

$H(\omega) = H(z)\Big|_{z=e^{j\omega}}$ provided the ROC for $H(z)$ contains the unit circle

* Effects of poles/zeros on Frequency Response
  - geometrical interpretation
    - zeros "attenuate" neighboring freq.
    - poles "accentuate" " "

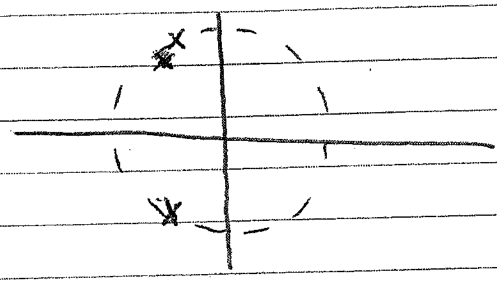
* Pole-zero Placement Design of Filters
  - Notch Filters



$H_{FIR}(z) = G\left[1 - (2\cos\omega_0)z^{-1} + z^{-2}\right]$

G ← choose to set $|H(\omega_1)| = 1$ for some ω_1

- Oscillator



→ Oscill. amp may depend on freq.
→ we made it constant by appropriately scaling the impulse input

- Comb Filter (Periodic Filters)

$$H(z) \longrightarrow H(\omega)$$
$$H_N(z) = H(z^N) \longrightarrow H(N\omega)$$

- Periodic Notches
  Periodic Peaking Filters

$$\boxed{1 - z^{-N} \quad \& \quad 1 + z^{-N}}$$

- All Pass Filters

→ $|H(\omega)| = 1$ for all ω
- used mainly for phase compensation

$$H_{ap}(z) = z^{-N} \frac{A(z^{-1})}{A(z)}$$

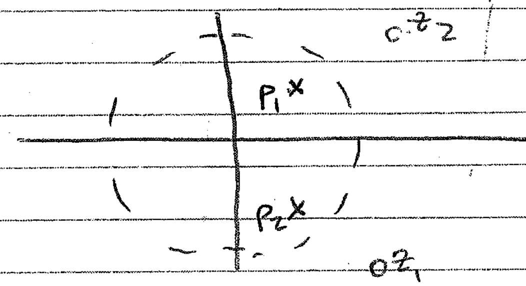
Ex: $H_{ap}(z) = \dfrac{\left[r^2 - (2r\cos\omega_0)z^{-1} + z^{-2}\right]}{1 - (2r\cos\omega_0)z^{-1} + r^2 z^{-2}}$ ← reverse of den(

← resonator

$= r^2 \dfrac{\left[1 - \left(\frac{2}{r}\cos\omega_0\right)z^{-1} + \frac{1}{r^2}z^{-2}\right]}{\underbrace{\left[1 - (2r\cos\omega_0)z^{-1} + r^2 z^{-2}\right]}_{}}$

gain factor!

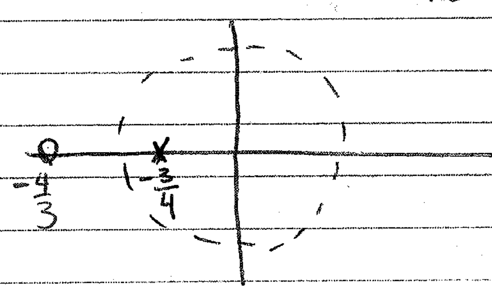
$G|H(\omega_0)| = \frac{1}{r^2}$



$p_1 = re^{j\omega_0} \Rightarrow z_1 = \frac{1}{p_1} = \frac{1}{r}e^{-j\omega_0}$

$p_2 = re^{-j\omega_0} \rightarrow z_2 = \frac{1}{p_2} = \frac{1}{r}e^{j\omega_0}$

Ex:



If you have real poles, its inverse zero is on the real axis

Write Denom → then write Num for All pass, Num. is opp Den

$H(z) = \dfrac{\left(\frac{3}{4} + z^{-1}\right)}{\left(1 + \frac{3}{4}z^{-1}\right)}$

- **Min Phase Sys**

  Causal, stable system with zeros inside (<u>not on</u>) the unit circle.

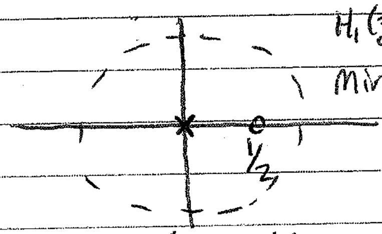
  $\Rightarrow$ Poles & zeros are <u>inside</u> the unit circle

* Min. group delay

* " phase deviation from zero phase

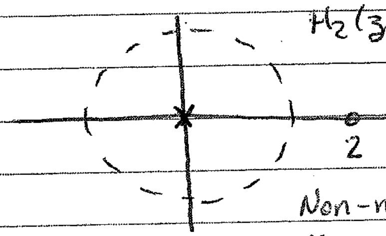
* Quickest energy delivery

Ex:



$H_1(z) = (1 - \frac{1}{2}z^{-1})$

Min. Phase



$H_2(z) = (z^{-1} - \frac{1}{2})$
$= (-\frac{1}{2} + z^{-1})$

Non-min Phase & Max phase

$h_1(n) = \{ \underset{\uparrow}{1}, -\frac{1}{2} \}$

$h_2(n) = \{ -\frac{1}{2}, 1 \}$

Reflect the zero about the unit circle
$\Rightarrow$ gives same $|H_1(\omega)| = |H_2(\omega)|$

$$E_{1N} = \sum_{n=0}^{N} |h_1(n)|^2 \qquad E_{2N} = \sum_{n=0}^{N} |h_2(n)|^2$$

$$\boxed{E_{1N} \geq E_{2N} \text{ for Min Phase}}$$

* Non-minimum Phase Sys. (causal & stable) can be decomposed into a cascade of min. phase and all pass systems

$$\boxed{H(z) = H_{ap}(z)\, H_{min}(z)}$$

* Watch out to make ~~H_{ap}~~ $|H_{ap}(\omega)| = 1$ and associate proper gains with $H_{ap}(z)$ and $H_{min}(z)$

Inverse systems

$H(z) \cdot H_I(z) = 1$, provided ROC_H & ROC_{H_I} intersection is non-null.

The inverse of a min. phase sys will always be min. phase too

Mag - Compensation

$$x(n) \rightarrow \boxed{H(z)} \xrightarrow{y(n)} \boxed{\frac{1}{H_{min}(z)}} \rightarrow y_1(n)$$

$$|Y_1(\omega)| = |X(\omega)|$$

$$\theta_{Y_1}(\omega) = \theta_X(\omega) + \theta_{H_{ap}}(\omega)$$

Verify:

$$x(n) \rightarrow \boxed{H(z)} \rightarrow y(n)$$

$$x(n) \rightarrow \underbrace{\boxed{H_{ap}(z)} \rightarrow \boxed{H_{min}(z)}}_{H(z)} \xrightarrow{y(n)} \boxed{\frac{1}{H_{min}(z)}} \rightarrow y_1(n)$$

$$x(n) \rightarrow \boxed{H_{ap}(z)} \rightarrow y_1(n)$$

Linear Phase Sequences

$$H(z) \longrightarrow H(\omega) = \underbrace{|H(\omega)|}_{\text{mag}} e^{j\overbrace{\theta_H(\omega)}^{\text{phase}}}$$

Linear phase $\Rightarrow \theta_H(\omega)$ is linear in ω

$\theta_H(\omega) = -\omega\alpha$, for α const.

Group Delay: $\tau(\omega) = -\dfrac{d\,\theta_H(\omega)}{d\omega} = \alpha \Rightarrow$ constant group delay

Generalized Linear Phase

$\Rightarrow \tau_H(\omega) = $ constant

$$H(\omega) = \underbrace{A(\omega)}_{\text{Ampl. spectrum}} e^{-j(\omega\alpha - \beta)}$$

- Real
- Bipolar ($\pm$)

$$\theta_H(\omega) = \begin{cases} -\omega\underbrace{\alpha + \beta}_{\text{const.}}, & A(\omega) > 0 \\ -\omega\alpha + \beta + \pi, & A(\omega) < 0 \end{cases}$$

$-1 = e^{j\pi}$

* essentially the same as linear phase, but more generalized

* ~~Most~~ Some FIR filters have linear phase

* Never have IIR with " "

$\boxed{\text{Causal: only some FIR have (generalized) linear phase}}$