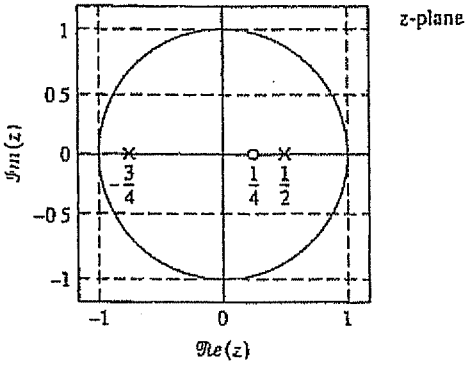
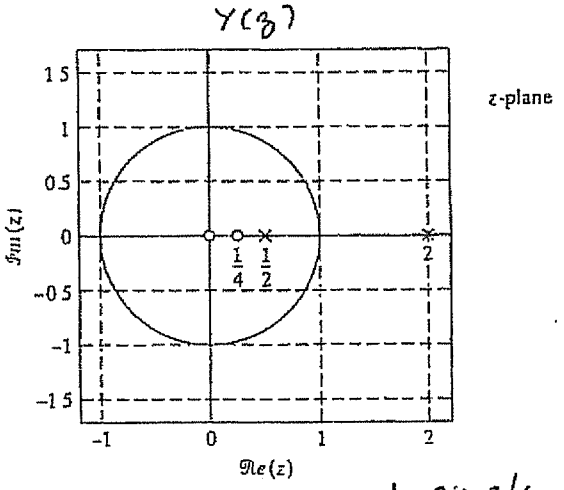
2. The signal y(n) is the output of an LTI system with impulse response h(n) for a given stable, input sequence x(n). Throughout the problem, assume that y(n) is stable. The pole-zero configurations of X(z) and Y(z) are shown below. (a) What is the ROC of Y(z)? (b) Is y(n) right-sided, left-sided or two-sided? (c) What is the ROC of X(z)? (d) Is x(n) a causal sequence? (e) Draw the pole-zero plot of H(z) and specify its ROC. (f) Is h(n) causal, anti-causal or two-sided?

X(z)



Y(z)



Stability implies that ROC includes unit circle.

(a) ROC of Y(z): $\frac{1}{2} < |z| < 2$

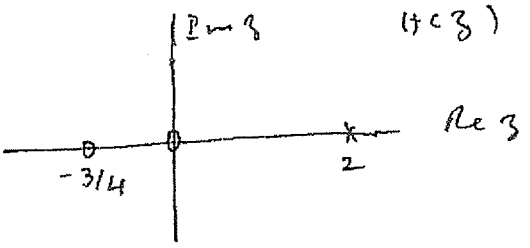
(b) y(n) is two-sided.

(c) ROC of X(z): $|z| > \frac{3}{4}$

(d) x(n) is causal.

(e) $H(z) = \frac{Y(z)}{X(z)} = \left[\frac{(1-\frac{1}{4}z^{-1})z}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}\right]\left[\frac{(1+\frac{3}{4}z^{-1})(1-\frac{1}{2}z^{-1})}{(1-\frac{1}{4}z^{-1})}\right]$

$= \frac{(1+\frac{3}{4}z^{-1})z}{(1-2z^{-1})}$



Valid ROC: $|z| < 2$

(f) h(n) is anti causal