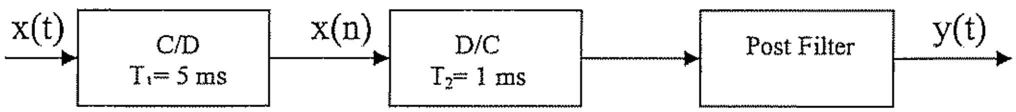
DSP EXAM 1 — 2004

1. (20 points)
Consider the simple signal processing system shown in the figure below. The sampling periods of the C/D (continuous-to-digital) and D/C converters are $T_1 = 5$ ms and $T_2 = 1$ ms, respectively. Determine the output y(t) of the system, if the input is given by

$$x(t) = 3\cos 100\pi t + 2\sin 250\pi t$$

The post-filter is an ideal low pass filter with a cutoff frequency of 500 Hz.



2. **(20 points)**

The input $x(n) = \{1, 2, -1, \gamma\}$ to a LTI system with an impulse response $\{1, \alpha, \beta, 2\}$ yields an output $y(n) = \{1, 4, 2, 0, 9, -4, 4\}$. What is the output $y_1(n)$ of the LTI system due to an input $x_1(n) = \{1, \underset{\wedge}{0}, -1\}$?

3. (20 points)

The following LCCDE describes the input-output relationship of a causal, LTI system; determine the impulse response, h (n) of the LTI system.

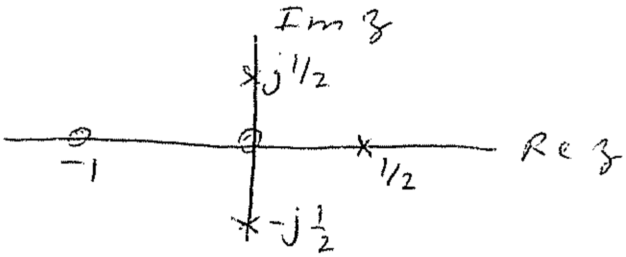
$$y(n) - \frac{1}{2} y(n-1) + \frac{1}{4} y(n-2) = x(n) - \frac{1}{4} x(n-1)$$

(Note: Your answer for h(n) should contain only real coefficients to receive credit.)

4. **(20 points)**
When the input to an LTI system is $x(n) = (1/3)^n u(n) + (2)^n u(-n-1)$, the corresponding output is given by $y(n) = 5 (1/3)^n u(n) - 5 (2/3)^n u(n)$. (a) Find the system transfer function H (z), and indicate the region of convergence, (b) Sketch the Direct Form II (canonical or minimum delays) implementation of the system, and (c) Is the system stable? Is it causal?

5. **(20 points)**

The z-transform of the sequence x (n) is denoted by X (z). The pole-zero plot of X (z) is shown in the figure below. There are 3 finite poles (z=1/2, +/-1/2 j), and 2 finite zeros (z=0, -1). Sketch the pole-zero plots of (a) $X_1(z)$, where $x_1(n) = x(1-n)$, and (b) $X_2(z)$, where $x_2(n) = (1/2)^n x(n)$. Gain factor is unity.



6. (20 points)
Determine the impulse response h(n) of the causal LTI system implemented as shown below, **without using z-transforms.**